Cooperative Spectrum Sensing using Improved *p*-norm Detector in Generalized *κ*-*μ* Fading Channel

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Abstract. The classical energy detection (CED) system is a well-known technique for spectrum sensing in cognitive radio. Generalized p-norm detector for spectrum sensing in additive white Gaussian noise (AWGN) has been shown to provide improved performance over CED under certain conditions. Further, improved algorithm exists which works better than the classical energy detection algorithm. The present paper takes into account the combined benefit of the p-norm energy detector and the improved algorithm for spectrum sensing for individual cognitive user in a cooperative spectrum sensing system to achieve a significant performance gain in both AWGN and generalized κ-μ fading channels over the cooperative/ non-cooperative CED scheme.

Keywords: p-norm energy detector · Energy detection · Cooperative spectrum sensing · Cognitive radio · κ-μ fading channel

1 Introduction

Cognitive radio (CR) is considered as a promising solution to the radio spectrum under-utilization. Spectrum sensing is the key technology that enables the secondary users (SUs) to access the licensed frequency bands without affecting the quality-of-service (QoS) of the primary users (PUs). Various spectrum sensing techniques have been suggested [1,2], which include Energy detector, Matched filtering, Cyclostationary detection etc. Among all these techniques, the classical energy detector (CED) is the most popular because of its low implementation cost and less complexity. However, the performance of the energy detector is limited by high susceptibility of the detection threshold to noise uncertainty and interference level. An improved energy detector (IED) has been proposed [3], which outperforms the CED in AWGN channel with almost same algorithmic complexity without the need for a-priori information about the PU's signal format.

Another interesting improvement strategy for energy detection based on p-norm detector was first proposed by Chen [4], in which the classical energy

M. Weichold et al. (Eds.): CROWNCOM 2015, LNICST 156, pp. 225–234, 2015.
DOI: 10.1007/978-3-319-24540-9_18

detector was modified by replacing the squaring operation of the signal amplitude by arbitrary positive power p. The optimal p value depends on system parameter settings viz. the probability of false alarm, the average signal-to noise ratio, and the sample size in order to achieve a higher probability of detection. The application of p-norm detector for spectrum sensing in fading channel and diversity reception has been well investigated recently [5]. The performance of p-norm detector for cooperative spectrum sensing has been carried out in [6], where an optimized value of p and sensing threshold of each CR is obtained by minimizing the total probability of error.

In the present work, we endeavor to evaluate the maximum achievable performance gain in a cooperative sensing system where each individual secondary user utilizes the combined benefit of both the optimized p-norm detector and the IED algorithm for spectrum sensing in generalized κ-μ fading channel. It is difficult to obtain analytically the optimized p-value for a given target performance criterion and therefore a numerical evaluation is adopted.

The organization of the paper is as follows: Section 2 provides the mathematical details of the classical energy detector, improved energy detector, p-norm energy detector and the improved p-norm energy detector with the derivation of the performance parameters. The performance of the improved p-norm detector in a generalized κ-μ fading channel is presented in section 3. Section 4 deals with the mathematical details of the cooperative spectrum sensing. Section 5 provides the detailed theoretical results of the improved p-norm energy detector as well as the practical design guidelines. Finally the conclusion is drawn in section 6.

2 Spectrum Sensing

The spectrum sensing may be modeled as a binary hypothesis testing problem as:

$$\begin{aligned} H_0 &: y[n] = w[n] \\ H_1 &: y[n] = h[n].s[n] + w[n] \end{aligned} \tag{1}$$

where $y[n]$ is the signal sample detected by the secondary user, $s[n]$ is the signal transmitted by the PU, $h[n]$ represents the channel fading coefficient, and $w[n]$ is a zero-mean additive white Gaussian noise (AWGN) with variance σ_w^2.

The hypotheses H_0 and H_1 correspond to the binary space, representing the absence and the presence of the PU respectively. In order to analyze the performance of the sensing scheme, the probability of false alarm, P_{fa}, and the probability of detection, P_d need to be evaluated. The parameters are defined as follows:

$$\begin{aligned} P_{fa} &= P\left(H_1 | H_0\right) \\ P_d &= P\left(H_1 | H_1\right) \end{aligned} \tag{2}$$

where $P(\cdot|\cdot)$ denotes the conditional probability. The expression for these probabilities are obtained in the next section.

2.1 Classical Energy Detector (CED)

In CED, if the received signal energy during a sensing event exceeds the predetermined threshold, the channel is considered as busy (H_1 is true), otherwise, the channel is idle (H_0 is true). The decision variable $T_i(y_i)$ at the i^{th} sensing event can be represented as:

$$T_i(y_i) = \frac{1}{N} \sum_{n=1}^{N} \left| \frac{y_i(n)}{\sigma_w} \right|^2 \tag{3}$$

where N is the number of samples per sensing event, $y_i(n)$ is the n^{th} received faded sample at the i^{th} sensing event and σ_w is the standard deviation of the additive white Gaussian noise. The decision rule can be adopted as:

$$\begin{aligned} H_0 : \quad & T_i(y_i) < \lambda \\ H_1 : \quad & T_i(y_i) \geq \lambda \end{aligned} \tag{4}$$

where λ is the decision threshold. For the number of samples $N \gg 1$, the decision variable can be well approximated as a Gaussian distribution [3], i.e.,

$$T_i(y_i) = \begin{cases} \mathcal{N}\left(1, \frac{2}{N}\right) & : H_0 \\ \mathcal{N}\left((1+\gamma), \frac{2}{N}(1+\gamma)^2\right) & : H_1 \end{cases} \tag{5}$$

where $\gamma = \frac{\sigma_s^2}{\sigma_w^2}$ is the signal-to-noise ratio (SNR) of the received signal, σ_s^2 being the signal power. For the AWGN channel, P_{fa}^{CED} and P_d^{CED} can be expressed as [3]:

$$P_{fa}^{CED} = Q\left(\frac{\lambda - 1}{\sqrt{2/N}}\right) \tag{6}$$

$$P_d^{CED} = Q\left(\frac{\lambda - (1+\gamma)}{\sqrt{(2/N)(1+\gamma)^2}}\right) \tag{7}$$

where, $Q(x) = \int_x^{\infty} e^{-t^2} dt$ represents the Gaussian tail probability. From (6), the expression for λ directly follows:

$$\lambda = \sqrt{2/N} Q^{-1}\left(P_{fa}^{CED}\right) + 1 \tag{8}$$

2.2 Improved Energy Detector (IED)

The improved energy detector (IED), proposed in [3], is a modified version of CED, that provides better detection results without much additional complexity.

In IED, the decision for the presence of the primary user is done based on the average of last L test statistics T_i^{avg} at the i^{th} interval, which is defined as:

$$T_i^{avg}(T_i) = \frac{1}{L}\sum_{l=1}^{L} T_{i-L+l}(y_{i-L+l}) \tag{9}$$

Out of these last L sensing events, $M \in [0, L]$ is the total number of events in which the primary signal was actually present. In IED algorithm, two additional checks are imposed to improve the detection probability as well as the probability of false alarm.

If $T_i(y_i) < \lambda$, the first additional check for $T_i^{avg}(T_i) > \lambda$ would improve the detection probability and the second additional check for $T_{i-1}(y_{i-1}) > \lambda$ would prevent the consequential false alarm degradation. Since $T_i^{avg}(T_i)$ is the average of independent and identically distributed Gaussian random variables, it is also normally distributed as:

$$T_i^{avg}(T_i) \sim \mathcal{N}(\mu_{avg}, \sigma_{avg}^2) \tag{10}$$

where, μ_{avg} and σ_{avg}^2 are obtained as [3]:

$$\begin{aligned} \mu_{avg} &= \frac{M}{L}(1+\gamma) + \frac{L-M}{L} \\ \sigma_{avg}^2 &= \frac{M}{L^2}\left(\frac{2}{N}(1+\gamma)^2\right) + \frac{L-M}{L^2}\left(\frac{2}{N}\right) \end{aligned} \tag{11}$$

Based on the above assumption, the probability of false alarm, P_{fa}^{IED} and the probability of detection, P_d^{IED} can easily be derived as:

$$\begin{aligned} P_{fa}^{IED} &= P_{fa}^{CED} + P_{fa}^{CED}(1 - P_{fa}^{CED})Q\left(\frac{\lambda_{IED} - \mu_{avg}}{\sigma_{avg}}\right) \\ P_d^{IED} &= P_d^{CED} + P_d^{CED}(1 - P_d^{CED})Q\left(\frac{\lambda_{IED} - \mu_{avg}}{\sigma_{avg}}\right) \end{aligned} \tag{12}$$

where λ_{IED} is the detection threshold in case of IED algorithm, that depends on the probability of false alarm, M, L and γ.

2.3 *p*-norm Energy Detector

The decision variable for the p-norm detector, $T_i^p(y_i)$ is obtained by modifying (3) as:

$$T_i^p(y_i) = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{y_i(n)}{\sigma_w}\right|^p \tag{13}$$

It may be noted that $p = 2$ in (13) leads to the CED case. The decision statistics may again be well approximated by Gaussian distribution for $N \gg 1$ as follows:

$$T_i^p(y_i) = \begin{cases} \mathcal{N}\left(\mu_{0,p}, \sigma_{0,p}^2\right) & : H_0 \\ \mathcal{N}\left(\mu_{1,p}, \sigma_{1,p}^2\right) & : H_1 \end{cases} \tag{14}$$

where $\mu_{0,p}$ and $\mu_{1,p}$ are the means and $\sigma_{0,p}^2$ and $\sigma_{1,p}^2$ are the variances of the decision variable under the hypotheses H_0 and H_1 respectively. The above parameters are defined as follows [4]:

$$\mu_{0,p} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \tag{15}$$

$$\mu_{1,p} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \left(\sqrt{1+\gamma}\right)^p \tag{16}$$

$$\sigma_{0,p}^2 = \frac{2^p \Gamma\left(\frac{2p+1}{2}\right)}{N\sqrt{\pi}} - \frac{2^p}{N\pi}\left\{\Gamma\left(\frac{p+1}{2}\right)\right\}^2 \tag{17}$$

$$\sigma_{1,p}^2 = \left[\frac{2^p \Gamma\left(\frac{2p+1}{2}\right)}{N\sqrt{\pi}} - \frac{2^p}{N\pi}\left\{\Gamma\left(\frac{p+1}{2}\right)\right\}^2\right](1+\gamma)^p \tag{18}$$

The probability of false alarm, P_{fa}^p and the probability of detection, P_d^p can be calculated as:

$$\begin{aligned} P_{fa}^p &= Q\left(\frac{\lambda_p - \mu_{0,p}}{\sigma_{0,p}}\right) \\ P_d^p &= Q\left(\frac{\lambda_p - \mu_{1,p}}{\sigma_{1,p}}\right) \end{aligned} \tag{19}$$

where λ_p is the detection threshold in case of p-norm detector that depends on the probability of false alarm, $\mu_{0,p}$ and $\sigma_{0,p}$.

2.4 Improved *p*-norm Energy Detector

By replacing the squaring operation of the signal amplitude in IED by an arbitrary positive power p, $T_i^{avg}(T_i^p)$ may be well approximated by Gaussian distribution as:

$$T_i^{avg}(T_i^p) = \mathcal{N}\left(\mu_{avg,p}, \sigma_{avg,p}^2\right) \tag{20}$$

where $\mu_{avg,p}$ and $\sigma_{avg,p}^2$ being the mean and the variance of the decision variable $T_i^{avg}(T_i^p)$ defined as follows [3]:

$$\begin{aligned} \mu_{avg,p} &= \frac{M}{L}\mu_{1,p} + \frac{L-M}{L}\mu_{0,p} \\ \sigma_{avg,p}^2 &= \frac{M}{L^2}\sigma_{1,p}^2 + \frac{L-M}{L^2}\sigma_{0,p}^2 \end{aligned} \tag{21}$$

Modifying (12) to the present case, one obtains the probability of false alarm, $P_{fa}^{IED,p}$ and the probability of detection, $P_{d}^{IED,p}$ in the following form:

$$P_{fa}^{IED,p} = P_{fa}^{p} + P_{fa}^{p}\left(1 - P_{fa}^{p}\right) Q\left(\frac{\lambda_{IED,p} - \mu_{avg,p}}{\sigma_{avg,p}}\right)$$
$$P_{d}^{IED,p} = P_{d}^{p} + P_{d}^{p}\left(1 - P_{d}^{p}\right) Q\left(\frac{\lambda_{IED,p} - \mu_{avg,p}}{\sigma_{avg,p}}\right) \tag{22}$$

where $\lambda_{IED,p}$ is the detection threshold for improved p-norm energy detector, which depends on the probability of false alarm

3 Spectrum Sensing over Generalized κ-μ Fading Channel

In case of fading channels, where the channel coefficient $h[n]$ varies, the probability of detection $P_{d}^{IED,p}$ in (22) gives a conditional probability for a given instantaneous signal-to-noise ratio, γ. To find the detection probability, this conditional probability should be averaged over the probability density function (pdf) of SNR i.e., $f(\gamma)$ [7]:

$$P_{d_f}^{IED,p} = \int_0^{\infty} P_{d}^{IED,p}(\gamma) f(\gamma) d\gamma \tag{23}$$

Here, $P_{d_f}^{IED,p}$ represents the detection probability over the fading channel using the improved p-norm energy detection scheme. The integral in (23) is computed using MATLAB. In the following, the κ-μ generalized fading model [8], is described for computational purpose while evaluating (23).

For the κ-μ fading channel, the pdf of SNR is given as [8]:

$$f_{\kappa-\mu}(\gamma) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp[\kappa\mu]\sqrt{\gamma\bar{\gamma}}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\mu}{2}} \times$$
$$\exp\left[-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right] I_{\mu-1}\left[2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right] \tag{24}$$

where, $I_v(.)$ is the modified Bessel function of the first kind of order v. In this distribution, $\kappa(> 0)$ represents the ratio between the total power in the dominant component and the total power in the scatter waves; $\mu(> 0)$ is related to the multipath clustering and $\bar{\gamma}$ is the average SNR. Table 1 provides the values of κ and μ, for which the κ-μ distribution converges to some well-known wireless channel distributions.

In Table 1, m is the Nakagami shape parameter and K is the Rician-K parameter.

Table 1. Values of κ and μ for different known distributions

Type of distribution	κ	μ
Nakagami-m	$\to 0$	m
Rayleigh	$\to 0$	1
Rician	K	1
One sided Gaussian	$\to 0$	0.5

4 Cooperative Spectrum Sensing

In cooperative spectrum sensing there are multiple secondary users (SU), each SU sends its autonomous decision to a fusion center (FC) and the final decision about the presence of primary user (PU) is done at FC. In this work we have assumed that all the SUs behave identically (regarding SNR and threshold). Furthermore, we focus on the use of hard-decision based fusion rules e.g. OR, MAJORITY, and AND rules in the analysis. Since the binary decisions (H_0 or H_1) of all SUs are independent, the probability of detection in a cooperative scenario can be represented by [1]:

$$P_d^{coop} = \sum_{j=n}^{U} \binom{U}{j} \left(P_{d,i}^{x}\right)^{j} \left(1 - P_{d,i}^{x}\right)^{U-j} \tag{25}$$

where $P_{d,i}^{x}$ is the probability of detection for i^{th} individual node and U is the total number of SUs. In case of AWGN channel, $P_{d,i}^{x} = P_{d,i}^{IED,p}$ and for fading channel $P_{d,i}^{x} = P_{d_f,i}^{IED,p}$. Considering the optimal fusion rule i.e., OR [1], the probability of detection in a co-operative scenario can be evaluated by putting $n = 1$. The expression for P_d^{IED} under OR fusion rule, therefore follows in a straightforward manner as:

$$P_{d,i,OR}^{x} = 1 - \left(1 - P_{d,i}^{x}\right)^{U} \tag{26}$$

The performance results of both, conventional and co-operative spectrum sensing in AWGN and fading channels are presented in the following section.

5 Results and Discussion

In this section, the results for the combined benefit of the improved algorithm as well as the p-norm detector are highlighted for cooperative spectrum sensing in κ-μ fading channel. For a given target false alarm probability, the threshold, λ is chosen for individual sensor node and for a given SNR, γ the optimal value of p is determined which yields the highest value of the probability of detection. The hard decision from the individual sensor node is sent to the FC, which combines the individual decisions using the OR rule.

To provide practical design guidelines for a spectrum sensing system with improved p-norm energy detector, Fig. 1 provides the surface plots for the probability of detection with the variation of p and the probability of false alarm for AWGN and κ-μ fading channel scenarios respectively for cooperative spectrum sensing, each for N = 100 and $\overline{\gamma}$ = - 5 dB. It is quite evident that p has a definitive role in order to achieve a higher detection probability in both AWGN and fading channel.

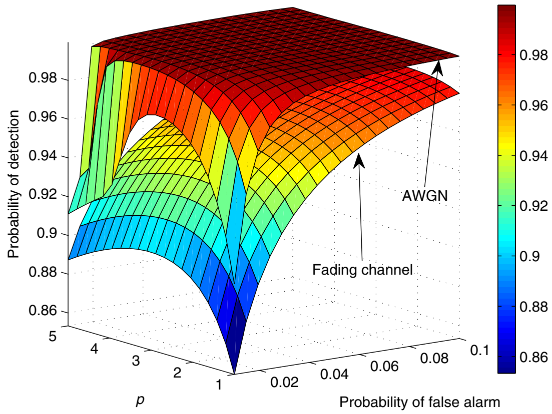

Fig. 1. Surface plot for the probability of detection as a function of p and probability of false alarm for IED over AWGN and κ-μ ($\kappa \rightarrow 0$, $\mu = 1$) fading for cooperative spectrum sensing with $N = 100$, $\overline{\gamma}$ = -5 dB, $U = 4$.

In Fig. 2, a comparison of the receiver operating characteristics (ROCs) for CED and improved p-norm algorithm i.e., IED with optimal p value has been depicted for cooperative as well as non-cooperative spectrum sensing in the κ-μ channel ($\kappa \rightarrow 0$, $\mu = 1$). The optimal p-value is determined from the surface plot in Fig. 1, such that for a given $P_{fa,target}$ and $\overline{\gamma}$, the probability of detection becomes maximum. It is clearly evident that the IED with optimal p, outperforms the CED, in fading scenarios for both cooperative and non-cooperative spectrum sensing. As the probability of false alarm increases, the difference in the performance gain of IED with optimal p decreases for cooperative spectrum sensing, but the algorithm still retains its superiority in performance over CED.

In Fig. 3, the variation of the probability of detection against SNR at a fixed target false alarm probability of 10^{-3} has been shown for cooperative scenario as well as single user case. The optimal value of p has been determined in the same manner as in Fig. 2.

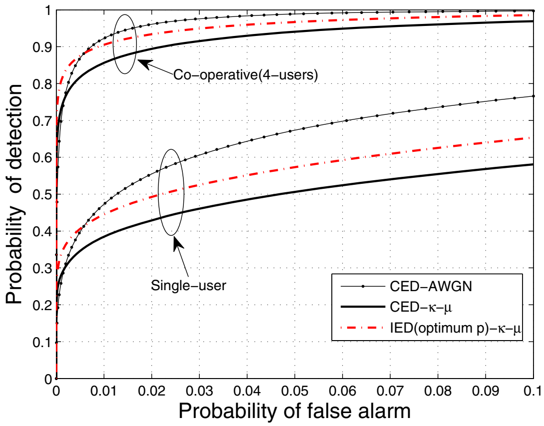

Fig. 2. ROCs for classical and improved p-norm energy detector over AWGN and κ-μ ($\kappa \to 0$, $\mu = 1$) fading channels in single user and cooperative scenarios with $N = 100$, $\overline{\gamma}$ = -5 dB and $U = 4$.

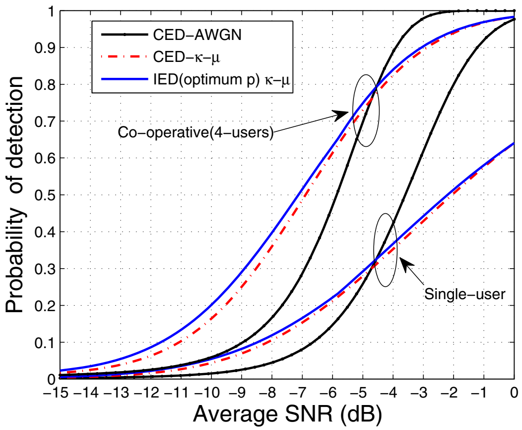

Fig. 3. Probability of detection as a variation of SNR for improved p-norm energy detector over κ-μ ($\kappa \to 0$, $\mu = 1$) fading in single user and cooperative scenario with $N = 100$, $P_{fa,target} = 10^{-3}$ and $U = 4$.

6 Conclusion

We have analyzed the sensing performance of an improved energy detector with the optimal p-norm value in a generalized κ-μ fading channel for cooperative spectrum sensing. The performance gain depends upon the various system design parameters e.g., SNR, the probability of false alarm and the number of samples per sensing event N. The IED algorithm outperforms CED in both AWGN and fading channels. With p-norm detector, an optimal p value ($\neq 2$) exists, that maximizes the detection probability over a significant range of SNRs with lower values of probability of false alarm. The study reveals that the combined benefit of both IED and p-norm detector results in significant performance gain in κ-μ fading channels for cooperative spectrum sensing.

Acknowledgments. The work has been carried out under the project, "Mobile Broadband Service Support over Cognitive Radio Networks," sponsored by Information Technology Research Academy (ITRA), Department of Electronics and Information Technology (DeitY), Govt. of India.

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