# Stability and Delay Analysis for Cooperative Relaying with Multi-access Transmission

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**Abstract.** We consider a cooperative relaying system with two source terminals, one full duplex relay, and a common destination. Each terminal has a local traffic queue while the relay has two relaying queues to store the relayed source packets. We assume that the source terminals transmit packets in orthogonal frequency bands. In contrast to previous work which assumes a time division multi-access cooperation strategy, we assume that the source terminals and the relay simultaneously transmit their packets to the common destination through a multi-access channel (MAC). A new cooperative MAC scheme for the described network is proposed. We drive an expression for the stable throughput and characterize the stability region of the network. Moreover, the fundamental trade-off between the delay and the stable throughput is studied. Numerical results reveal that the proposed protocol outperforms traditional time division multi-access strategies.

**Keywords:** Cooperative relaying · Multi-access channel · Stable throughput region · Queuing theory · Average delay

## 1 Introduction

In wireless networks, the transmission of a single node may successfully reach multiple nodes within its range, which is referred to as the wireless multicast advantage. As a result, intermediate nodes have the capability to capture the transmission and contribute to the communication by *cooperatively* relaying the data. This contribution enhances the aggregate throughput of the network and reduces the delay encountered by the packets of different nodes [1], [2]. Cooperative communication in wireless networks has been widely investigated. In [3], a time division multiple access (TDMA) policy is assumed, where a single relay cooperatively transmits the packets of the source nodes during idle time slots.

Recently, multi-packet reception (MPR) has received considerable attention in the literature. A generalized MPR model was first introduced in [4]. The number of successful transmissions in a time-slot was modelled as a random variable

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which is a function of the number of attempted transmissions. Also, multi-access channel (MAC) systems have been addressed in the literature in different contexts, most of which do not deal with cognitive or cooperative systems [5], [6]. Nevertheless, in [7], a MAC network with two primary transmitters and a single secondary node was considered with a symmetric configuration. The primary users, simultaneously, access the channel to deliver their packets to a common destination. The cognitive node transmits during idle time slots. The impact of the cognitive node with and without relaying capability was studied.

Several metrics have been considered for evaluating the performance of cooperative networks and the average packet delay is one of these metrics. In [8], the delay analysis for a cognitive relaying scenario was presented, using the moment generation function approach, where a full priority is given to the relaying queue. However, in [9], the delay analysis for randomized cooperation policy was studied where the secondary user serves either its own data or the primary packets with certain service probabilities. This policy enhances the secondary user delay at the expense of a slight degradation in the primary user delay.

In this paper, we investigate a cooperative scenario with one full duplex relay and two source terminals. Unlike most of the existing work, e.g., [7], [3], we assume that the source terminals transmit their packets using two orthogonal frequency bands. We assume that the receivers have perfect CSI. In contrast with previous work in [10], a new MAC cooperation scheme is proposed. Under this scheme, the relay transmits only if the destination can decode the message of the relay by treating the source terminal message as noise. The relay may exploit one or both frequency bands for transmission. For comparison purpose, we introduce a TDMA cooperation scheme where the source terminals and the relay transmit their packets over disjoint fractions of time. The comparison between the two schemes shows that the proposed MAC scheme outperforms the conventional TDMA scheme.

The remainder of the paper is organized as follows. Section 2 introduces the system model and the proposed cooperative strategies. Section 3 presents the analysis of the stable throughput region. The average delay characterization is provided in Section 4. Numerical results are then presented in Section 5, followed by the conclusion in Section 6.

## 2 System Model

We assume a network consisting of two source terminals ($s_1$ and $s_2$), one common relay ($r$), and one common destination ($d$), as shown in Fig. 1. The source terminals transmit their signals to the common destination using two orthogonal frequency bands donated by $w_1$ and $w_2$ for $s_1$ and $s_2$, respectively. All wireless links are assumed to be stationary, frequency non-selective, and Rayleigh block fading. The fading coefficients, $h_{m,n}$, where $m \in \{s_1, s_2, r\}$ and $n \in \{r, d\}$, are assumed to be constant during one slot duration, but change independently from

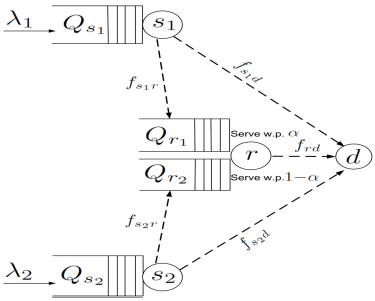


**Fig. 1.** System Model

one time slot to another according to a circularly symmetric complex Gaussian distribution with zero mean and variance $\rho^2_{m,n}$. All wireless links are corrupted by additive white Gaussian noise (AWGN) with zero mean and unit variance.

The $i$th source terminal, where $i \in \{1, 2\}$, transmits with fixed power $P_{s_i}$. An outage occurs when the rate $R$ is more than the instantaneous capacity of the link $(m, n)$. Each link is characterized by the probability

$$f_{mn} = \mathbb{P}\{R < \log_2(1 + P_m|h_{m,n}|^2)\} = \exp\left(-\frac{2^R - 1}{P_m \rho^2_{m,n}}\right) \tag{1}$$

which denotes the probability that the link $(m, n)$ is not in outage.

Time is slotted and the transmission of a packet takes exactly one slot duration. Each source terminal has an infinite queue to store its own incoming packets. Packet arrivals of both terminals are independent and stationary Bernoulli processes with means $\lambda_1$ and $\lambda_2$ (packets per slot) for $s_1$ and $s_2$, respectively.

The relay has two relaying queues ($Q_{r_1}$ and $Q_{r_2}$) to store the packets of the source terminals that are not successfully decoded at the destination. Let $Q_l^t$ denote the number of packets in the $l$th queue at the beginning of time slot $t$. The instantaneous evolution of the $l$th queue length is given by

$$Q_l^{t+1} = (Q_l^t - Y_l^t)^+ + X_l^t \tag{2}$$

where $l \in \{s_1, s_2, r_1, r_2\}$ and $(x)^+ = \max\{x, 0\}$. The binary random variables $Y_l^t$ and $X_l^t$, denote the departures and arrivals of $Q_l$ in time slot $t$, respectively, and their values are either 0 or 1.

We assume that the relay is full duplex, i.e., it can transmit and receive at the same time slot. In wireless networks when a node transmits and receives simultaneously on the same frequency, the problem of self-interference arises. Although there are some techniques that allow the possibility of perfect self-interference cancellation [11], in practice, there are currently several technological limitations and challenges that limit the accuracy and the effectiveness of self-interference cancellation [12]. Therefore, we assume that the relay can transmit and receive, simultaneously, over two *distinct* frequency bands. In addition, the relay also has the capability to receive or transmit packets on the two frequency bands simultaneously. It is worth noting that our network can be applied in the uplink of a cellular system where the source terminals are mobile nodes, the destination is a base station and the relay is a fixed node.

### 2.1 Cooperative MAC Scheme

Each source terminal transmits the packet at the head of its queue on its assigned frequency band whenever the queue is not empty. If the destination receives the packet successfully, it sends an acknowledgement message (ACK) which can be heard by both the terminal and the relay. If the destination does not succeed in receiving the packet correctly but the relay does, then the relay stores this packet at the end of its queue and sends an ACK to the source terminal. The source terminal drops the transmitted packet when it hears an ACK from the destination or the relay, otherwise, it retransmits the packet in the next time slot. The feedback messages are assumed to be error-free as short length packets and low rate codes can be employed in the feedback channel.

We assume that the destination knows the state of the channels from the sources and the relays, i.e., $h_{m,d}$, where $m \in \{s_1, s_2, r\}$. Note that this assumption is well-justified as the system can dedicate a small portion at the beginning of each time slot to transmit a short training sequence to the destination to be used for channel estimation[1]. We assume that the average channel gain between the source and the relay is higher than that between the source and the destination. In absence the of the relay, the source terminal wastes power when it transmits a packet that the destination can not successfully decode. However, the relay might still be able to decode that packet and this provides a diversity gain to the source terminals. According to the CSI, the destination decides the reception/transmission policy of the relay and sends it through a short error-free message to the relay at the beginning of each time slot[2]. The reception/transmission policy is described as follows

- The relay stores the transmitted packet from $s_i$, where $i \in \{1, 2\}$, if the destination can not decode this packet successfully.
- The relay transmits a packet on $w_i$, when it is not receiving packets on that band and the destination can decode the packet of the relay by treating $s_i$ as noise.

If the relay transmits on both frequency bands simultaneously, a packet from each of the relaying queues is served. When the relay transmits on only one frequency band, a packet is served from $Q_{r_1}$ with probability $\alpha$ or from $Q_{r_2}$ with probability $1 - \alpha$. We do not assume that the destination controls the source terminals. We lose the diversity gain provided by the relay if the source transmits, only, according to the source-destination channel.

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[1] The nodes re-transmit the training sequence whenever the channel changes.

[2] The side communication between the destination node and the relay and the amount of required training for CSI estimation is outside the scope of this paper.

Let $g_{r,d}^{s_i}$ denote the probability that the destination decodes the packet of the relay by treating $s_i$ as noise. Therefore,

$$\begin{aligned} g_{rd}^{s_i} &= \mathbb{P}\Big\{R < \log_2\Big(1+\frac{P_r|h_{r,d}|^2}{P_s|h_{s_i,d}|^2+1}\Big)\Big\} \\ &= \exp\Big(-\frac{2^R-1}{\rho_{r,d}^2 P_r}\Big)\frac{\rho_{r,d}^2 P_r}{\rho_{r,d}^2 P_r+(2^R-1)\rho_{s_i,d}^2 P_s} \end{aligned} \tag{3}$$

where $P_r$ denotes the power transmitted by the relay per frequency band. Note that if the source terminal is not transmitting simultaneously with the relay, e.g., when the queue of the source is empty, the destination will be able to successfully decode the transmission of the relay with a higher probability than that in (3). It is obvious that there is an interaction between the queues of the source terminals and that of the relay because the probability of successful transmission of the relay depends on the queue state of the source terminals. Since, the analysis of the average delay of interacting queues is difficult [13], we resort to the use of a dominant system where $s_i$ transmits dummy packet whenever the relay is transmitting on $w_i$ [7]. In other words, $s_1$ transmits a dummy packet if the relay is transmitting on $w_1$ and $Q_{s_1}$ is empty. Similarly, $s_2$ transmits a dummy packet if the relay is transmitting on $w_2$ and $Q_{s_2}$ is empty. The dominant system decouples the interaction between the queues and provides an upper bound on the the delay of the original system.

It is worth noting from the given description of the proposed policy that the system at hand is non work-conserving. A system is considered work-conserving if it is not idle whenever it has packets [14]. This condition is violated when the relay randomly selects to transmit a packet from a queue which is empty, while the other queue is non-empty. We resort to a non-conserving policy for its mathematical tractability.

### 2.2 Cooperative TDMA Scheme

The main difference between the two schemes is in the way the nodes utilize the available resources (time and frequency). Here, the cooperation policy depends on a TDMA frame work where $s_1$ and $r_1$ transmit their packets on $w_1$ only while $s_2$ and $r_2$ transmit using $w_2$. Each of $s_i$ and $r_i$ transmits in fixed fraction of time donated by $m_{s_i}$ and $m_{r_i}$ for $s_i$ and $r_i$, respectively, where $m_{s_i}+m_{r_i}=1$. Based on the cooperation policy described, there is no interaction between the queues because all nodes transmit over orthogonal resources.

## 3 Stable Throughput Region

A fundamental performance measure of a communication network is the stability of its queues. The stability of the overall system requires the stability of each individual queue. We can apply Loynes' theorem to check the stability of a queue [15]. Loynes' theorem states that if the arrival process and the service process of a queue are strictly stationary, then the queue is stable if and only if the average service rate is greater than the average arrival rate of the queue.

### 3.1 The Stability Analysis of MAC Scheme

A packet departs $Q_{s_i}$ if it is successfully decoded by at least one node, i.e., the destination or the relay. Thus, the average service rate of $Q_{s_i}$ is given by

$$\mu_i = f_{s_i d} + (1 - f_{s_i d}) f_{s_i r} \tag{4}$$

Thus, for stability of $Q_{s_i}$, the following condition must be satisfied

$$\lambda_i < f_{s_i d} + f_{s_i r}(1 - f_{s_i d}) \tag{5}$$

A packet arrives at $Q_{r_1}$ if the following two conditions are met. First, if an outage occurs in the link between $s_1$ and the destination node while no outage occurs in the link between $s_1$ and the relay. Second, $Q_{s_1}$ is not empty which has a probability of $\lambda_1/\mu_1$. Thus, the average arrival rate of $Q_{r_1}$ is given by

$$\lambda_{r_1} = (1 - f_{s_1 d}) f_{s_1 r} \frac{\lambda_1}{\mu_1} \tag{6}$$

A packet departs $Q_{r_1}$ if the relay transmits on both frequency bands, simultaneously, which happens with probability $p_1 g_{rd}^{s_1} g_{rd}^{s_2}$ or the relay transmits on a single frequency which happens with probability $p_1(\overline{g_{sd}^{s_1}} g_{rd}^{s_2} + g_{rd}^{s_1}\overline{g_{rd}^{s_2}}) + p_2 g_{rd}^{s_2} + p_3 g_{rd}^{s_1}$ and $Q_{r_1}$ is selected to transmit a packet which happens with probability $\alpha$. Thus, the service rate of $Q_{r_1}$ is given by

$$\mu_{r_1} = p_1 g_{rd}^{s_1} g_{rd}^{s_2} + \alpha(p_1 \overline{g_{sd}^{s_1}} g_{rd}^{s_2} + p_1 g_{rd}^{s_1} \overline{g_{rd}^{s_2}} + p_2 g_{rd}^{s_2} + p_3 g_{rd}^{s_1}) \tag{7}$$

where $p_1 = f_{s_1 d} f_{s_2 d}$, $p_2 = \overline{f_{s_1 d}} f_{s_2 d}$, $p_3 = f_{s_1 d} \overline{f_{s_2 d}}$, and $\overline{x} = 1 - x$.

For the stability of $Q_{r_1}$, the service rate must be higher than the arrival rate, i.e., $\lambda_{r_1} < \mu_{r_1}$, and hence, we have

$$\lambda_1 < \frac{\mu_{r_1}}{(1 - f_{s_1 d}) f_{s_1 r}} \mu_1 \tag{8}$$

Applying exactly the same analysis for $Q_{r_2}$, we get

$$\lambda_2 < \frac{\mu_{r_2}}{(1 - f_{s_2 d}) f_{s_2 r}} \mu_2 \tag{9}$$

where

$$\mu_{r_2} = p_1 g_{rd}^{s_1} g_{rd}^{s_2} + \overline{\alpha}(p_1 \overline{g_{sd}^{s_1}} g_{rd}^{s_2} + p_1 g_{rd}^{s_1} \overline{g_{rd}^{s_2}} + p_2 g_{rd}^{s_2} + p_3 g_{rd}^{s_1}) \tag{10}$$

From (5), (8) and (9), it is obvious that to guarantee the stability of the system the followings must be satisfied

$$\lambda_i < \min\{\mu_i, \mu_{u_i}\} \tag{11}$$

$$\text{where} \quad \mu_{u_i} = \frac{\mu_{r_i}}{(1 - f_{s_i d}) f_{s_i r}} \mu_i \quad i \in \{1, 2\} \tag{12}$$

The stable throughput of the system is constrained by the stability of the queues of the source terminals as long as $\mu_i \leq \mu_{u_i}$. The effect of $\alpha$ appears only when the value of $\mu_{u_i}$ is less than $\mu_i$. Let $\alpha_1$ denote the value of $\alpha$ that satisfies $\mu_{u_1} = \mu_1$

$$\alpha_1 = \min\left\{1, \frac{(1-f_{s_1d})f_{s_1r}-p_1 g_{rd}^{s_1} g_{rd}^{s_2}}{p_1(\overline{g_{rd}^{s_1}} g_{rd}^{s_2}+g_{rd}^{s_1}\overline{g_{rd}^{s_2}})+p_2 g_{rd}^{s_2}+p_3 g_{rd}^{s_1}}\right\} \tag{13}$$

and $\alpha_2$ denote the value of $\alpha$ that satisfy $\mu_{u_2} = \mu_2$

$$\alpha_2 = \max\left\{0, 1-\frac{(1-f_{s_2d})f_{s_2r}-p_1 g_{rd}^{s_1} g_{rd}^{s_2}}{p_1(\overline{g_{rd}^{s_1}} g_{rd}^{s_2}+g_{rd}^{s_1}\overline{g_{rd}^{s_2}})+p_2 g_{rd}^{s_2}+p_3 g_{rd}^{s_1}}\right\} \tag{14}$$

Therefore the interesting values of $\alpha$ are between $\alpha_2$ and $\alpha_1$ because above $\alpha_1$ or below $\alpha_2$ the stable throughput, $\lambda_i$, is constant and equals to $\mu_i$.

### 3.2 The Stability Analysis of TDMA Scheme

We follow the same steps as those in the MAC scheme. A packet departs $Q_{s_i}$ in the assigned time slot if it is successfully decoded by at least one node. Thus, the average service rate of $Q_{s_i}$ is given by

$$\mu_i = m_{s_i}(f_{s_id} + f_{s_ir}(1 - f_{s_id})) \tag{15}$$

For $Q_{s_i}$ stability, the following condition must be satisfied

$$\lambda_i < m_{s_i}(f_{s_id} + f_{s_ir}(1 - f_{s_id})) \tag{16}$$

A packet arrives at $Q_{r_i}$ when $s_i$ transmits on the assigned time slot and an outage occurs in the direct link from $s_i$ to the destination node while no outage occurs in the link between $s_i$ and the relay, yet, $Q_{s_i}$ is not empty. Thus, the average arrival rate of $Q_{r_i}$ is given by

$$\lambda_{r_i} = m_{s_i}\left(\frac{\lambda_i}{\mu_i}(1 - f_{s_id})f_{s_ir}\right) \tag{17}$$

A packet departs $Q_{r_i}$ if there is no outage in the link between the relay and the destination. Thus, the average service rate of $Q_{r_i}$ is given by

$$\mu_{r_i} = m_{r_i} f_{rd} \tag{18}$$

For stability of $Q_{r_i}$, $\lambda_{r_i} < \mu_{r_i}$, which yields

$$\lambda_i < \frac{\mu_{r_i}}{m_{s_i}(1 - f_{s_id})f_{s_ir}}\mu_i \tag{19}$$

From (16) and (19), the system is stable if

$$\lambda_i < \min\{\mu_i, \mu_{u_i}\} \tag{20}$$

$$\mu_{u_i} = \frac{\mu_{r_i}}{m_{s_i}(1 - f_{s_id})f_{s_ir}}\mu_i \quad i \in \{1, 2\} \tag{21}$$

It is obvious from (19) that the maximum stable throughput depends on $m_{s_i}$. We formulate an optimization problem to calculate the maximum achievable stable throughput for both source terminals. From (20), $\lambda_i$ is a concave function in the parameter $m_{s_i}$ as it is the minimum between two affine functions, $\mu_i$ and $\mu_{s_i}$ [16]. For the $i$th source terminal, the optimization problem is given by

$$\begin{aligned} \underset{m_{s_i}}{\text{maximize}} \quad & \min\{\mu_i, \mu_{u_i}\} \\ \text{subject to} \quad & 0 \leq m_{s_i} \leq 1 \end{aligned} \tag{22}$$

The optimal solution of this problem, $m^*_{s_i}$, can be easily calculated because $\mu_i$ is monotonically increasing in $m_{s_i}$, while $\mu_{u_i}$ is monotonically decreasing in $m_{s_i}$. Therefore, $m^*_{s_i}$ is obtained at $\mu_i = \mu_{u_i}$ and is given by

$$m^*_{s_i} = \frac{f_{rd}}{f_{s_i r}(1 - f_{s_i d}) + f_{rd}} \tag{23}$$

## 4 Average Delay Characterization

In this section, we present the delay analysis for both cooperative schemes. Then, we investigate the fundamental trade-off between the average delay and the stable throughput for $s_1$ and $s_2$.

### 4.1 Delay Analysis of MAC Scheme

If a packet is directly delivered to the destination then this packet experiences a queueing delay at the source only. This event occurs for the $i$th source with probability $\epsilon_i = \frac{f_{s_i d}}{f_{s_i d} + f_{s_i r} - f_{s_i d} f_{s_i r}}$, which is the the probability that the packet is successfully decoded by the destination given that it is dropped from $Q_{s_i}$. If the first successful transmission for this packet is not to the destination, then the packet experiences two delays; a queuing delay at $Q_{s_i}$ in addition to the queuing delay at $Q_{r_i}$. This event occurs with probability $1 - \epsilon_i$. Therefore, the average delay is given by

$$D_i = T_{s_i} + \ (1 - \epsilon_i)\ T_{r_i} \tag{24}$$

where $T_{s_i}$ and $T_{r_i}$ denote the average queueing delays at $s_i$ and $r_i$, respectively. Since the arrival rates at $Q_{s_i}$ and $Q_{r_i}$ are given by $\lambda_i$ and $\lambda_{r_i}$, respectively, then applying Little's law yields

$$T_{s_i} = N_i/\lambda_i, \ \ T_{r_i} = N_{r_i}/\lambda_{r_i} \tag{25}$$

where $N_i$ and $N_{r_i}$ denote the average queue size of $Q_{s_i}$ and $Q_{r_i}$, respectively. The dominant system, described before, de-couples the interaction between the queues. Thus, we can easily calculate $N_{s_i}$ and $N_{r_i}$ by observing that $Q_{s_i}$ and $Q_{r_i}$ are discrete-time M/M/1 queues with Bernoulli arrivals and geometrically distributed service rates. Then, by applying the Pollaczek-Khinchine formula [17], we obtain $N_i$ and $N_{r_i}$ as

$$N_i = \frac{-\lambda_i^2 + \lambda_i}{\mu_i - \lambda_i}, \quad N_{r_i} = \frac{-\lambda_{r_i}^2 + \lambda_{r_i}}{\mu_{r_i} - \lambda_{r_i}} \tag{26}$$

Substituting (25) and (26) in (24), we can write the average queueing delay for the $i$th source terminal as

$$D_i = \frac{1-\lambda_i}{\mu_i - \lambda_i} + \frac{f_{s_i r}(1-f_{s_i d})}{f_{s_i d} + f_{s_i r} - f_{s_i d} f_{s_i r}} \frac{1-\lambda_{r_i}}{\mu_{r_i} - \lambda_{r_i}} \tag{27}$$

### 4.2 Delay Analysis of TDMA Scheme

Since the source terminals and the relay transmit their packets over orthogonal resources, there is no interaction between the queues. Using exactly the same analysis as that used in the MAC scheme, we can write the average queueing delay for the $i$-th source terminal as

$$D_i = \frac{1-\lambda_i}{\mu_i - \lambda_i} + \frac{f_{s_i r}(1-f_{s_i d})}{f_{s_i d} + f_{s_i r} - f_{s_i d} f_{s_i r}} \frac{1-\lambda_{r_i}}{\mu_{r_i} - \lambda_{r_i}} \tag{28}$$

$$\text{where } \mu_i = m_{s_i}(f_{s_i d} + f_{s_i r}(1-f_{s_i d})) \tag{29}$$

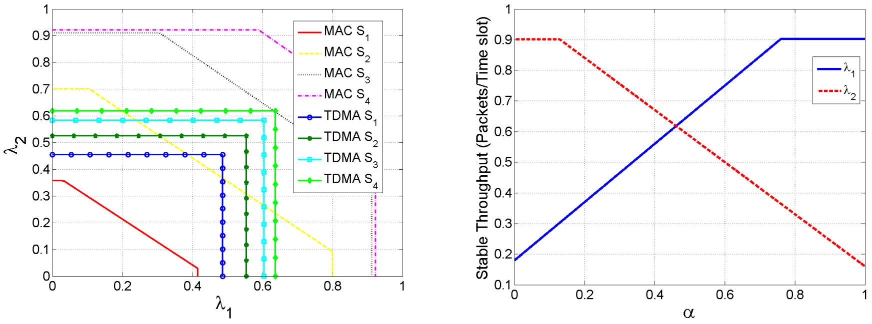
$$\lambda_{r_i} = m_{s_i}\left(\frac{\lambda_i}{\mu_i}(1-f_{s_i d}) f_{s_i r}\right) \tag{30}$$

$$\mu_{r_i} = m_{r_i} f_{rd}, \quad i \in \{1,2\} \tag{31}$$

## 5 Numerical Results

In this section, we investigate the performance of the proposed cooperative schemes. First, we show the effect of changing the direct link channel gain, between the sources and the destination, on the stability region. Next, we demonstrate the effect of varying $\alpha$ on the maximum stable throughput for the MAC scheme. Furthermore, we characterize the fundamental trade-off between the average delay and the stable throughput for both source terminals. Finally, we demonstrate effect of varying $\alpha$ on the delay experienced by the packets of $s_1$ and $s_2$ and validate our results via queue simulation.

In Fig. 2a, we plot the stable throughput region of the studied schemes for different direct link channel conditions. Hereafter, the system parameters are chosen as follows: $P_r = P_{s_i} = 6, R = 1$, $\rho^2_{s_1,r} = 0.8$, $\rho^2_{s_2,r} = 0.86$, and we define four different sets each contains a channel condition for the direct links according to the following: $S_1 = \{\rho^2_{s_1,d} = 0.14, \rho^2_{s_2,d} = 0.1\}$, $S_2 = \{\rho^2_{s_1,d} = 0.2, \rho^2_{s_2,d} = 0.16\}$, $S_3 = \{\rho^2_{s_1,d} = 0.27, \rho^2_{s_2,d} = 0.2\}$, and $S_4 = \{\rho^2_{s_1,d} = 0.32, \rho^2_{s_2,d} = 0.28\}$. In Fig. 2a, it is obvious that the MAC scheme provides the worst performance for both terminals for low direct channel gains. The poor direct link causes slow emptying




(a) Stable throughput region for different direct link channels

(b) Stable throughput for $s_1$ and $s_2$

**Fig. 2.** Stable Throughputs

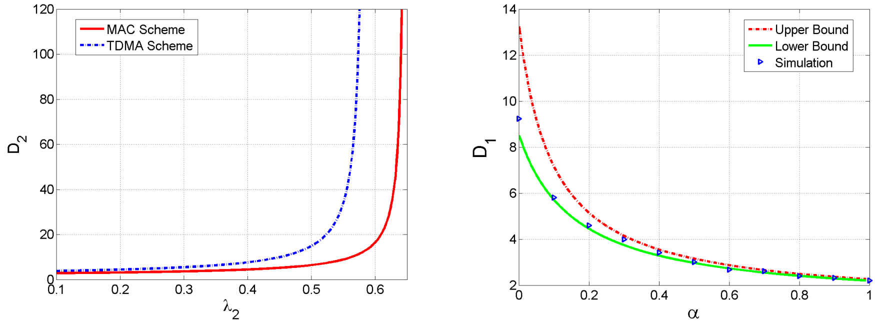
of the source queues and a very few relay transmission opportunities. In this case, it is more efficient to use the TDMA scheme to achieve higher throughput. As the direct link channel gain increases, the performance of the MAC scheme improves and outperforms the TDMA scheme which becomes inefficient due to the division of the available degrees of freedom.

Next, we show the effect of varying $\alpha$ on the stability of the MAC scheme. We use $S_2$ for the direct channel condition. In Fig. 2b, we plot the stable throughput versus $\alpha$ for $s_1$ and $s_2$. Increasing the value of $\alpha$ increases the maximum stable arrival rate at $s_1$ while decreasing $\alpha$ increases the maximum stable arrival rate at $s_2$. This result is intuitive, since increasing the value of $\alpha$ gives more chance for transmitting the packets of $s_1$ at the cooperative queues and this reduces the amount of cooperation that the $s_2$ experiences from the relay.

Using (13) and (14), we can compute the values of $\alpha_1$ and $\alpha_2$ as $\alpha_1$=0.76, $\alpha_2$=0.13. It is clear from the figure that the stable throughput for $s_1$, $\lambda_1$, becomes constant when the value of $\alpha$ exceed $\alpha_1$ because $\mu_{u_1}$ becomes greater than $\mu_1$ which is constant and does not depend on $\alpha$ and this emphasize the results obtained in (11) and (13). It is exactly the same for $s_2$ when the system operates with value of $\alpha$ below $\alpha_2$.

Next, we characterize a fundamental trade-off that arises between the average delay and the stable throughput for $s_1$ and $s_2$. Given that the system is stable, the throughput of any node is equal to its packet arrival rate. Thus, increasing the throughput means injecting more packets into the system which yields a higher delay. In Fig. 3a, we illustrate the delay throughput trade-off for $s_2$. We plot the average delay versus the stable throughput for the proposed cooperative schemes. The system parameters are chosen as follows: $P_r$=$P_{s_i}$=6, $R$=1, $\rho^2_{s_1,r}$=0.8, $\rho^2_{s_2,r}$=0.86, $S_3$, and for a fair comparison we choose $\lambda_1$=0.6, which is the maximum stable stable throughput for the TDMA scheme at $S_3$. The trade-off is obvious where as the throughput increases the delay also increases.

The MAC scheme sustains the stability of the system up to $\lambda_2 \approx 0.65$, while in the TDMA scheme the system is unstable with $\lambda_2 \approx 0.58$. These values appear clearly in Fig. 2a where at $\lambda_1$=0.6 the maximum stable throughput for $s_2$ in the MAC scheme is $\lambda_2 \approx 0.65$ while in the TDMA scheme is $\lambda_2 \approx 0.58$.




(a) Delay-throghput trade-off at $s_2$

(b) Average delay experienced by the packets of $s_1$

**Fig. 3.** Average Delay

Finally, we demonstrate effect of varying $\alpha$ on the delay of the packets of $s_1$ in the MAC scheme. In Fig. 3b, it is clear that the results obtained through simulations are close to the expressions derived in (27). The gap between the upper bound and the queue simulation emerges due to the dominant system where the nodes transmit dummy packets which affect the average delay experienced by the packets. We also introduce a new dominant where the relay transmits its packet over new frequency bands. This dominant system provides a lower bound on the delay of the original system and we can calculate the delay for this dominant system by substituting in (27) by

$$\mu_{r_1} = p_1 f_{rd} + \alpha (p_2 f_{rd} + p_3 f_{rd}) \tag{32}$$

$$\mu_{r_2} = p_1 f_{rd} + \overline{\alpha} (p_2 f_{rd} + p_3 f_{rd}) \tag{33}$$

Moreover, given $\lambda_1$ and $\lambda_2$, it is clear that as $\alpha$ increases $D_1$ decreases and this matches the result stated in (27).

## 6 Conclusion

In this paper, we have proposed a novel randomized MAC cooperative policy where a full duplex relay can efficiently transmit the packets of two source terminals. We have characterized the stable throughput region for the proposed cooperative scheme in addition to a TDMA scheme. The results indicate that the MAC scheme can provide significant gain over the TDMA scheme in the case of high direct channel gain. Moreover, we have also addressed the throughput delay trade-off. The results show that the MAC scheme achieves higher stable throughput than that of the TDMA scheme.

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