An Efficient Switching Threshold-Based Scheduling Protocol for Multiuser Cognitive AF Relay Networks with Primary Users Using Orthogonal Spectrums

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Abstract. In this paper, we propose and evaluate the performance of multiuser switched diversity (MUSwiD) cognitive amplify-and-forward (AF) relay networks with multiple primary receivers using orthogonal spectrums. Using orthogonal spectrum bands aims to mitigate the interference between users in wireless networks. The spectrum of primary receiver whose channel results in the best performance for the secondary system is shared with secondary users. To reduce the channel estimation load in the secondary cell, the MUSwiD selection scheme is used to select among secondary users. In this scheme, the user whose end-to-end (e2e) signal-to-noise ratio (SNR) satisfies a predetermined switching threshold is scheduled to receive data from the source instead of the best user. In the analysis, an upper bound on the e2e SNR of a user is used in deriving of analytical approximations of the outage probability and average symbol error probability (ASEP). The performance is also studied at the high SNR regime where the diversity order and coding gain are derived. The derived expressions are verified by Monte-Carlo simulations. Results illustrate that the diversity order of the studied MUSwiD cognitive AF relaying network is the same as its non-cognitive counterpart. Unlike the existing papers where the same spectrum band is assumed to be shared by the primary receivers, our findings demonstrate that increasing the number of primary receivers in the proposed scenario enhances the system performancevia improving the coding gain.

Keywords: Amplify-and-forward · Multiuser cognitive relay network · Switching threshold · Orthogonal spectrums

1 Introduction

Cognitive radio is an important tool used to improve the spectrum resource utilization efficiency in wireless networks [1]. Several cognitive radio paradigms have

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been proposed in [2], among which is the underlay scheme. This scheme allows users in a secondary cell to utilize the frequency bands of users in a primary cell only if the interference is below a certain threshold. Beside the cognitive radio networks, a lot of research has been recently done on relay network which is used to deal with the multipath fading problem in wireless systems [3].

In the area of decode-and-forward (DF) cognitive relay networks (CRNs), closed-form expressions were derived in [4] for the outage and error probabilities of DF CRNs considering various relay selection scenarios. The outage and symbol error probabilities of amplify-and-forward (AF) CRNs with opportunistic and partial-relay selection schemes were evaluated in [5]. In [6], the error rate performance of an AF CRN was studied using the partial-relay selection scheme. The outage performance of an AF CRN with multiple primary users was recently studied in [7]. In addition to deriving the ergodic channel capacity, Bao *et al.* evaluated in [8] some lower bounds for the outage and error rate probabilities of AF CRNs assuming Rayleigh fading channels. Recently, the outage performance of opportunistic AF and DF CRNs with multiple secondary users and direct link was studied in [9].

Currently, the performance of CRNs with multiple secondary users is attracting a lot of researchers to work on such important topic. In [10], the secondary user was selected to achieve the largest secondary rate while satisfying primary rate target. In [11], the outage performance of AF and DF CRNs was studied assuming multiple secondary sources, single secondary relay and destination, and multiple primary receivers. The secondary source which maximizes the SNR at the destination combiner output considering the presence of the direct link is selected to send its message.

As can be seen, the only considered scenario in CRNs is the one where the multiple primary receivers utilize the same spectrum band. Another important scenario that could be seen in such systems is the one where the primary receivers use orthogonal spectrums. This situation could be seen in long term evolution (LTE) networks where the orthogonal frequency division multiple access (OFDMA) technique is used in the downlink transmission. Another application is in IEEE 802.22 wireless regional area networks (WRANs) where the OFDMA is a candidate access method for these networks. Furthermore, it is noticed that the mostly used scheduling scheme in multiuser CRNs is the opportunistic scheduling. A drawback of this scheme is the heavy load of channel estimations it requires in selecting the best secondary user among the available users. An efficient candidate which can reduce the channel estimation load between the secondary relay and users and the system complexity is the multiuser switched diversity (MUSwiD) selection scheme [12]. In this scheme, each user triggers a feedback only when its channel quality is greater than a certain threshold. Therefore, only the users with good channel quality are worth being considered to be scheduled [13].

According to authors knowledge, the scenario of cognitive AF relay networks with multiuser switched diversity selection and multiple primary receivers using orthogonal spectrums has not been presented yet. The contributions of this paper are as follows. *i*) We propose the new scenario of CRNs with multiple primary receivers using orthogonal spectrum bands. *ii*) Also, we introduce the MUSwiD user selection scheme to select among secondary users in the proposed scenario. *iii*) We provide a comprehensive analysis for evaluating the performance of the proposed scenario where some analytical approximations are derived for the outage probability and average symbol error probability (ASEP) for the independent non-identically distributed (i.n.i.d.) generic case of users channels. Furthermore, we study the behavior at the high SNR regime where the diversity order and coding gain are derived.

The rest of this paper is organized as follows. Section 2 presents the system and channel models. The performance evaluation is conducted in Section 3. Section 4 provides the asymptotic performance analysis. Some simulation and numerical results are discussed in Section 5. Finally, Section 6 concludes the paper.

2 System and Channel Models

Consider a dual-hop cognitive AF relay network consisting of one secondary source S, one AF secondary relay R, K secondary destinations or users D_k ($k = 1, \ldots, K$), and M primary receivers P_m ($m = 1, \ldots, M$) using orthogonal frequency bands. All nodes are assumed to be equipped with single antenna and the communication is assumed to operate in a half-duplex mode. Secondary users need to share the spectrum with the primary receiver whose channel satisfies the interference constraint and results in a best performance for the secondary system[1]. The communications take place in two phases. In the first phase, the secondary source sends its message x to relay under a transmit power constraint which guarantees that the interference with the primary receivers does not exceed a threshold $\mathcal{I}_{\mathsf{p}}$. As a result, the source S must transmit at a power given by $P_{\mathsf{s}} = \mathcal{I}_{\mathsf{p}} / \min\limits_{m} |g_{\mathsf{s},m}|^2$, $m = 1, \ldots, M$, where $g_{\mathsf{s},m}$ is the channel coefficient of the $\mathsf{S} \to \mathsf{P}_m$ link. In the second phase, R amplifies the received message from S with a variable gain G and forwards the amplified message to K users. The transmit power at R must also satisfy the interference constraint, it is defined as $P_{\mathsf{R}} = \mathcal{I}_{\mathsf{p}} / \min\limits_{m} |g_{\mathsf{r},m}|^2$, $m = 1, \ldots, M$, where $g_{\mathsf{r},m}$ is the channel coefficient of the $\mathsf{R} \to \mathsf{P}_m$ link. Hence, the received message at D_k from R is given by $y_{\mathsf{r},k} = \sqrt{P_{\mathsf{s}}} G h_{\mathsf{r},k} h_{\mathsf{s},\mathsf{r}} x + G h_{\mathsf{r},k} n_{\mathsf{s},\mathsf{r}} + n_{\mathsf{r},k}$, where $h_{\mathsf{s},\mathsf{r}}$ and $h_{\mathsf{r},k}$ are the channel coefficients of the $\mathsf{S} \to \mathsf{R}$ and $\mathsf{R} \to \mathsf{D}_k$ links, respectively, $n_{\mathsf{s},\mathsf{r}}$ and $n_{\mathsf{r},k}$ are the additive white Gaussian noise (AWGN) terms at R and D_k, respectively, with a power of N_0. We assume that the channel information of all links can

[1] Getting the best behavior of the secondary system in the sense of selecting the primary receiver which allows the secondary users to transmit at their max. power.

be perfectly estimated by the secondary users[2]. Also, it is assumed that the interference from the primary user is neglected[3]. As we are using a channel-state-information (CSI)-assisted AF relaying, the gain G can be expressed as $G^2 = 1\Big/\left(\min_m |g_{\mathsf{r},m}|^2\right)\left[\frac{|h_{\mathsf{s,r}}|^2}{\left(\min_m |g_{\mathsf{s},m}|^2\right)} + \frac{N_0}{\mathcal{I}_\mathsf{p}}\right]$. Thus, the end-to-end (e2e) SNR of D_k can be written as [8]

$$\gamma_{\mathsf{S-R-D}_k} = \frac{\frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{s,r}}|^2}{\min_m |g_{\mathsf{s},m}|^2}\frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{r},k}|^2}{\min_m |g_{\mathsf{r},m}|^2}}{\frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{s,r}}|^2}{\min_m |g_{\mathsf{s},m}|^2} + \frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{r},k}|^2}{\min_m |g_{\mathsf{r},m}|^2} + 1} \leq \gamma_k^{\mathsf{up}} = \min\Bigg(\underbrace{X/Y}_{\gamma_1}, \underbrace{X_k}_{\gamma_{2_k}}\Bigg), \tag{1}$$

where $X = \frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{s,r}}|^2$, $Y = \min_m |g_{\mathsf{s},m}|^2$, and $X_k = \frac{\frac{\mathcal{I}_\mathsf{p}}{N_0}|h_{\mathsf{r},k}|^2}{\min_m |g_{\mathsf{r},m}|^2}$. The MUSwiD user scheduling is achieved by selecting the user with the e2e SNR γ_k^{up} that satisfies a predetermined switching threshold. In the upcoming analysis, all channel coefficients are assumed to undergo i.n.i.d. Rayleigh fading and hence, the channel gains $|g_{\mathsf{s},m}|^2$, $|h_{\mathsf{s,r}}|^2$, $|h_{\mathsf{r},k}|^2$, and $|g_{\mathsf{r},m}|^2$ follow exponential distribution with mean powers $\mu_{\mathsf{s,m}}$, $\Omega_{\mathsf{s,r}}$, $\Omega_{\mathsf{r},k}$, and $\mu_{\mathsf{r},m}$, respectively.

Referring to Figure 1, the MUSwiD selection scheme works as follows. In each scheduling period, the relay probes the secondary users in a sequential way so only a single user has an opportunity to send a feedback at one time. For each user to decide whether to send a feedback or not, a single feedback threshold is used for all users. This threshold could be assumed to be constant or it could be calculated to optimize a certain performance measure[4]. The order of the users is set by the relay and sent to all users each scheduling period. The second or even the k^{th} user will not send any feedback signal to the relay unless it does not receive a flag from the previous user in the sequence within a certain time duration[5]. Suppose the users are arranged in a certain order, the first user compares its channel quality with the threshold. If it is higher than the threshold, the first user sends a feedback to the relay and a flag to other users signaling its presence. Otherwise, the first user keeps silent and the second (next) user compares its channel quality against the threshold. Again, if it exceeds the threshold, the second user sends a feedback to the relay and a flag to other users signaling its presence, otherwise the third user will get a chance. Once the relay detects a feedback from any user, it immediately selects

[2] Secondary users can know the channel information of the primary user by either a direct reception of pilot signals from a primary user [14].

[3] The interference is assumed to be represented by noise as in the case where the primary transmitters signal is generated by random Gaussian codebooks [15].

[4] In this paper, the switching threshold is numerically calculated to optimize the e2e outage probability. Also, a simple method is mentioned in Section 4 to obtain approximate but accurate values for the optimum switching threshold.

[5] The time duration of the feedback channel is not long and hence, the MUSwiD scheduling scheme does not cause additional delay to the scheduling process [16].

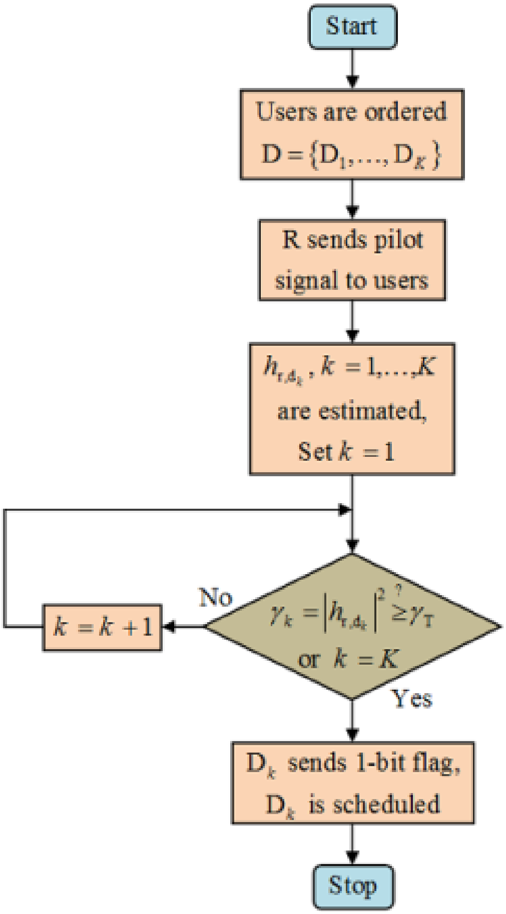

Fig. 1. Flowchart of the proposed MUSwiD user scheduling.

that user for the subsequent data reception and the whole user selection process ends. This process continues till a suitable user is found or all users are examined and found unacceptable. In this case, the MUSwiD scheme selects the last user for simplicity. To achieve fairness among users, the feedback sequence can be changed continuously. The feedback in MUSwiD systems is reduced significantly into only one feedback channel per resource unit instead of per-user feedback channels. Also, a user sends feedback only ahead of the resource units that it will be allocated instead of sending feedback for all resource units. This provides considerable savings in battery life of mobile terminals.

3 Performance Analysis

3.1 Outage Probability

The outage probability is defined as the probability that the SNR at the scheduled user γ_{up} goes below a predetermined outage threshold γ_{out}, i.e., $P_{\mathsf{out}} = \Pr\left[\gamma_{\mathsf{up}} \leq \gamma_{\mathsf{out}}\right]$, where Pr[.] denotes the probability operation.

Lemma 1. *L.1 The outage probability for MUSwiD cognitive CSI-assisted AF relay system with multiple primary receivers using orthogonal spectrums is given by*

$$
\begin{aligned}
P_{\mathsf{out}} \simeq & \sum_{i=0}^{K-1}\left\{\frac{(1-\exp(-\zeta_{\mathrm{tot}}\gamma_{\mathsf{T}}))}{\zeta_{\mathrm{tot}}} + \sum_{k=0}^{K-1}(-1)^{k+1}\sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1}\prod_{t=0}^{k}\frac{\left(1+\lambda_{2_{n_t}}\gamma_{\mathsf{T}}\right)^{-1}}{\Delta_1} - \left(1+\lambda_{2_i}\gamma_{\mathsf{out}}\right)^{-1}\right.\\
& \left.\times\left[\frac{(1-\exp(-(\lambda_1\gamma_{\mathsf{out}}+\zeta_{\mathrm{tot}})\gamma_{\mathsf{T}}))}{\lambda_1\gamma_{\mathsf{out}}+\zeta_{\mathrm{tot}}} + \sum_{k=0}^{K-1}(-1)^{k+1}\sum_{\substack{n_0<\ldots<n_k\\ n(.)\neq i}}^{K-1}\prod_{t=0}^{k}\frac{(1-\exp(-\Delta_1\gamma_{\mathsf{T}}))}{\left(1+\lambda_{2_{n_t}}\gamma_{\mathsf{T}}\right)\Delta_1}\right]\right\}(\pi_i\zeta_{\mathrm{tot}})\\
& + \sum_{l=0}^{K-1}\pi_l\zeta_{\mathrm{tot}}\left(\sum_{q=0}^{K}\frac{(-1)^q}{q!}\sum_{m_1,\ldots,m_q}^{K}\prod_{z=1}^{q}\frac{(1-\exp(-\Delta_1\gamma_{\mathsf{T}}))}{\left(1+\lambda_{2_{m_z}}\gamma_{\mathsf{T}}\right)\Delta_1} + \sum_{w=0}^{K-1}\pi_{((l-w))_K}\left[(1+\lambda_{2_l}\gamma_{\mathsf{T}})^{-1}\right.\right.\\
& \times\left\{\frac{\exp(-(\lambda_1\gamma_{\mathsf{T}}+\zeta_{\mathrm{tot}})\gamma_{\mathsf{T}})}{\lambda_1\gamma_{\mathsf{T}}+\zeta_{\mathrm{tot}}} + \sum_{p=0}^{w-1}(-1)^{p+1}\sum_{v_0<\ldots<v_p}^{w-1}\prod_{g=0}^{p}\frac{\exp(-\Delta_2\gamma_{\mathsf{T}})}{\Delta_3\Delta_2}\right\} - \left\{\frac{\exp(-(\lambda_1\gamma_{\mathsf{out}}+\zeta_{\mathrm{tot}})\gamma_{\mathsf{T}})}{\lambda_1\gamma_{\mathsf{out}}+\zeta_{\mathrm{tot}}}\right.\\
& \left.\left.\left.+\sum_{p=0}^{w-1}(-1)^{p+1}\sum_{v_0<\ldots<v_p}^{w-1}\prod_{g=0}^{p}\frac{\exp(-\Delta_4\gamma_{\mathsf{T}})}{\Delta_3\Delta_4}\right\}\left(1+\lambda_{2_l}\gamma_{\mathsf{out}}\right)^{-1}\right]\right),
\end{aligned} \tag{2}
$$

where $\zeta_{\mathrm{tot}} = \sum_{m=1}^{M}\zeta_{\mathsf{s},m}$, $\Delta_1 = \lambda_1\gamma_{\mathsf{T}} + \zeta_{\mathrm{tot}}$, $\Delta_2 = \sum_{u=0}^{p}\lambda_{2_{((l-w+v_u))_K}} + \lambda_1\gamma_{\mathsf{T}} + \zeta_{\mathrm{tot}}$, $\Delta_3 = 1+\lambda_{2_{((l-w+v_g))_K}}\gamma_{\mathsf{T}}$, *and* $\Delta_4 = \sum_{u=0}^{p}\lambda_{2_{((l-w+v_u))_K}} + \lambda_1\gamma_{\mathsf{out}} + \zeta_{\mathrm{tot}}$.

Proof. lease see Appendix.

3.2 Average Symbol Error Probability

The ASEP can be expressed in terms of the cumulative distribution function (CDF) of γ_{up}, $F_{\gamma_{\mathsf{up}}}(\gamma) = P_{\mathsf{out}}(\gamma_{\mathsf{out}} = \gamma)$ as

$$
\mathrm{ASEP} \simeq \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^{\infty}\frac{\exp(-b\gamma)}{\sqrt{\gamma}}F_{\gamma_{\mathsf{up}}}(\gamma)d\gamma, \tag{3}
$$

where a and b are modulation-specific constants.

Lemma 2. *L.2 The ASEP for MUSwiD cognitive CSI-assisted AF relay system with multiple primary receivers using orthogonal spectrums is given by*

$$
\begin{aligned}
\mathrm{ASEP} \simeq{}& \frac{a\sqrt{b}}{2\sqrt{\pi}}\zeta_{\mathrm{tot}} \Bigg\{ \sum_{i=0}^{K-1} \pi_i \Bigg(\frac{\sqrt{\pi}}{\sqrt{b}\zeta_{\mathrm{tot}}} + \sum_{k=0}^{K-1} (-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k \\ n(.)\neq i}} \prod_{t=0}^{k} \frac{\left(1+\lambda_{2_{n_t}}\gamma_{\mathsf{T}}\right)^{-1}}{\vartheta_1} - \Bigg[\left(-\frac{\lambda_1}{\lambda_{2_i}} + \zeta_{\mathrm{tot}}\right)^{-1} \Gamma(1/2)(\lambda_{2_i})^{-1/2} \\
&\times \Bigg\{ \exp\left(\frac{b}{\lambda_{2_i}}\right) \Bigg(\Gamma\left(1/2, b/\lambda_{2_i}\right) - \exp\left(-\left(\zeta_{\mathrm{tot}} - \frac{\lambda_1}{\lambda_{2_i}}\right)\gamma_{\mathsf{T}}\right) \Gamma\left(1/2, (\lambda_1\gamma_{\mathsf{T}}+b)/\lambda_{2_i}\right) \Bigg) + \left(\frac{\lambda_{2_i}\zeta_{\mathrm{tot}}}{\lambda_1}\right)^{-1/2} \exp\left(\frac{b\zeta_{\mathrm{tot}}}{\lambda_1}\right) \\
&\times \Bigg(\Gamma\left(1/2, (\lambda_1\gamma_{\mathsf{T}}+b)\zeta_{\mathrm{tot}}/\lambda_1\right) - \Gamma\left(1/2, b\zeta_{\mathrm{tot}}/\lambda_1\right) \Bigg) \Bigg\} + \sum_{k=0}^{K-1} (-1)^{k+1} \sum_{\substack{n_0<\ldots<n_k \\ n(.)\neq i}} \prod_{t=0}^{k} \frac{\left(1-\exp\left(-\vartheta_1\gamma_{\mathsf{T}}\right)\right)}{\left(1+\lambda_{2_{n_t}}\gamma_{\mathsf{T}}\right)\vartheta_1} \Bigg] \Bigg) + \sum_{l=0}^{K-1} \pi_l \\
&\times \Bigg(\sum_{q=0}^{K} \frac{(-1)^q}{q!} \sum_{m_1,\ldots,m_q}^{K} \prod_{z=1}^{q} \frac{\exp\left(-\vartheta_1\gamma_{\mathsf{T}}\right)}{\left(1+\lambda_{2_{m_z}}\gamma_{\mathsf{T}}\right)\vartheta_1} + \sum_{w=0}^{K-1} \pi_{((l-w))_K} \Bigg[\left(1+\lambda_{2_l}\gamma_{\mathsf{T}}\right)^{-1} \Bigg\{ \frac{\exp\left(-\vartheta_1\right)}{\vartheta_1} + \sum_{p=0}^{w-1} (-1)^{p+1} \sum_{v_0,\ldots,v_p}^{w-1} \\
& \prod_{g=0}^{p} \frac{\exp\left(-\vartheta_3\gamma_{\mathsf{T}}\right)}{\left(1+\lambda_{2_{((l-w+v_g))_K}}\gamma_{\mathsf{T}}\right)\vartheta_3} \Bigg\} - \Bigg\{ \left(-\frac{\lambda_1}{\lambda_{2_l}} + \zeta_{\mathrm{tot}}\right)^{-1} \Gamma(1/2) \frac{\exp\left(-\zeta_{\mathrm{tot}}\gamma_{\mathsf{T}}\right)}{(\lambda_{l2})^{1/2}} \Bigg(\exp\left(\frac{\lambda_1\gamma_{\mathsf{T}}+b}{\lambda_{2_l}}\right) \Gamma\left(1/2, \left(\lambda_1\gamma_{\mathsf{T}}+b\right)/\lambda_{2_l}\right) \\
&- \left(\frac{\lambda_{2_l}\zeta_{\mathrm{tot}}}{\lambda_1}\right)^{-1/2} \exp\left(\frac{(\lambda_1\gamma_{\mathsf{T}}+b)\zeta_{\mathrm{tot}}}{\lambda_1}\right) \Gamma\left(1/2, (\lambda_1\gamma_{\mathsf{T}}+b)\zeta_{\mathrm{tot}}/\lambda_1\right) \Bigg) \Bigg\} + \sum_{p=0}^{w-1} (-1)^{p+1} \prod_{g=0}^{p} \frac{\exp\left(-\vartheta_1\gamma_{\mathsf{T}}\right)}{1+\lambda_{2_{((l-w+v_g))_K}}\gamma_{\mathsf{T}}} \\
&\times \Bigg\{ \left(-\frac{\lambda_1}{\lambda_{2_l}} + \zeta_{\mathrm{tot}}\right)^{-1} \Gamma(1/2)(\lambda_{2_l})^{-1/2} \Bigg(\exp\left(\frac{\lambda_1\gamma_{\mathsf{T}}+b}{\lambda_{2_l}}\right) \Gamma\left(1/2, \left(\lambda_1\gamma_{\mathsf{T}}+b\right)/\lambda_{2_l}\right) - \exp\left(\frac{(\lambda_1\gamma_{\mathsf{T}}+b)\zeta_{\mathrm{tot}}}{\lambda_1}\right) \\
&\times \left(\frac{\lambda_{2_l}\zeta_{\mathrm{tot}}}{\lambda_1}\right)^{-1/2} \Gamma\left(1/2, (\lambda_1\gamma_{\mathsf{T}}+b)\zeta_{\mathrm{tot}}/\lambda_1\right) \Bigg) \Bigg\} \Bigg] \Bigg) \Bigg\},
\end{aligned}
\tag{4}
$$

where $\Gamma(.,.)$ is the incomplete Gamma function defined in [19, Eq.(8.350.2)], $\vartheta_1 = \lambda_1\gamma_{\mathsf{T}} + \zeta_{\mathrm{tot}}$ and $\vartheta_3 = 2\lambda_1\gamma_{\mathsf{T}} + \zeta_{\mathrm{tot}}$.

Proof. y replacing γ_{out} with γ in (2) and using the partial fraction expansion and the integration in (3) and with the help of [19, Eq.(3.361.2)] and [19, Eq.(3.383.10)], we get (4).

4 Asymptotic Performance Analysis

To get more insights about the system performance and simplify the results, we study the behavior at high SNR values where the outage probability can be expressed as $P_{\mathsf{out}} \approx (G_{\mathsf{c}}\mathrm{SNR})^{-G_{\mathsf{d}}}$, where G_{c} and G_{d} denote the coding gain and diversity order of the system, respectively [17]. In the upcoming analysis, the $\mathsf{S} \rightarrow \mathsf{P}_m$ links, the $\mathsf{R} \rightarrow \mathsf{D}_k$ links, and the $\mathsf{R} \rightarrow \mathsf{P}_m$ links are assumed to be identical, that is $(\zeta_{\mathsf{s},1} = \ldots = \zeta_{\mathsf{s},M} = \zeta_{\mathsf{s,p}})$, $(\lambda_{\mathsf{r},1} = \ldots = \lambda_{\mathsf{r},K} = \lambda_{\mathsf{r,d}})$, and $(\zeta_{\mathsf{r},1} = \ldots = \zeta_{\mathsf{r},M} = \zeta_{\mathsf{r,p}})$, respectively. The conditional CDF of γ_{up} is given for the identical case of users channels as

$$
F_{\gamma_{\mathsf{up}}}(\gamma|Y) = \begin{cases} \left[F_{\gamma^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right]^{K-1} F_{\gamma^{\mathsf{up}}}(\gamma|Y), & \gamma < \gamma_{\mathsf{T}}; \\ \sum_{j=0}^{K-1} \left[F_{\gamma^{\mathsf{up}}}(\gamma|Y) - F_{\gamma^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right] & \\ \times \left[F_{\gamma^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right]^{j} + \left[F_{\gamma^{\mathsf{up}}}(\gamma_{\mathsf{T}}|Y)\right]^{K}, & \gamma \geq \gamma_{\mathsf{T}}, \end{cases} \tag{5}
$$

where $F_{\gamma^{\mathsf{up}}}(\gamma|Y)$ is the CDF of γ_k^{up} conditioned on $Y = \min_m |g_{\mathsf{s},m}|^2, m = 1, \ldots, M$ and it can be expressed for the case of identical first and second hop channels as

$$F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) = 1 - \frac{\exp(-\lambda_1 \gamma Y)}{(1 + \lambda_2 \gamma)}, \tag{6}$$

where λ_1 is as defined in the Appendix and $\lambda_2 = 1 \Big/ \left(\sum_{m=1}^{M} \zeta_{\mathsf{r},m} \Omega_{\mathsf{r,d}} \frac{\mathcal{I}_{\mathsf{p}}}{N_0} \right)$. As $\frac{\mathcal{I}_{\mathsf{p}}}{N_0} \to \infty$, the CDF in (6) simplifies to $F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) \approx \lambda_1 Y \gamma$. Upon substituting this CDF in (5) and following the same procedure as in the Appendix, the outage probability at high SNR values can be evaluated with the help of [19, Eq. (3.351.1)] and [19, Eq.(3.351.2)] and recalling that $\lambda_1 = 1 \Big/ \left(\Omega_{\mathsf{s,r}} \frac{\mathcal{I}_{\mathsf{p}}}{N_0} \right)$ as

$$P_{\mathsf{out}}^{\infty} = \left\{ \frac{\Omega_{\mathsf{s,r}} M \zeta_{\mathsf{s,p}}}{\Gamma(2, M\zeta_{\mathsf{s,p}} \gamma_{\mathsf{T}}) (\gamma_{\mathsf{out}} - \gamma_{\mathsf{T}})} \frac{\mathcal{I}_{\mathsf{p}}}{N_0} \right\}^{-1}. \tag{7}$$

Upon substituting the asymptotic outage probability in (3) and with the help of [19, Eq.(3.351.3)] and recalling that $\lambda_1 = 1 \Big/ \left(\Omega_{\mathsf{s,r}} \frac{\mathcal{I}_{\mathsf{p}}}{N_0} \right)$, the ASEP can be obtained at high SNR values as

$$\mathrm{ASEP}^{\infty} = \left\{ \left(\Xi \left[\frac{\Gamma\left(\frac{3}{2}\right)}{b^{3/2}} - \frac{\Gamma\left(\frac{1}{2}\right)}{b^{1/2}} \gamma_{\mathsf{T}} \right] \right)^{-1} \frac{\mathcal{I}_{\mathsf{p}}}{N_0} \right\}^{-1}, \tag{8}$$

where $\Xi = \frac{a\sqrt{b}\,\Gamma(2, M\zeta_{\mathsf{s,p}}\gamma_{\mathsf{T}})}{2\sqrt{\pi} M \zeta_{\mathsf{s,p}} \Omega_{\mathsf{s,r}}}$.

A simple but an accurate method to find approximate optimum switching threshold is by using $\min\left(\frac{\frac{\mathcal{I}_{\mathsf{p}}}{N_0} \Omega_{\mathsf{s,r}}}{(M\zeta_{\mathsf{s,p}})^{-1}}, \frac{\frac{\mathcal{I}_{\mathsf{p}}}{N_0} \Omega_{\mathsf{r,d}}}{(M\zeta_{\mathsf{r,p}})^{-1}} \right)$.

5 Simulation and Numerical Results

We can see from figure 2 that the asymptotic results perfectly converge to the analytical results as well as the Monte-Carlo simulations. It is obvious also that the used bound on the e2e SNR is indeed very tight; especially, at the high SNR region. Furthermore, we can see from this figure that the MUSwiD selection scheme has nearly the same performance as the opportunistic scheduling for very low SNR region; whereas, as we go further in increasing SNR, the opportunistic scheduling is clearly outperforming the MUSwiD scheme, as expected. In addition, we can see that for the MUSwiD scheme as K increases, the system performance becomes more enhanced; especially, at the range of SNR values that are comparable to the switching threshold γ_{T}. More importantly, for $K = 2$, 3, and 4, it is obvious that at both low and high SNR values, all curves asymptotically converge to the same behavior and no gain is achieved in the system performance with having more users. This is expected since when γ_{T} takes values much smaller or much larger than the average SNR, the system asymptotically

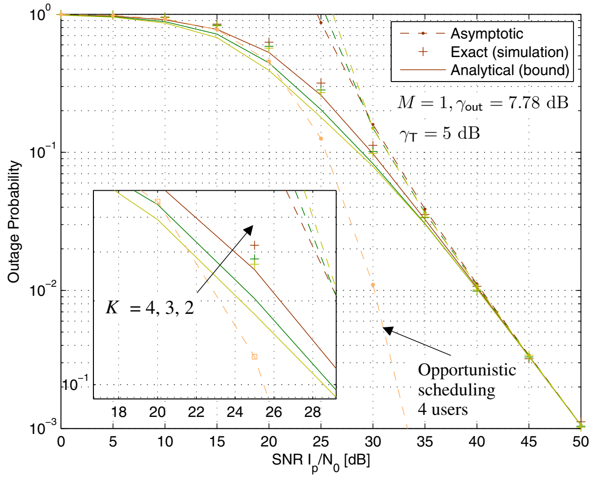

Fig. 2. P_{out} vs SNR for different values of K and $\mu_{\text{s,p}} = 30$, $\Omega_{\text{s,r}} = 0.8$, $\Omega_{\text{r,p}} = 0.1$, and $\Omega_{\text{r},k} = 0.7$ for $k = 1, \ldots, 4$.

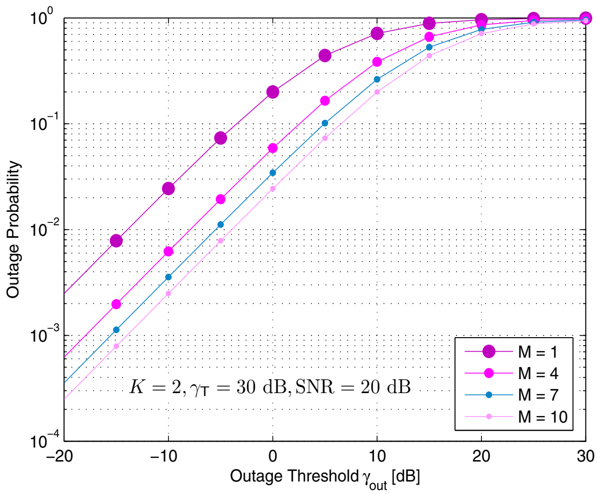

Fig. 3. P_{out} vs γ_{out} for different values of M and $\mu_{\text{s},m} = 20$ for $m = 1, \ldots, M$, $\Omega_{\text{s,r}} = 0.8$, $\mu_{\text{r},m} = 0.01$ for $m = 1, \ldots, M$, and $\Omega_{\text{r},k} = 0.9$ for $k = 1, 2$.

converges to the case of two users and hence, having more users will have no effect on the system performance. Finally, the effectiveness of the MUSwiD scheme is in the reduction of CSI feedback load it offers compared to the opportunistic scheduling. In order to achieve this effectiveness with a slight reduction in the

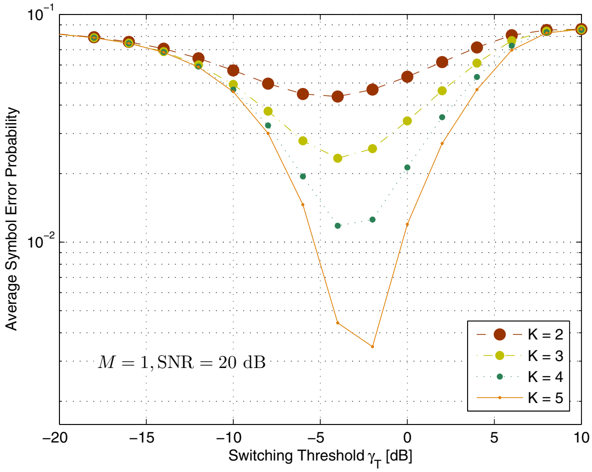

Fig. 4. ASEP vs γ_T for different values of K and $\mu_{\mathsf{s,p}} = 20$, $\Omega_{\mathsf{s,r}} = 0.1$, $\mu_{\mathsf{r,p}} = 0.3$, and $\Omega_{\mathsf{r},k} = 0.2$ for $k = 1, \ldots, 5$.

multiuser diversity gain, γ_T should be chosen to be close to the average SNR. Due to their features and performance, MUSwiD systems are actually attractive options for practical implementation in emerging mobile broadband communication systems [16].

It is obvious from Figure 3 that as γ_{out} increases, worse the achieved performance. Furthermore, it is clear from this figure that the best performance is achieved with the maximum number of primary users M.

We can see from Figure 4 that increasing K leads to a significant gain in system performance; especially, in the range of γ_T values that are comparable to the average value of γ_k^{up}. On the other hand, as γ_T becomes much smaller or much larger than the average value of γ_k^{up}, the improvement in performance decreases, as all curves asymptotically converge to the case of two users. This is due to the fact that, if the average value of γ_k^{up} is very small compared to γ_T, all users will be unacceptable most of the time. Whereas, if it is very high compared to γ_T, all users will be acceptable and one user will be scheduled most of the time. Thus, having more secondary users in both cases will add no gain to the system performance.

The average number of channel estimations versus switching threshold γ_T is illustrated in Figure 5 for the case of 4 secondary users. The average number of channel estimations for the MUSwiD selection scheme is provided in [20]. We can see from this figure that as the channels of all users are required for its operation, the opportunistic scheduling is always of need for 4 channel estimations. On the other hand, we can see that the MUSwiD selection scheme needs to estimate at most 3 channels because when the first 3 users are found unacceptable, the

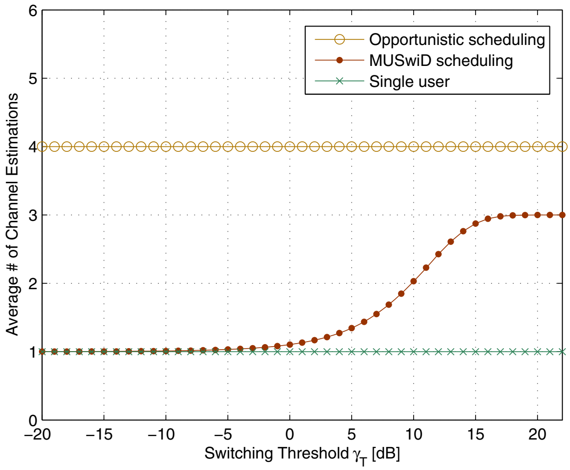

Fig. 5. Average number of channel estimations of the MUSwiD scheduling in comparison with the opportunistic scheduling with $K = 4$ and an average power/user path = 10 dB.

last checked user will be scheduled by the central unit regardless of its quality. Also, we can notice from this figure that as γ_{T} increases, the average number of channel estimations of users increases since it is more difficult to find a user with an acceptable quality.

6 Conclusion

The new scenario of MUSwiD cognitive AF relay network with multiple primary receivers using orthogonal spectrums was proposed in this paper. Analytical and asymptotic approximations for the outage and average symbol error probabilities were derived. Results illustrated that the diversity order of the proposed scenario is the same as its non-cognitive counterpart. Unlike the existing papers where the same spectrum band is shared by the primary receivers, increasing the number of primary receivers in the proposed scenario enhances the system behavior.

Appendix
Proof of Lemma 1

Herein, we first apply the conditional statistics on the fading channel from S to P. The CDF of γ_k^{up} conditioned on $Y = \min_m |g_{\mathsf{s},m}|^2, m = 1, \ldots, M$ can be written as

$$F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) = 1 - (1 - F_{\gamma_1}(\gamma|Y))\left(1 - F_{\gamma_{2_k}}(\gamma|Y)\right). \quad (9)$$

It is easy to see that

$$F_{\gamma_1}(\gamma|Y) = 1 - \exp(-\lambda_1 \gamma Y), \tag{10}$$

$$\begin{aligned} F_{\gamma_{2_k}}(\gamma|Y) &= \int_0^\infty F_{|h_{\mathsf{r},k}|^2}\left(\frac{N_0\gamma}{\mathcal{I}_\mathsf{p}}x\right) f_{\min\limits_m |g_{\mathsf{r},m}|^2}(x)\,dx \\ &= 1 - (1+\lambda_{2_k}\gamma)^{-1}, \end{aligned} \tag{11}$$

where $\lambda_1 = 1\Big/\left(\Omega_{\mathsf{s},\mathsf{r}}\frac{\mathcal{I}_\mathsf{p}}{N_0}\right)$, $\lambda_{2_k} = 1\Big/\left(\sum_{m=1}^{M}\zeta_{\mathsf{r},m}\Omega_{\mathsf{r},k}\frac{\mathcal{I}_\mathsf{p}}{N_0}\right)$, and the PDF $f_{\min\limits_m |g_{\mathsf{r},m}|^2}(x)$ is given by

$$f_{\min\limits_m |g_{\mathsf{r},m}|^2}(x) = \sum_{m=1}^{M}\zeta_{\mathsf{r},m}\exp\left(-\sum_{m=1}^{M}\zeta_{\mathsf{r},m}x\right), \tag{12}$$

where $\zeta_{\mathsf{r},m} = 1/\mu_{\mathsf{r},m}$.
Upon substituting (10) and (11) in (9), we get

$$F_{\gamma_k^{\mathsf{up}}}(\gamma|Y) = 1 - \frac{\exp(-\lambda_1\gamma Y)}{(1+\lambda_{2_k}\gamma)}. \tag{13}$$

Upon substituting (13) in the conditional CDF of γ_{up} provided by [17], we get

$$F_{\gamma_{\mathsf{up}}}(\gamma|Y) = \begin{cases} \sum_{i=0}^{K-1}\pi_i\left(1-\frac{\exp(-\lambda_1\gamma Y)}{(1+\lambda_{2_i}\gamma)}\right)\underbrace{\prod_{\substack{k=0\\k\neq i}}^{K-1}\left(1-\frac{\exp(-\lambda_1\gamma_\mathsf{T} Y)}{(1+\lambda_{2_k}\gamma_\mathsf{T})}\right)}_{\mathcal{P}_1}, & \gamma<\gamma_\mathsf{T}; \\ \sum_{l=0}^{K-1}\pi_l\Bigg\{\underbrace{\prod_{q=1}^{K}\left(1-\frac{\exp(-\lambda_1\gamma_\mathsf{T} Y)}{(1+\lambda_{2_q}\gamma_\mathsf{T})}\right)}_{\mathcal{P}_2}+\sum_{w=0}^{K-1}\pi_{((l-w))_K} & \\ \times\left[\left(1-\frac{\exp(-\lambda_1\gamma Y)}{(1+\lambda_{2_l}\gamma)}\right)-\left(1-\frac{\exp(-\lambda_1\gamma_\mathsf{T} Y)}{(1+\lambda_{2_l}\gamma_\mathsf{T})}\right)\right]\underbrace{\prod_{p=0}^{w-1}\left(1-\frac{\exp(-\lambda_1\gamma_\mathsf{T} Y)}{\left(1+\lambda_{2_{((l-w+p))_K}}\gamma_\mathsf{T}\right)}\right)}_{\mathcal{P}_3}\Bigg\}, & \gamma\geq\gamma_\mathsf{T}, \end{cases} \tag{14}$$

where K is the number of secondary destinations, γ_T is a predetermined switching threshold, $\pi_i, i = 0,\ldots,K-1$ is the probability that the i^{th} destination or user is chosen as given by [17], and $((l-w))_K$ denotes $l-w$ modulo K.
With the help of the product identities in [18] and [8], the terms $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ in (14) can be simplified as follows

$$\mathcal{P}_1 = 1 + \sum_{k=0}^{K-1}(-1)^{k+1}\sum_{\substack{n_0<\ldots<n_k\\n_{(.)}\neq i}}^{K-1}\prod_{t=0}^{k}\frac{\exp(-\lambda_1\gamma_\mathsf{T}Y)}{(1+\lambda_{2_{n_t}}\gamma_\mathsf{T})}, \tag{15}$$

where $\sum_{\substack{n_0<\ldots<n_k\\n_{(.)}\neq i}}^{K-1}$ is a short-hand notation for $\sum_{\substack{n_0=0\\n_0\neq i}}^{K-k-1}\sum_{\substack{n_1=n_0+1\\n_1\neq i}}^{K-k}\cdots\sum_{\substack{n_k=n_{k-1}+1\\n_k\neq i}}^{K-1}$.

$$\mathcal{P}_2 = \sum_{q=0}^{K} \frac{(-1)^q}{q!} \sum_{m_1,\ldots,m_q}^{K} \prod_{z=1}^{q} \frac{\exp\left(-\lambda_1 \gamma_{\mathsf{T}} Y\right)}{\left(1+\lambda_{2_{m_z}} \gamma_{\mathsf{T}}\right)}, \tag{16}$$

where $\sum_{m_1,\ldots,m_q}^{K}$ is a short-hand notation for $\sum_{\substack{m_1=\ldots=m_q=1 \\ m_1\neq\ldots\neq m_q}}^{K-k-1}$.

$$\mathcal{P}_3 = 1 + \sum_{p=0}^{w-1} (-1)^{p+1} \sum_{v_0<\ldots<v_p}^{w-1} \prod_{g=0}^{p} \frac{\exp\left(-\lambda_1 \gamma_{\mathsf{T}} Y\right)}{\left(1+\lambda_{2_{((l-w+v_g))_K}} \gamma_{\mathsf{T}}\right)}, \tag{17}$$

where $\sum_{v_0<\ldots<v_p}^{w-1}$ is a short-hand notation for $\sum_{v_0=0}^{w-p-1} \sum_{v_1=v_0+1}^{w-p} \cdots \sum_{v_p=v_{p-1}+1}^{w-1}$. Up to now, the outage probability can be expressed as

$$P_{\mathsf{out}} \simeq \int_0^\infty F_{\gamma_{\mathsf{up}}}\left(\gamma|Y\right) f_Y(y) dy, \tag{18}$$

where the PDF $f_Y(y)$ is given by

$$f_Y(y) = \sum_{m=1}^{M} \zeta_{\mathsf{s},m} \exp\left(-\sum_{m=1}^{M} \zeta_{\mathsf{s},m} y\right), \tag{19}$$

where $\zeta_{\mathsf{s},m} = 1/\mu_{\mathsf{s},m}$. Upon substituting (15), (16), and (17) in (14) and using (18), the outage probability can be evaluated as in (2).

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