An Efficient Secondary User Selection Scheme for Cognitive Networks with Imperfect Channel Estimation and Multiple Primary Users

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Abstract. In this paper, we study the performance of multiuser cognitive generalized order user scheduling networks with multiple primary users and imperfect channel estimation. The utilized generalized order user selection scheme is efficient in situations where a user other than the best user is erroneously selected by the scheduling unit for data reception as in imperfect channel estimation or outdated channel information conditions. In this scheme, the secondary user with the second or even the N^{th} best signal-to-noise ratio (SNR) is assigned the system resources in a downlink channel. In our paper, closed-form expressions are derived for the outage probability, average symbol error probability (ASEP), and ergodic channel capacity assuming Rayleigh fading channels. Also, to get more insights about the system performance, the behavior is studied at the high SNR regime where the diversity order and coding gain are derived and analyzed. The achieved results are verified by Monte-Carlo simulations. Main results illustrate that the number of primary users affects the secondary system performance through affecting only the coding gain. Also, findings illustrate that a zero diversity gain is achieved by the system and a noise floor appears in the results when the secondary user channels are imperfectly estimated. Finally, results show that the generalized order user scheduling in cognitive networks has exactly the same diversity order as when implemented in the non-cognitive counterparts.

Keywords: Multiuser cognitive networks · Generalized order user scheduling · Imperfect channel estimation · Rayleigh fading

1 Introduction

The multiuser diversity is achieved by taking advantage of the channel fading variations in wireless networks. More specifically, it was shown that selecting the user with the best instantaneous channel each transmitting or receiving time increases the chance of having the communication to occur over a good

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channel. As a way for improving the spectrum utilization efficiency in wireless networks, the cognitive radio has been proposed in [1]. In such networks, the secondary or cognitive users can share the spectrum with primary users via underlay, overlay, or interweave paradigms [2]. The underlay paradigm which is adopted in this paper allows secondary users to share the spectrum of primary users if the interference between them is below a certain threshold.

The opportunistic scheduling where the user with the best instantaneous channel is always selected by the scheduling unit for data communications was used in [3] to select among secondary users. The multiuser diversity gain and bit error rate for multiple access, broadcast, and parallel access channels in cognitive radio networks with opportunistic scheduling were derived in [4]. Recently, Wang *et al.* proposed in [5] a limited feedback based underlay spectrum sharing scheme where the opportunistic scheduling was used to select among secondary users in a downlink transmission. The opportunistic scheduling was also used in [6] to select among secondary users in an uplink transmission. Most recently, the secondary users were allowed to utilize the spectrum of primary users in an opportunistic way in [7]. The performance of multiuser cognitive radio networks with opportunistic scheduling and multiple primary receivers was studied in [8]. All the aforementioned papers assumed perfectly estimated channels.

Some scheduling fairness and power control schemes were presented in [9] for multiuser cognitive networks with opportunistic scheduling among the secondary users. In [10], Khan *et al.* derived the exact outage and error rate probabilities for multiuser cognitive networks with opportunistic scheduling and Nakagami-m fading channels. In general, two important issues need to be considered when designing any multiuser network: the sum-rate capacity and fairness among users. The maximum-rate schedulers such as the generalized order user scheduling maximizes the sum capacity at the expense of unfairness among users; whereas, the proportional fair user selection scheme satisfies fairness among users at the expense of system sum-rate [11]. Therefore, the selection of the scheduling scheme depends on the system requirements and nature of the system. Although the proportional fair scheduling could be helpful for users of weak channels, the loss happens in the throughput when this scheduling scheme is used can be large in situations where users are scattered across the cell [12].

From our reading to the literature on the area of multiuser cognitive networks, we noticed that the most commonly used secondary user selection scheme in these networks is the opportunistic scheduling. In this scheme, the user with the best instantaneous channel is selected every time for data transmission or reception. Also, we noticed that most of the papers on multiuser cognitive networks assumed perfectly estimated channels and ignored the effect of imperfect channel estimation on the system performance. There exists several situations in wireless networks where the opportunistic scheduling could fail, among which is the presence of imperfect channel state information where the scheduling unit could fail in error in selecting the best user among the available users and in the presence of outdated channel information where the user which was the best at the selection time instant could not be the best at the transmission time instant. An efficient selection scheme which can deal with such situations is the generalized order user scheduling. In this scheme, the user with the second or even the N^{th} best channel is selected instead of the best user for transmitting or receiving data. This scheme was firstly proposed in literature to select among antennas [13], then, it was presented to select among relays in relay networks [14], and recently, it was used to select among users in multiuser relay networks [15]. Most of the previous papers consider the opportunistic scheduling and perfectly estimated channels.

In this paper, we study the performance of multiuser cognitive generalized order user selection network with multiple primary receivers in the presence of imperfect channel estimation. In the considered scheme, the secondary user with the first, the second, or even the N^{th} best signal-to-noise ratio (SNR) is allowed by the scheduling unit for data reception. Closed-form expressions are derived for the outage probability, average symbol error probability (ASEP), and ergodic channel capacity assuming independent non-identically distributed (i.n.i.d.) generic case of Rayleigh fading channels. Furthermore, the performance is studied at the high SNR regime where approximate expressions are derived for the outage probability and ASEP in addition to the derivation of the diversity order and coding gain of the system. The effect of number of primary users, number of secondary users, and channel estimation error on the system performance is illustrated via providing some simulation and numerical examples.

This paper is organized as follows. Section 2 presents the system and channel models. The exact performance evaluation is conducted in Section 3. Section 4 provides the asymptotic performance analysis. Some simulation and numerical results are discussed in Section 5. Finally, Section 6 concludes the paper.

2 System and Channel Models

The system under consideration consists of one secondary source S, K secondary destinations or users D_k ($k = 1, \ldots, K$), and M primary receivers P_m ($m = 1, \ldots, M$) using the same frequency band. All nodes are assumed to be equipped with single antenna. The secondary source sends its message x to K users under a transmit power constraint which guarantees that the interference with the primary users does not exceed a threshold $\mathcal{I}_\mathsf{p}$. To satisfy the primary interference constraint, the source S must transmit at a power given by $P_\mathsf{s} = \mathcal{I}_\mathsf{p}/\max_m |g_{\mathsf{s},m}|^2$, $m = 1, \ldots, M$, where $g_{\mathsf{s},m}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{P}_m$ link. Therefore, the message at the k^{th} destination D_k from the source S is given by $y_{\mathsf{s},k} = \sqrt{P_\mathsf{s}} h_{\mathsf{s},k} x + n_{\mathsf{s},k}$, where $h_{\mathsf{s},k}$ is the channel coefficient of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link and $n_{\mathsf{s},k}$ represents the additive white Gaussian noise (AWGN) term at D_k with a power of N_0. We assume that perfect channel information including the interference channel is available at the secondary source[1].

[1] Secondary source can know the channel information of the primary users by either a direct reception of pilot signals from primary users [16], or by exchange of channel information between primary and secondary users through a band manager [17].

Also, we assume that no interference is introduced from the primary user on the secondary receivers[2]. All channel coefficients are assumed to be Rayleigh distributed, so the channel gains $|g_{\mathsf{s},m}|^2$ and $|h_{\mathsf{s},k}|^2$ follow exponential distribution with mean powers $\mu_{g_{\mathsf{s}},m}$ and $\Omega_{h_{\mathsf{s},k}}$, respectively.

The channel coefficient of the $\mathsf{S} \rightarrow \mathsf{D}_k$ channel can be written as [19]

$$h_{\mathsf{s},k} = \hat{h}_{\mathsf{s},k} + e_{h_{\mathsf{s},k}}, \tag{1}$$

where $\hat{h}_{\mathsf{s},k}$ is the estimate of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link and $e_{h_{\mathsf{s},k}}$ is the channel estimation error, which is assumed to be complex Gaussian with zero mean and variance $\sigma^2_{e_{h_{\mathsf{s},k}}} = \Omega_{h_{\mathsf{s},k}} - \mathbb{E}[|\hat{h}_{\mathsf{s},k}|^2]$, with $\mathbb{E}[.]$ denoting the expectation operator. Also, $\hat{h}_{\mathsf{s},k}$ is complex Gaussian with zero mean and variance $\Omega_{\hat{h}_{\mathsf{s},k}} = \Omega_{h_{\mathsf{s},k}} + \sigma^2_{e_{h_{\mathsf{s},k}}}$. The above definition also applies to the $\mathsf{S} \rightarrow \mathsf{P}_m$ channel, i.e., $\hat{g}_{\mathsf{s},m} \sim \mathcal{CN}\big(0, \mu_{\hat{g}_{\mathsf{s},m}} = \mu_{g_{\mathsf{s},m}} + \sigma^2_{e_{g_{\mathsf{s},m}}}\big)$.

Upon using the values $h_{\mathsf{s},k} = \hat{h}_{\mathsf{s},k} + e_{h_{\mathsf{s},k}}$ and $g_{\mathsf{s},m} = \hat{g}_{\mathsf{s},m} + e_{g_{\mathsf{s},m}}$, the signal at the k^{th} user can be rewritten as

$$y_{\mathsf{s},k} = \sqrt{\frac{\mathcal{I}_{\mathsf{p}}}{W + \sigma^2_{eW}}}\hat{h}_{\mathsf{s},k} x + \sqrt{\frac{\mathcal{I}_{\mathsf{p}}}{W + \sigma^2_{eW}}} e_{h_{\mathsf{s},k}} x + n_{\mathsf{s},k}, \tag{2}$$

where $W = \max_m |\hat{g}_{\mathsf{s},m}|^2$, $m = 1, \ldots, M$ and σ^2_{eW} is the variance of the channel estimation error associated with the channel estimate $\max_m \hat{g}_{\mathsf{s},m}$.

From (2), the SNR of the $\mathsf{S} \rightarrow \mathsf{D}_k$ link can be easily obtained after simple manipulations as

$$\gamma_{\mathsf{S}-\mathsf{D}_k} = \frac{\bar{\gamma}|\hat{h}_{\mathsf{s},k}|^2}{W + \sigma^2_{eW} + \bar{\gamma}\sigma^2_{e_{h_{\mathsf{s},k}}}} = \gamma_k, \tag{3}$$

where $\bar{\gamma} = \mathcal{I}_{\mathsf{p}}/N_0$. The generalized order user scheduling is performed by choosing the user which has the N^{th} best SNR γ_k. The estimation error variance can be made small by transmitting large number of pilots at medium to high SNRs [19][3].

3 Exact Performance Analysis

In this section, closed-form expressions are derived for the outage probability, ASEP, and ergodic channel capacity.

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[2] This assumption is valid when the primary transmitter is in a location far from the secondary receiver [18].

[3] The variance of the channel estimation error can be also assumed to be inversely proportional to SNR as 1/SNR.

3.1 Outage Probability

In this section, we derive the outage probability of the considered system. The outage probability is defined as the probability that the SNR at the selected destination γ_{Sel} goes below a predetermined outage threshold γ_{out}, i.e., $P_{\mathsf{out}} = \Pr\left[\gamma_{\text{Sel}} \leq \gamma_{\mathsf{out}}\right]$, where $\Pr[.]$ denotes the probability operation.

Theorem 1. *The outage probability for multiuser cognitive generalized order user selection network with multiple primary users and imperfect channel estimation is given by*

$$P_{\mathsf{out}} = M\zeta_{\mathsf{s,p}} \sum_{i=1}^{M-1} \binom{M-1}{i} (-1)^i$$
$$\times \sum_{l=1}^{K} \lambda_{\mathsf{s},l} \sum_{\mathcal{P}} \left[\left\{ ((i+1)\zeta_{\mathsf{s,p}})^{-1} - \left(\Delta_1 \gamma_{\mathsf{out}} + (i+1)\zeta_{\mathsf{s,p}}\right)^{-1} \right\} (\Delta_1)^{-1} + \sum_{j=1}^{K-N} (-1)^j \right.$$
$$\left. \times \sum_{s_1<\ldots<s_j} (\Delta_2)^{-1} \left\{ ((i+1)\zeta_{\mathsf{s,p}})^{-1} - \left(\Delta_2 \gamma_{\mathsf{out}} + (i+1)\zeta_{\mathsf{s,p}}\right)^{-1} \right\} \right]. \quad (4)$$

Proof. To evaluate the outage probability, the cumulative distribution function (CDF) of γ_{Sel} is required to be obtained first. Herein, we first apply the conditional statistics on the fading channel from S to P_{Sel}, where P_{Sel} is the primary receiver who has the best channel with S. The CDF of the SNR in (3) conditioned on $W = \max\limits_{m} |\hat{g}_{\mathsf{s},m}|^2$, $m = 1, \ldots, M$ can be easily obtained as

$$F_{\gamma_k}\left(\gamma | W\right) = 1 - \exp\left(-\lambda_{\mathsf{s},k} \gamma W\right), \quad (5)$$

where $\lambda_{\mathsf{s},k} = \left(\sigma_{e_W}^2 + \sigma_{e_{h_{\mathsf{s},k}}}^2 \bar{\gamma} + 1\right) \Big/ \left(\Omega_{\hat{h}_{\mathsf{s},k}} \bar{\gamma}\right)$.

The conditional probability density function (PDF) of the selected secondary user is given by [20]

$$f_{\gamma_{\text{Sel}}}(\gamma|W) = \sum_{l=1}^{K} f_{\gamma_l}(\gamma|W) \sum_{\mathcal{P}} \prod_{j=1}^{K-N} F_{\gamma_{i_j}}(\gamma|W) \prod_{w=K-N+1}^{K-1} \left(1 - F_{\gamma_{i_w}}(\gamma|W)\right), \quad (6)$$

where $\sum_{\mathcal{P}}$ denotes the summation over all $n!$ permutations $(i_1, i_2, \ldots, i_K)$ of $(1, 2, \ldots, K)$ and N is the order of the selected user. Upon substituting (5) in (6), and using the binomial rule and applying the identity

$$\prod_{j=1}^{K-N} (1 - t_j) = 1 + \sum_{j=1}^{K-N} (-1)^j \sum_{s_1<\ldots<s_j} \prod_{n=1}^{j} t_{s_n}, \quad (7)$$

with $\sum_{s_1<\ldots<s_j}$ being a short hand-notation for $\sum_{s_1=1}^{K-N-j+1} \sum_{s_2=s_1+1}^{K-N-j+2} \cdots \sum_{s_j=s_{j-1}+1}^{K-N}$, (6) can be rewritten as

$$f_{\gamma_{\text{Sel}}}(\gamma|W) = \sum_{l=1}^{K} \lambda_{\mathsf{s},l} W \sum_{\mathcal{P}} \left[\exp\left(-\Delta_1 W\gamma\right) + \sum_{j=1}^{K-N} (-1)^j \sum_{s_1<\ldots<s_j} \exp\left(-\Delta_2 W\gamma\right) \right], \quad (8)$$

where $\Delta_1 = \sum_{w=K-N+1}^{K-1} \lambda_{\mathsf{s},i_w}$ and $\Delta_2 = \Delta_1 + \sum_{n=1}^{j} \lambda_{\mathsf{s},s_n} + \lambda_{\mathsf{s},l}$.
Assuming identical $\mathsf{S} \to \mathsf{P}_m$ channels, that is $\mu_{\hat{g}_{\mathsf{s},1}} = \ldots = \mu_{\hat{g}_{\mathsf{s},M}} = \mu_{\hat{g}_{\mathsf{s},\mathsf{p}}}$, the CDF and PDF of W are respectively given by

$$\begin{aligned} F_W(w) &= \left[F_{|g_{\mathsf{s},\mathsf{p}}|^2}(w)\right]^M = \left[1 - \exp\left(-\zeta_{\mathsf{s},\mathsf{p}} w\right)\right]^M, \\ f_W(w) &= M f_{|g_{\mathsf{s},\mathsf{p}}|^2}(w) \left[F_{|g_{\mathsf{s},\mathsf{p}}|^2}(w)\right]^{M-1} \\ &= M\zeta_{\mathsf{s},\mathsf{p}} \sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \exp\left(-(i+1)\zeta_{\mathsf{s},\mathsf{p}} w\right), \end{aligned} \quad (9)$$

where $\zeta_{\mathsf{s},\mathsf{p}} = 1/\mu_{\hat{g}_{\mathsf{s},\mathsf{p}}}$.

Up to now, the PDF of γ_{Sel} can be obtained using $\int_0^\infty f_{\gamma_{\text{Sel}}}(\gamma|W) f_W(w) dw$ as follows

$$\begin{aligned} f_{\gamma_{\text{Sel}}}(\gamma) = \sum_{i=0}^{M-1} \frac{\binom{M-1}{i}}{(-1)^{-i}} \sum_{l=1}^{K} \lambda_{\mathsf{s},l} \sum_{\mathcal{P}} \Bigg[\Big(\Delta_1\gamma + (i+1)\zeta_{\mathsf{s},\mathsf{p}}\Big)^{-2} \\ + \sum_{j=1}^{K-N} (-1)^j \sum_{s_1<\ldots<s_j} \Big(\Delta_2\gamma + (i+1)\zeta_{\mathsf{s},\mathsf{p}}\Big)^{-2} \Bigg] M\zeta_{\mathsf{s},\mathsf{p}}, \end{aligned} \quad (10)$$

where [21, Eq.(3.381.4)] has been used in getting (10). The outage probability can be obtained by integrating (10) using $\int_0^{\gamma_{\mathsf{out}}} f_{\gamma_{\text{Sel}}}(z) dz$ as given in (4).

3.2 Average Symbol Error Probability

In this section, we derive the ASEP of the considerd system. The ASEP can be written in terms of the CDF of γ_{Sel}, $F_{\gamma_{\text{Sel}}}(\gamma) = P_{\mathsf{out}}(\gamma_{\mathsf{out}} = \gamma)$ as

$$\text{ASEP} = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{\exp\left(-b\gamma\right)}{\sqrt{\gamma}} F_{\gamma_{\text{Sel}}}(\gamma) d\gamma, \quad (11)$$

where a and b are modulation-specific parameters.

By replacing γ_{out} with γ in (4), and with the help of [21, Eq.(3.381.4)] and [21, Eq. (3.383.10)] and after some simple steps, the ASEP can be obtained as follows

$$
\begin{aligned}
\mathrm{ASEP} = &\sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \\
&\times \sum_{l=1}^{K} \lambda_{\mathsf{s},l} \sum_{\mathcal{P}} \left[\frac{1}{\Delta_1} \left\{ \frac{((i+1)\zeta_{\mathsf{s,p}})^{-1}}{b^{1/2}} - \frac{\exp\left(\frac{b(i+1)\zeta_{\mathsf{s,p}}}{\Delta_1}\right) \Gamma\left(1/2, \frac{b(i+1)\zeta_{\mathsf{s,p}}}{\Delta_1}\right)}{\left(\Delta_1 (i+1)\zeta_{\mathsf{s,p}}\right)^{1/2}} \right\} \right. \\
&+ \sum_{j=1}^{K-N} (-1)^j \sum_{s_1<\ldots<s_j} \\
&\left. \times \frac{1}{\Delta_2} \left\{ \frac{((i+1)\zeta_{\mathsf{s,p}})^{-1}}{b^{1/2}} - \frac{\exp\left(\frac{b(i+1)\zeta_{\mathsf{s,p}}}{\Delta_2}\right) \Gamma\left(1/2, \frac{b(i+1)\zeta_{\mathsf{s,p}}}{\Delta_2}\right)}{\left(\Delta_2 (i+1)\zeta_{\mathsf{s,p}}\right)^{1/2}} \right\} \right] \frac{a\sqrt{b}}{2} M \zeta_{\mathsf{s,p}},
\end{aligned}
\tag{12}
$$

where $\Gamma(.,.)$ is the incomplete Gamma function defined in [21, Eq.(8.350.2)].

3.3 Ergodic Channel Capacity

In this section, we derive the ergodic channel capacity of the considered system. The channel capacity can be written in terms of the PDF of γ_{Sel} as

$$C = \frac{1}{\ln(2)} \int_0^\infty \ln(1+\gamma) f_{\gamma_{\mathrm{Sel}}}(\gamma) d\gamma. \tag{13}$$

Upon substituting (10) in (13), and with the help of [21, Eq.(4.291.15)], the ergodic channel capacity can be obtained as

$$
\begin{aligned}
C = &\sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \sum_{l=1}^{K} \lambda_{\mathsf{s},l} \sum_{\mathcal{P}} \left[\frac{\ln\left(\frac{\Delta_1}{(i+1)\zeta_{\mathsf{s,p}}}\right)}{\Delta_1(\Delta_1 - (i+1)\zeta_{\mathsf{s,p}})} \right. \\
&\left. + \sum_{j=1}^{K-N} (-1)^j \sum_{s_1<\ldots<s_j} \frac{\ln\left(\frac{\Delta_2}{(i+1)\zeta_{\mathsf{s,p}}}\right)}{\Delta_2\left(\Delta_2 - (i+1)\zeta_{\mathsf{s,p}}\right)} \right] \frac{M\zeta_{\mathsf{s,p}}}{\ln(2)}.
\end{aligned}
\tag{14}
$$

4 Asymptotic Performance Analysis

To get more insights about the system behavior and to simplify the achieved expressions, we study in this section the performance at the high SNR regime where simple approximate expressions are derived for the outage probability and ASEP in addition to the derivation of the diversity order and coding gain of the system.

4.1 Outage Probability

The outage probability can be expressed at the high SNR regime as $P_{\mathsf{out}} \approx (G_{\mathsf{c}}\mathrm{SNR})^{-G_{\mathsf{d}}}$, where G_{c} and G_{d} denote the coding gain and diversity order of the system, respectively [22]. Obviously, G_{c} represents the horizontal shift in the outage probability performance relative to the benchmark curve $(\mathrm{SNR})^{-G_{\mathsf{d}}}$ and G_{d} refers to the increase in the slope of the outage probability versus SNR curve [22, Ch.14]. The parameters on which the diversity order depends will affect the slope of the outage probability curves and the parameters on which the coding gain depends will affect the position of the curves. In the upcoming analysis, the secondary users are assumed to have identical channels, that is $\lambda_{\mathsf{s},1} = \ldots = \lambda_{\mathsf{s},K} = \lambda_{\mathsf{s,d}} = \left(\sigma^2_{e_W} + \sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma} + 1\right)/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}}$, and the channels from the secondary source to primary users are also assumed to be identical $\zeta_{\mathsf{s},1} = \ldots = \zeta_{\mathsf{s},M} = \zeta_{\mathsf{s,p}}$. The PDF of the selected N^{th} best user is given for identical users' channels as

$$f_{\gamma_{\mathrm{Sel}}}(\gamma|W) \approx \binom{K-1}{N-1} K f_{\gamma_{\mathsf{d}}}(\gamma|W) \left(F_{\gamma_{\mathsf{d}}}(\gamma|W)\right)^{K-N} \left(1 - F_{\gamma_{\mathsf{d}}}(\gamma|W)\right)^{N-1}. \quad (15)$$

As $\bar{\gamma} \to \infty$, the CDF in (5) simplifies to $F_{\gamma_{\mathsf{d}}}(\gamma|W) \approx \lambda_{\mathsf{s,d}} W\gamma$ and accordingly, the PDF simplifies to $f_{\gamma_{\mathsf{d}}}(\gamma|W) \approx \lambda_{\mathsf{s,d}} W$. Upon substituting the approximated CDF and PDF in (15) and following the same procedure as in Section 3.1, the outage probability at high SNR values can be evaluated with the help of [21, Eq.(3.351.3)] as

$$\begin{aligned} P_{\mathsf{out}}^{\infty} =& K\binom{K-1}{N-1}\left(\lambda_{\mathsf{s,d}}\right)^{K-N+1} M \sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i \\ &\times \sum_{k=0}^{N-1}\binom{N-1}{k}(-1)^k \left(\lambda_{\mathsf{s,d}}\right)^k (k+K-N)! \\ &\times \left((i+1)\zeta_{\mathsf{s,p}}\right)^{-(k+K-N+1)} \left(\gamma_{\mathsf{out}}\right)^{k+K-N+1}. \end{aligned} \quad (16)$$

The result in (16) is still dominant for the first term of the summation $k = 0$. With $\lambda_{\mathsf{s,d}} = \left(\sigma^2_{e_W} + \sigma^2_{e_{h_{\mathsf{s,d}}}}\bar{\gamma} + 1\right)/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}}$, we may end up with three main cases. These cases are determined by the status of the estimation process of primary and secondary users' channels:

Case 1: $\sigma^2_{e_W} = \sigma^2_{e_{h_{\mathsf{s,d}}}} = 0$ (perfect channel estimation)

For this case, $\lambda_{\mathsf{s,d}}$ simplifies to $1/\bar{\gamma}\Omega_{\hat{h}_{\mathsf{s,d}}}$ and the outage probability can be simplified as

$$\begin{aligned} P_{\mathsf{out}}^{\infty} =& \left\{ \left(\chi \sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i \left((i+1)\zeta_{\mathsf{s,p}}\Omega_{\hat{h}_{\mathsf{s,d}}}\right)^{-(K-N+1)} \right.\right. \\ &\left.\left. \times \left(\gamma_{\mathsf{out}}\right)^{K-N+1} \right)^{\frac{-1}{(K-N+1)}} \bar{\gamma} \right\}^{-K-N+1}, \end{aligned} \quad (17)$$

where $\chi = K(K-N)!\binom{K-1}{N-1}M$.

Case 2: $\sigma^2_{e_{h_{s,d}}} \neq 0$ (imperfect channel estimation of secondary users' channels)
Here, the numerator of $\lambda_{s,d}$ can be approximated by $(\sigma^2_{e_{h_{s,d}}} \bar{\gamma})$ and hence, $\lambda_{s,d}$ simplifies to $\sigma^2_{e_{h_{s,d}}} / \Omega_{\hat{h}_{s,d}}$. As a result, the outage probability can be simplified as

$$P^{\infty}_{out} = \chi \sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \left(\frac{(i+1)\zeta_{s,p} \Omega_{\hat{h}_{s,d}}}{\sigma^2_{e_{h_{s,d}}} \gamma_{out}} \right)^{-(K-N+1)}. \quad (18)$$

Case 3: $\sigma^2_{eW} = \sigma^2_{e_{h_{s,d}}} = 1/\mathrm{SNR} = 1/\bar{\gamma}$ (imperfect channel estimation)
For this case, $\lambda_{s,d}$ simplifies to $1/\bar{\gamma}\Omega_{\hat{h}_{s,d}}$. As a result, the outage probability can be simplified as obtained in (17) with the same coding gain and diversity order.

4.2 Average Symbol Error Probability

The asymptotic ASEP for the studied system can be obtained by replacing γ_{out} by γ in (16) and then substituting the result in (11). Upon doing that, and with the help of [21, Eq.(3.381.4)], we can easily get the following three cases:

Case 1: $\sigma^2_{eW} = \sigma^2_{e_{h_{s,d}}} = 0$ (perfect channel estimation)
For this case, the asymptotic ASEP can be obtained as

$$\mathrm{ASEP}^{\infty} = \left\{ \left(\chi \sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \left((i+1)\zeta_{s,p} \Omega_{\hat{h}_{s,d}} \right)^{-(K-N+1)} \frac{\Gamma(K-N+3/2)}{(b)^{K-N+3/2}} \right)^{\frac{-1}{(K-N+1)}} \bar{\gamma} \right\}^{-(K-N+1)}. \quad (19)$$

Case 2: $\sigma^2_{e_{h_{s,d}}} \neq 0$ (imperfect channel estimation of secondary users' channels)
Here, the asymptotic ASEP can be obtained as

$$\mathrm{ASEP}^{\infty} = \chi \sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^i \left(\frac{(i+1)\zeta_{s,p} \Omega_{\hat{h}_{s,d}}}{\sigma^2_{e_{h_{s,d}}}} \right)^{-(K-N+1)} \frac{\Gamma(K-N+3/2)}{(b)^{K-N+3/2}}. \quad (20)$$

Case 3: $\sigma^2_{eW} = \sigma^2_{e_{h_{s,d}}} = 1/\mathrm{SNR} = 1/\bar{\gamma}$ (imperfect channel estimation)
For this case, the asymptotic ASEP can be obtained to be similar to that found in (19).

It clear from (17), (19) that the multiuser cognitive generalized order user selection network with multiple primary users using the same spectrum band and imperfect channel estimation has a coding gain that is affected by several parameters such as K, N, $\Omega_{\hat{h}_{s,d}}$, M, $\zeta_{s,p}$, and γ_{out}; while the diversity order is constant at $K-N+1$. This is valid for the case where the channels are

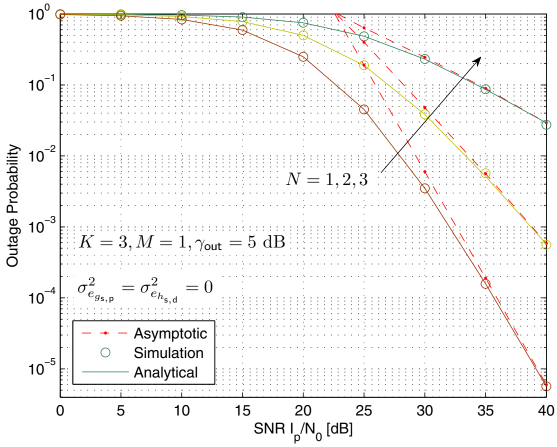

Fig. 1. P_{out} vs. SNR for different values of N and $\mu_{\hat{g}_{\mathrm{s,p}}} = 15$ dB, $\Omega_{\hat{h}_{\mathrm{s},k}} = 0$ dB$, k = 1, \ldots, 3$.

perfectly estimated. Also, this applies when the estimation errors are inversely proportional to SNR as shown in Case 3. On the other hand, when the channels are imperfectly estimated with constant estimation errors, it is obvious from (18), (20) that the system has zero diversity order and a coding gain that is affected by the same previous parameters but now with the effect of the channel estimation error $\sigma^2_{e_{h_{\mathrm{s,d}}}}$.

5 Simulation and Numerical Results

In this section, the achieved expressions are validated by Monte-Carlo simulations and some numerical examples are provided to illustrate the impact of several parameters on the system performance.

The effect of order of selected user N on the outage performance is illustrated in Figure 1. We can see from this figure that the asymptotic and analytical results perfectly match with Monte-Carlo simulations. Also, we can notice that as N increases, the diversity order of the system decreases and the system performance is more degraded. On the other hand, as N decreases, the diversity order increases and hence, better the achieved performance. These results on the diversity order of the generalized order selection scheme were achieved also when this scheme was implemented in non-cognitive systems.

Figure 2 studies the effect of number of primary users M on the system performance. Clearly, as M increases, worse the achieved performance. This is expected as having more primary users increases the probability of finding primary users of stronger channels and hence, having secondary users of smaller

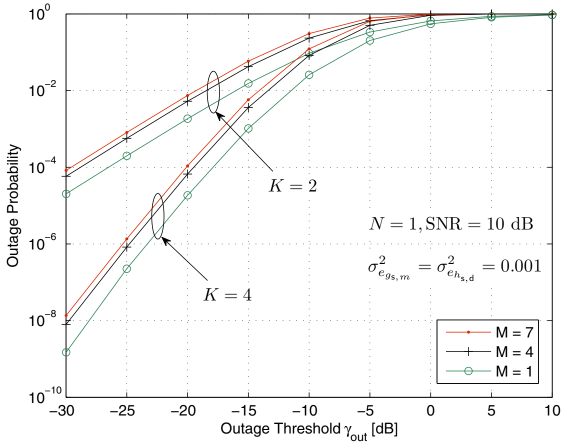

Fig. 2. P_{out} vs. outage threshold for different values of M, K and $\mu_{\hat{g}_{s,m}} = 15$ dB, $m = 1, \ldots, M$, $\Omega_{\hat{h}_{s,k}} = 0$ dB, $k = 1, \ldots, 4$.

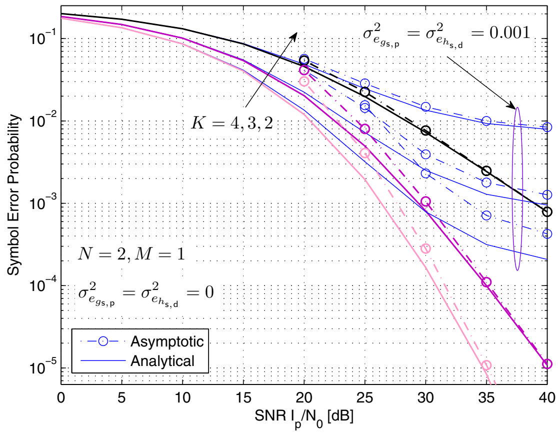

Fig. 3. ASEP vs. SNR for different values of K and $\mu_{\hat{g}_{s,p}} = 15$ dB, $\Omega_{\hat{h}_{s,k}} = 0$ dB, $k = 1, \ldots, 4$.

transmit power which degrades the system performance. As expected, more secondary users ($K = 4$) gives better performance compared to the case where $K = 2$.

The error probability performance of the studied system is shown in Figure 3 for different numbers of secondary users K. The figure is plotted for two cases: perfect channel estimation and imperfect channel estimation with constant estimation error variance. Again, it is clear that the asymptotic and analytical

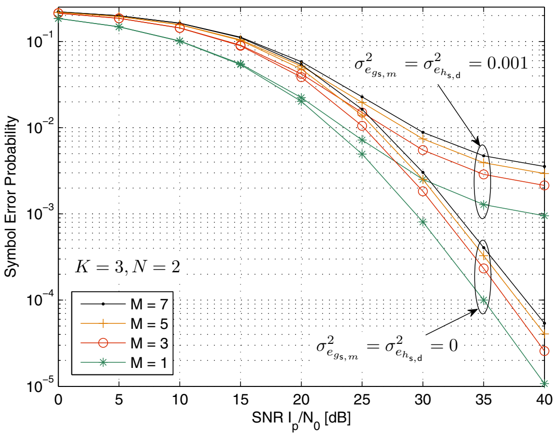

Fig. 4. ASEP vs. SNR for different values of M and $\mu_{\hat{g}_{\mathsf{s},m}} = 15$ dB, $m = 1, \ldots, M$, $\Omega_{\hat{h}_{\mathsf{s},k}} = 0$ dB, $k = 1, \ldots, 3$.

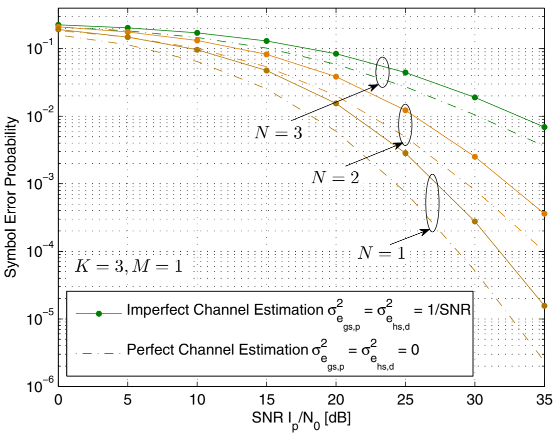

Fig. 5. ASEP vs. SNR for different values of N and $\mu_{\hat{g}_{\mathsf{s},\mathsf{p}}} = 15$ dB, $\Omega_{\hat{h}_{\mathsf{s},k}} = 0$ dB, $k = 1, \ldots, 3$.

results perfectly match with Monte-Carlo simulations. Also, we can see from this figure that for the case of perfect channel estimation ($\sigma^2_{e_{h_{\mathsf{s},\mathsf{p}}}} = \sigma^2_{e_{h_{\mathsf{s},\mathsf{d}}}} = 0$), as K increases, the diversity order of the system increases and the system performance is more enhanced. Also, it is clear that as K decreases, the diversity order decreases and hence, worse the achieved performance. On the other hand, in the presence of channel estimation error ($\sigma^2_{e_{h_{\mathsf{s},\mathsf{p}}}} = \sigma^2_{e_{h_{\mathsf{s},\mathsf{d}}}} = 0.001$), zero diversity gain

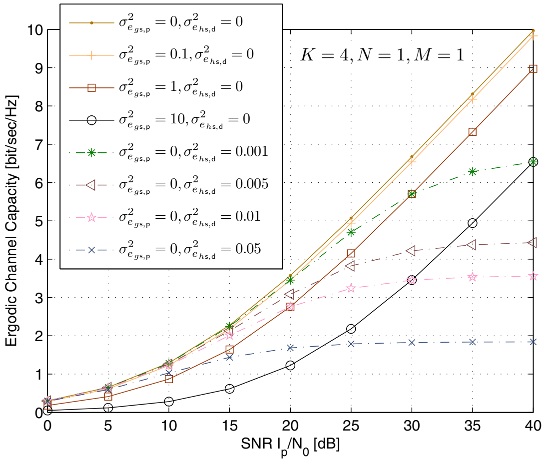

Fig. 6. Capacity vs. SNR for different values of $\sigma^2_{e_{g_{\mathsf{s,p}}}}$, $\sigma^2_{e_{h_{\mathsf{s,d}}}}$ and $\mu_{\hat{g}_{\mathsf{s,p}}} = 15$ dB, $\Omega_{\hat{h}_{\mathsf{s},k}} = 0$ dB, $k = 1, \ldots, 4$.

is achieved by the system and a noise floor appears in the results due to the imperfect channel estimation effect on the system behavior. This can be easily concluded from the asymptotic results where the diversity order of the system becomes zero when $\sigma^2_{e_{h_{\mathsf{s,d}}}} \neq 0$. In such case, any further increase in the SNR will add no enhancement to the system behavior.

Figure 4 studies the effect of number of primary users M on the error rate performance for the cases of perfect and imperfect channel estimations. Again, as M increases, worse the coding gain and hence, worse the achieved behavior. Also, as mentioned regarding Figure 3, it is clear in this figure that with imperfect channel estimation, the diversity gain of the system reaches zero and a noise floor appears in the results. This was illustrated in Case 2 of the asymptotic analysis section. Clearly, the diversity order of the system is not affected by the parameter M.

The effect of order of selected secondary user on the error probability performance is studied in Figure 5. The figure includes two cases: perfect channel estimation and imperfect channel estimation with an estimation error variance that is inversely proportional to SNR. The effect of channel estimation error on the system performance is obvious in this figure where worse behavior is achieved compared to the case where the channels are perfectly estimated. More importantly, for the case of imperfect channel estimation and as the variance of channel estimation error is assumed to be inversely proportional to SNR, the system can still achieve full diversity order when N decreases. This is also the case when the channels are perfectly estimated.

Figure 6 shows the ergodic channel capacity of the system for different values of $\sigma^2_{e_{g_{\mathsf{s,p}}}}$, $\sigma^2_{e_{h_{\mathsf{s,d}}}}$. Two cases are shown in this figure: imperfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{P}$ link with perfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{D}$ link; and perfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{P}$ link with imperfect channel estimation of the $\mathsf{S} \rightarrow \mathsf{D}$ link. For the first case where $\sigma^2_{e_{g_{\mathsf{s,p}}}}$ is taking different values and $\sigma^2_{e_{h_{\mathsf{s,d}}}} = 0$, the system capacity or performance keeps enhancing as SNR increases. On the other hand, when $\sigma^2_{e_{g_{\mathsf{s,p}}}} = 0$ and $\sigma^2_{e_{h_{\mathsf{s,d}}}}$ is taking different values, a noise floor appears in all results of this case. The behavior of the system in the two cases is expected as in the first case, the power of the channel estimation error of the $\mathsf{S} \rightarrow \mathsf{P}$ link $\sigma^2_{e_{g_{\mathsf{s,p}}}}$ is not affecting the SNR as clear from the asymptotic results; whereas, the power of the channel estimation error of the $\mathsf{S} \rightarrow \mathsf{D}$ link $\sigma^2_{e_{h_{\mathsf{s,d}}}}$ is a multiplied factor by the SNR in this case.

6 Conclusion

In this paper, we evaluated the performance of multiuser cognitive generalized order user selection network with multiple primary receivers and imperfect channel estimation. Closed-form expressions were derived for the outage probability, average symbol error probability, and ergodic channel capacity assuming Rayleigh fading channels. Furthermore, the system performance was evaluated at the high SNR values. Main results showed that the number of primary receivers affects the system performance through affecting only the coding gain. Also, findings illustrated that zero diversity gain is achieved by the system and a noise floor appears in the results when the channels of secondary users are imperfectly estimated. Finally, results showed that the imperfect estimation of primary receivers' channels affects the system performance via affecting only the coding gain.

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