Fractional Low Order Cyclostationary-Based Spectrum Sensing in Cognitive Radio Networks

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Abstract. In this paper, we study the problem of cyclostationary spectrum sensing in cognitive radio networks based on cyclic properties of linear modulations. For this purpose, we use fractional order of observations in cyclic autocorrelation function (CAF). We derive the generalized likelihood ratio (GLR) for designing the detector. Therefore, the performance of this detector has been improved compared to previous detectors. We also find optimum value of the fractional order of observations in additive Gaussian noise. The exact performance of the GLR detector is derived analytically as well. The simulation results are presented to evaluate the performance of the proposed detector and compare its performance with their counterpart, so to illustrate the impact of the optimum value of fractional order over performance improvement of these detectors.

Keywords: Cognitive radio · Spectrum sensing · Cyclostationary signal · Fractional low order

1 Introduction

Increasing need for bandwidth in telecommunication and limited environmental resources lead us to take advantage of other system's spectrum. In spectrum sensing, cognitive radio networks monitor the status of the frequency spectrum by observing their surroundings to exploit the unused frequency bands. There are several methods of spectrum sensing which need different and extra information about the primary user (PU) signal, such as accuracy and implementation complexity [1]. The most important methods are matched filter, energy detection, eigenvalues-based detection, detection based on the covariance matrix and cyclostationary based detection.

Among those, cyclostationary-based detector is one of the best way of spectrum sensing in terms of performance and robustness against environmental parameters like ambient noise. In the context of cyclostationary-based spectrum

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sensing, in [2,3], this detector has been investigated for one specific cyclic frequency. The authors in [2] have reviewed collaborative case and have demonstrated channel fading effects in its performance. The authors in [4–6] have used multiple cyclic frequencies for detection of PU signal and improvement the detection performance has been shown. Furthermore, several research such as [2,7,8] have been conducted where the benefit of using cyclostationary-based detectors in the collaborative systems are investigated. It is known that cyclostationary-based detectors have poor performance for situations where the environment is impulsive noisy and to compensate, the CAF with fractional order of observations are used [9–11]. In these works, the problem of fractional order of observations, is investigated in Alpha stable noisy environment.

In this paper, we provide a spectrum sensing method which benefits of PU signal's cyclostationary property and improve performance of cyclostationary-based detector in different practical cases and noise models. We suggest using fractional order of observed signals. We assume an additive Gaussian noise, thought the results could be extended for the other model of ambient noises. For this purpose, we formulate the spectrum sensing as a binary hypothesis testing problem and then derive the corresponding GLR detectors for the different practical scenarios. Then we investigate the optimum value of fractional order which results in best performance in related cases.

The remaining of the paper is organized as follows. In Section 2, we introduce the system model and the assumptions. In Section 3, we derive cyclostationary-based detectors in different scenarios for signal and noise prameters. In Section 4, we study the performance of the proposed detectors. The optimization of the performance of the proposed detectors is presented in Section 6. The simulation results are provided in Section 7 and finally Section 8 summarizes the conclusions.

Notation: Lightface letters denote scalars. Boldface lower-and upper-case letters denote column vectors and matrices, respectively. x(.) is the entries and $\mathbf{x}_i$ is sub-vector of vector $\mathbf{x}$. The inverse of matrix $\mathbf{A}$ is $\mathbf{A}^{-1}$. The $M \times M$ identity matrix is $\mathbf{I}_M$. Superscripts $*$, T and H are the complex conjugate, transpose and Hermitian (conjugate transpose), respectively. $\mathbb{E}[.]$ is the statistical expectation. $\mathcal{N}(\mathbf{m}, \mathbf{P})$ denotes Gaussian distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{P}$. $Q(x)$ is Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty exp\left(\frac{-u^2}{2}\right) du$.

2 System Model

Suppose a cognitive radio network in which PU and secondary user (SU) equipped with a single antenna. For presentation, it's assumed that the PU signal is transmitted with linear modulation such that

$$s(t) = \sum_{i=-\infty}^{\infty} d_i p(t - iT_{\mathrm{P}}), \tag{1}$$

where d_i is the PU data and $p(t)$ is shaping pulse in the PU transmitter. We suppose PU data, d_i, is a random variable with zero-mean Gaussian distribution, $\mathcal{N}(0, \sigma_s^2)$. For the shaping pulse, a rectangular pulse with unit amplitude and time spread T_P is assumed. Received signal in SU has been sampled with sampling rate of $f_s = \frac{1}{T_s}$. The wireless channel between PU transmitter and SU is assumed to be a flat fading channel with additive Gaussian noise and the channel gain. The random variable $w(n) \sim \mathcal{N}(0, \sigma_w^2)$ denotes noise samples and we assume noise and PU signal samples are mutually independent. Therefore observed signal samples in SU under two hypotheses can be shown as follows,

$$\begin{cases} \mathcal{H}_0: & x(n) = w(n), \\ \mathcal{H}_1: & x(n) = hs(n) + w(n), \end{cases} \tag{2}$$

where h is channel gain between the PU and SU antennas. It is assumed that the channel gain is constant during the sensing time. CAF for the SU observed signal samples is defined based on the correlation between samples and their complex conjugate with lag time $\tau_i < T_\mathrm{P}$. The CAF for fractional order is defined as,

$$R_{xx^*}^{\alpha}(\tau_i) = \frac{1}{N} \sum_{n=0}^{N-1} x^p(n) x^{*p}(n+\tau_i) e^{-j2\pi\alpha n}, \tag{3}$$

where p is fractional order $0 < p < 1$, $\alpha \in \{\frac{k}{T_\mathrm{P}}, k = 1, 2, ...\}$ is cyclic frequency for linear modulation which is assumed to be known to SU and $\tau_i, i = 1, \ldots, M$s is M lag times where the CAF is calculated.

We introduce vector $\mathbf{r}_{xx^*}^{\alpha}$ consisting of CAF real parts for M different lag times as,

$$\mathbf{r}_{xx^*}^{\alpha} = [Re(R_{xx^*}^{\alpha}(\tau_1)), ..., Re(R_{xx^*}^{\alpha}(\tau_M))]^T. \tag{4}$$

By considering central limit theorem (CLT), since the CAF is summation of N random variables, according to [12], for sufficiently large number of observation samples, each member of vector $\mathbf{r}_{xx^*}^{\alpha}$ has Gaussian distribution . Thus, we have,

$$\mathbf{r}_{xx^*}^{\alpha} \sim \begin{cases} \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0) & \text{for} \quad \mathcal{H}_0, \\ \mathcal{N}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1) & \text{for} \quad \mathcal{H}_1. \end{cases} \tag{5}$$

where $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ can be calculated for any given p. In Section 5 for the known noise and signal variance these values are computed.

3 Cyclostationary-Based Detectors

SUs use different detection methods in spectrum sensing to make decision about PU's presence. In this section, we assume SU determines PUs situation based on cyclostationary properties of PU signal in which the SU has knowledge about cyclic frequency of observation signal by consideration of different scenarios. These scenarios are investigated in following subsections.

3.1 Known Signal and Noise Variance

Since in (5) covariance matrices under two hypotheses are unknown, we have to use their estimations to construct the likelihood ratio (LR) function which results in a GLR detector. Covariance matrices estimation have been calculated under two hypotheses in Appendix. It has been shown that both of the covariance matrices have same estimation. Thus, $\widehat{\boldsymbol{\Sigma}_0} = \widehat{\boldsymbol{\Sigma}_1} = \boldsymbol{\Sigma}$. Now for the LR function, we have,

$$LR(\mathbf{r}^{\alpha}_{xx^*}) = \exp\{\boldsymbol{\mu}_0^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_0 - \boldsymbol{\mu}_1^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_1 + 2\mathbf{r}^{\alpha^T}_{xx^*} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)\} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta. \tag{6}$$

By incorporating the constant terms into threshold and taking logarithm in (6), we obtain,

$$\mathrm{T}_{sub1} = \mathbf{r}^{\alpha^T}_{xx^*} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1, \tag{7}$$

where $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ can be calculated. It can be seen that detector is the weighted summation of CAF real part for different lag times $\tau_i, i = 1, 2, ..., M$.

3.2 Known Noise Variance, Unknown Signal Variance

The mean of (4), when SU has just knowledge about noise variance, can be derived under null hypothesis according to section 5.1. But as mentioned, signal variance is unknown and thus, mean of the CAF real parts under alternative hypothesis cannot be calculated. In this situation, we can use Hotelling-test [13,17], because we definitely know that the mean under two hypotheses are different. Suppose, $L > M + 1$ given vector $\mathbf{r}^{\alpha}_{xx^*}$ in a vector are considered together, $\mathbf{r} = [\mathbf{r}^{\alpha}_{xx^*}(1), \mathbf{r}^{\alpha}_{xx^*}(2), ..., \mathbf{r}^{\alpha}_{xx^*}(L)]$. Statistical distribution of this vector under hypothesis $\mathcal{H}_j, j = 0, 1$ can be written in the form below,

$$f(\mathbf{r}|\mathcal{H}_j) = \frac{\exp\{-\frac{1}{2} tr([\frac{1}{L}\boldsymbol{\Psi} + (\bar{\mathbf{r}} - \boldsymbol{\mu}_j)(\bar{\mathbf{r}} - \boldsymbol{\mu}_j)^T] \boldsymbol{\Sigma}_j^{-1})\}}{(2\pi)^{\frac{LM}{2}} |\boldsymbol{\Sigma}_j|^{\frac{L}{2}}}, \tag{8}$$

where $\bar{\mathbf{r}} = \frac{1}{L}\sum_{i=1}^{L} \mathbf{r}^{\alpha}_{xx^*}(i)$ and $\boldsymbol{\Psi} = \sum_{i=1}^{L} (\mathbf{r}^{\alpha}_{xx^*}(i) - \bar{\mathbf{r}})(\mathbf{r}^{\alpha}_{xx^*}(i) - \bar{\mathbf{r}})^T$, under alternative hypothesis, $\bar{\mathbf{r}}$ is estimate of $\boldsymbol{\mu}_1$ and the statement inside the bracket of function $tr(.)$ is the estimation covariance matrix under two hypotheses. Thus after eliminating the constants we have,

$$\Lambda = \frac{|\frac{1}{L}(\boldsymbol{\Psi} + L(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)^T)|^{\frac{L}{2}}}{|\frac{1}{L}\boldsymbol{\Psi}|^{\frac{L}{2}}} = |\mathbf{I} + L\boldsymbol{\Psi}^{-1}(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)^T|^{\frac{L}{2}}. \tag{9}$$

By using the matrix determinant lemma that computes the determinant of the sum of an invertible matrix $\mathbf{I}$ and the dyadic product, $\boldsymbol{\Psi}^{-1}(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)^T$,

$$\Lambda = \left(1 + L(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)^T \boldsymbol{\Psi}^{-1} (\bar{\mathbf{r}} - \boldsymbol{\mu}_0)\right)^{\frac{L}{2}} = (1 + \mathrm{T}_{sub2})^{\frac{L}{2}}. \tag{10}$$

Since Λ is the strictly ascending function of T_{sub2}, therefore, T_{sub2} can be considered as a statistic.

$$\mathrm{T}_{sub2} = L(\bar{\mathbf{r}} - \boldsymbol{\mu}_0)^T \boldsymbol{\Psi}^{-1} (\bar{\mathbf{r}} - \boldsymbol{\mu}_0) \tag{11}$$

3.3 Unknown Signal and Noise Variance

In this situation, by considering covariance matrices estimation as (A-4), we have two Gaussian distribution by same covariance matrices and different mean under two hypotheses. If estimation is used for means of CAF real parts under both hypotheses, due to equality of estimation under two hypotheses the result of GLR test does not give any information to make decision. Thus, mean of CAFs for various lag time is considered as statistic and compared with a proper threshold.

$$\mathrm{T}_{sub3} = \frac{1}{M} \sum_{m=1}^{M} Re(R_{xx^*}^{\alpha}(\tau_m)) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_3. \tag{12}$$

4 Analytical Performance

In this section, we evaluate the performance of our proposed cyclostationary-based detectors in terms of detection and false alarm probabilities, P_{d} and P_{fa}, respectively.

4.1 Analytical Performance of $\mathbf{T}_{sub1}$

We should derive statistical distribution of (7) under two hypotheses. We can rewrite (7) as follows,

$$\mathrm{T}_{sub1} = (\mathbf{r}_{xx^*}^{\alpha^T} \boldsymbol{\Sigma}^{-\frac{1}{2}})(\boldsymbol{\Sigma}^{-\frac{1}{2}}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)) = \widetilde{\mathbf{r}}_{xx^*}^{\alpha^T} \mathbf{w} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_1, \tag{13}$$

where $\mathbf{w} = \boldsymbol{\Sigma}^{-\frac{1}{2}}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$ and $\widetilde{\mathbf{r}}_{xx^*}^{\alpha} = \boldsymbol{\Sigma}^{-\frac{1}{2}} \mathbf{r}_{xx^*}^{\alpha}$ which is distributed as Gaussian under two hypotheses, i.e.,

$$\widetilde{\mathbf{r}}_{xx^*}^{\alpha} | \mathcal{H}_\nu \sim \mathcal{N}(\mathbf{m}_\nu, \mathbf{I}_M), \ \nu = 0, 1, \tag{14}$$

where $\mathbf{m}_\nu = \boldsymbol{\Sigma}^{-\frac{1}{2}} \boldsymbol{\mu}_\nu$. As we can see in (13), our detector is a linear combination of independent Gaussian random variables mentioned in (14). Therefore, mean of statistic is,

$$\mu_{\mathrm{T}_{sub1}|\mathcal{H}_\nu} = \sum_{i=1}^{M} m_\nu(i) w(i), \ \nu = 0, 1. \tag{15}$$

And similarly variance has been derived,

$$\sigma^2_{\mathrm{T}_{sub1}|\mathcal{H}_\nu} = \sum_{i=1}^{M} w^2(i),\ \ \nu = 0, 1. \tag{16}$$

Then, the false alarm and detection probabilities can be calculated.

$$P_{\mathrm{fa}} = P\left[\mathrm{T}_{sub1} > \eta_1 | \mathcal{H}_0\right] = Q\left(\frac{\eta_1 - \mu_{\mathrm{T}_{sub1}|\mathcal{H}_0}}{\sigma_{\mathrm{T}_{sub1}|\mathcal{H}_0}}\right) \tag{17}$$

If β is maximum acceptable probability false alarm, then threshold of detector can be set, $\eta_1 = F^{-1}_{\mathrm{T}_{sub1}|\mathcal{H}_0}(\beta) = Q^{-1}(\beta) \times \sigma_{\mathrm{T}_{sub1}|\mathcal{H}_0} + \mu_{\mathrm{T}_{sub1}|\mathcal{H}_0}$. Similarly for probability of detection, we have,

$$P_{\mathrm{d}} = P\left[\mathrm{T}_{sub1} > \eta_1 | \mathcal{H}_1\right] = Q\left(\frac{\eta_1 - \mu_{\mathrm{T}_{sub1}|\mathcal{H}_1}}{\sigma_{\mathrm{T}_{sub1}|\mathcal{H}_1}}\right). \tag{18}$$

4.2 Analytical Performance of $\mathbf{T}_{sub2}$

We should derive statistical distribution of (11) under two hypotheses. According to [13], the asymptotic distribution of (11) under null hypothesis is central chi-squared with M degrees of freedom. Thus, probability of false alarm is as follows,

$$P_{\mathrm{fa}} = P\left[\mathrm{T}_{sub2} > \eta_2 | \mathcal{H}_0\right] = 1 - \frac{\gamma\left(\frac{M}{2}, \frac{\eta_2}{2}\right)}{\Gamma\left(\frac{M}{2}\right)}, \tag{19}$$

where $\Gamma(.)$ and $\gamma(.,.)$ are Gamma and lower incomplete Gamma function, respectively. The asymptotic distribution of (11) under alternative hypothesis is non-central chi-squared with noncentrality parameter, λ. Probability of detection is as follows,

$$P_{\mathrm{d}} = P\left[\mathrm{T}_{sub2} > \eta_2 | \mathcal{H}_1\right] = Q_{\frac{M}{2}}(\sqrt{\lambda}, \sqrt{\eta_2}), \tag{20}$$

where $Q(.,.)$ is Marcum Q-function and non-centrality parameter is, $\lambda = \frac{L}{2}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)^T \boldsymbol{\Sigma}_1^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$.

4.3 Analytical Performance of $\mathbf{T}_{sub3}$

Because (12) is a linear combination of Gaussian random variables, therefore, T_{sub3} distribution is Gaussian under two hypotheses. According to Appendix 8, mean and variance of (12) can be calculated. Thus, probability of false alarm and detection are as follow,

$$P_{\mathrm{fa}} = Q\left(\frac{\eta_3 - \mu_{\mathrm{T}_{sub3}|\mathcal{H}_0}}{\sigma_{\mathrm{T}_{sub3}|\mathcal{H}_0}}\right), \tag{21}$$

$$P_{\mathrm{d}} = Q\left(\frac{\eta_3 - \mu_{\mathrm{T}_{sub3}|\mathcal{H}_1}}{\sigma_{\mathrm{T}_{sub3}|\mathcal{H}_1}}\right). \tag{22}$$

5 Calculation of $\mathbf{r}_{xx^*}^{\alpha}$ Means

In this section, we have provided computations for expectation of $\mathbf{r}_{xx^*}^{\alpha}$ under two hypotheses when all variables are known.

5.1 Null Hypothesis

In this subsection, we investigate mean of $\mathbf{r}_{xx^*}^{\alpha}$ under null hypothesis. By consideration of noise samples independency, expectation of (3) can be easily derived for ith lag time as follows,

$$\mathbb{E}[R_{xx^*}^{\alpha}(\tau_i)|\mathcal{H}_0] = \frac{1}{N}\sum_{n=0}^{N-1}\mathbb{E}[w^p(n)]\mathbb{E}[w^{*p}(n+\tau_i)]e^{-j2\pi\alpha n}. \tag{23}$$

pth moment of Gaussian random variable has been calculated in Appendix, since $w(n)$ is zero mean Gaussian random variable, therefore,

$$\mathbb{E}[R_{xx^*}^{\alpha}(\tau)|\mathcal{H}_0] = \frac{e^{-j\pi\alpha(N-1)}}{N}\frac{sin(\pi\alpha N)}{sin(\pi\alpha)}\frac{(-2)^p\pi\sigma_n^{2p}}{\Gamma^2\left(\frac{1-p}{2}\right)}. \tag{24}$$

Mean of (4) for $i = 1, .., M$,

$$\mu_0(i) = \frac{sin(\pi\alpha N)}{Nsin(\pi\alpha)}\frac{\pi(2\sigma_n^2)^p}{\Gamma^2\left(\frac{1-p}{2}\right)}cos(\pi(\alpha(1-N)+p)). \tag{25}$$

5.2 Alternative Hypothesis

As mentioned earlier, each of the observation samples at SU is distributed as,

$$X = x(n) \sim \mathcal{N}(0, h^2p^2\sigma_s^2 + \sigma_n^2) \triangleq \mathcal{N}(0, \sigma_1^2). \tag{26}$$

Now, we assume random variable Y to be the ith lag time of observation samples which is distributed same as X, i.e., $Y = x(n+\tau_i)$. It can be easily demonstrated that correlation coefficient between X and Y is,

$$r = \frac{\mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)}{\sigma_1 \times \sigma_1} = \frac{h^2}{\sigma_1^2}\mathbb{E}[s(t)s(t+\tau_i)] = \frac{h^2p^2\sigma_s^2}{\sigma_1^2}, \tag{27}$$

which reveals that X and Y are correlated. Thus, X and Y have joint Gaussian distribution, $\mathcal{N}(0, 0, \sigma_1^2, \sigma_1^2, r)$. To determine the mean of CAF under alternative hypothesis, we need to calculate $\mathbb{E}[X^pY^p] = \mathbb{E}[Z^p] = \mathbb{E}[T]$. First we must derive probability density function (PDF) of Z which is product X and Y. i.e.,

$$f_Z(z) = \int_0^{\infty}\frac{1}{x}f_{XY}(x, \frac{z}{x})dx - \int_{-\infty}^{0}\frac{1}{x}f_{XY}(x, \frac{z}{x})dx. \tag{28}$$

$$\frac{(x\sigma_1\sqrt{2(1-r^2)})^p}{j^p\sqrt{2\sigma_1^2}} e^{\left(-\frac{x^2}{2\sigma_1^2(1-r^2)}\right)} \sum_{k=0}^{\infty} [\frac{\left(\frac{-p}{2}\right)^{\overline{k}}}{\Gamma\left(\frac{1-p}{2}\right)\left(\frac{1}{2}\right)^{\overline{k}} k!} - \frac{\sqrt{2} jrx \left(\frac{1-p}{2}\right)^{\overline{k}}}{\Gamma\left(-\frac{p}{2}\right)\sigma_1\sqrt{1-r^2}\left(\frac{3}{2}\right)^{\overline{k}} k!}] \times$$
$$(-\frac{r^2x^2}{2\sigma_1^2(1-r^2)})^k = \sum_{k=0}^{\infty}\left[A(r,\sigma_1,k,p)x^{2k+p} - B(r,\sigma_1,k,p)x^{2k+p+1}\right] e^{\left(-\frac{x^2}{2\sigma_1^2(1-r^2)}\right)} \tag{33}$$

In second step, we can declare distribution of T as function of Z PDF, as follows,

$$f_T(t) = \frac{1}{p} t^{\frac{1}{p}-1} f_Z(t^{\frac{1}{p}}). \tag{29}$$

And thus, for computation of T mean, we have,

$$\mathbb{E}[T] = \int_0^{\infty}\int_{-\infty}^{\infty} \frac{t^{\frac{1}{p}}}{px} f_{XY}(x, \frac{t^{\frac{1}{p}}}{x}) dtdx - \int_{-\infty}^{0}\int_{-\infty}^{\infty} \frac{t^{\frac{1}{p}}}{px} f_{XY}(x, \frac{t^{\frac{1}{p}}}{x}) dtdx. \tag{30}$$

Common part of above equation is derived in following expression,

$$\int_{-\infty}^{\infty} \frac{t^{\frac{1}{p}}}{px} f_{XY}(x, \frac{t^{\frac{1}{p}}}{x}) dt = \frac{\exp\left(-\frac{x^2}{2\sigma_1^2}\right)}{px2\pi\sigma_1^2\sqrt{1-r^2}} \int_{-\infty}^{\infty} t^{\frac{1}{p}} \exp\left\{-\frac{\left(t^{\frac{1}{p}} - rx^2\right)^2}{2x^2\sigma_1^2(1-r^2)}\right\} dt. \tag{31}$$

Integral expression in equation (31) is in the form of p-th moment of Gaussian random variable with respectively mean and variance rx^2 and $x^2\sigma_1^2(1-r^2)$ that is calculated in Appendix. Therefore,

$$\int_{-\infty}^{\infty} \frac{t^{\frac{1}{p}}}{px} f_{XY}(x, \frac{t^{\frac{1}{p}}}{x}) dt = \frac{(x\sigma_1\sqrt{(1-r^2)})^p}{j^p\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(2-r^2)x^2}{4\sigma_1^2(1-r^2)}\right) D_p\left(\frac{jrx}{\sigma_1\sqrt{1-r^2}}\right). \tag{32}$$

Result of replacement Apendix equations in (32) also some calculations and simplifications, has led to (33), which is at the top of next page. In (33),

$$A(r,\sigma_1,k,p) = \frac{\left(\frac{-p}{2}\right)^{\overline{k}} (\sigma_1\sqrt{2(1-r^2)})^p r^{2k}}{\Gamma\left(\frac{1-p}{2}\right)\left(\frac{1}{2}\right)^{\overline{k}} k!\sqrt{2\sigma_1^2} j^p (2\sigma_1^2(r^2-1))^k}, \tag{34}$$

$$B(r,\sigma_1,k,p) = \frac{\sqrt{2^{p+1}}\left(\frac{1-p}{2}\right)^{\overline{k}} (\sigma_1\sqrt{(1-r^2)})^{p-2k-1} r^{2k+1}}{\Gamma\left(-\frac{p}{2}\right)\left(\frac{3}{2}\right)^{\overline{k}} k! j^{p-1}(-2)^k}. \tag{35}$$

Finally, from (36) and according to [14], mean of T is derived in the next page. Therefore, ith member of $\boldsymbol{\mu}_1$ for $i = 1, ..., M$ is,

$$\mu_1(i) = \frac{sin(\pi\alpha N)}{N sin(\pi\alpha)} cos(\pi\alpha(N-1))\mathbb{E}[T] \tag{37}$$

$$\mathbb{E}[T] = \sum_{k=0}^{\infty}(1+(-1)^p)\Bigg[A(r,\sigma_1,k,p)2^{2k+p+1}(2\sigma_1^2(1-r^2))^{\frac{2k+p+1}{2}}\frac{\Gamma(2k+p+1)\sqrt{\pi}}{\Gamma(\frac{2k+p+2}{2})} - B(r,\sigma_1,k,p)2^{2k+p+2}(2\sigma_1^2(1-r^2))^{\frac{2k+p+2}{2}}\frac{\Gamma(2k+p+2)\sqrt{\pi}}{\Gamma(\frac{2k+p+3}{2})}\Bigg] \quad (36)$$

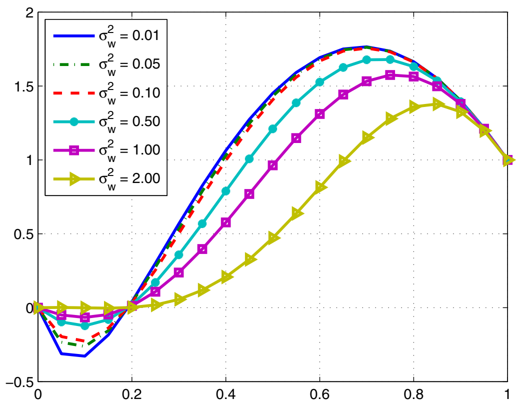

Fig. 1. Normalized difference of means for ith lag time

6 Performance Optimization

To optimize the performance of proposed detector and obtain an appropriate threshold by using the Neyman-Pearson criterion, we have to maximize the probability of detection respect to fractional order of observations, p. The difference between the null and alternative is just in the mean value while their covariance matrix is estimated to be similar. Therefore, since $\mathbf{r}_{xx^*}^{\alpha}$ has Gaussian distribution, for maximizing the probability of detection, statistical means difference between two hypotheses should be maximized.

$$p = \arg \max_{0<\mathrm{p}<1} \{\mu_1(\mathrm{i}) - \mu_0(\mathrm{i})\}, \quad (38)$$

where i denotes ith lag time.

Therefore, for a specific value of p, if the difference between the means of null and alternative hypotheses is maximized, it can be concluded that the performance has improved. Due to complex relations obtained for the means in (25) and (37), differentiation and solve the result of its equation for this purpose is not possible, however, with the help of numerical results, we can obtain the optimal amount of fractional order, p.

In Fig. 1, difference of means under two hypotheses for a certain lag time is plotted versus changes of p for various value of noise variance, σ_w^2. In this figure,

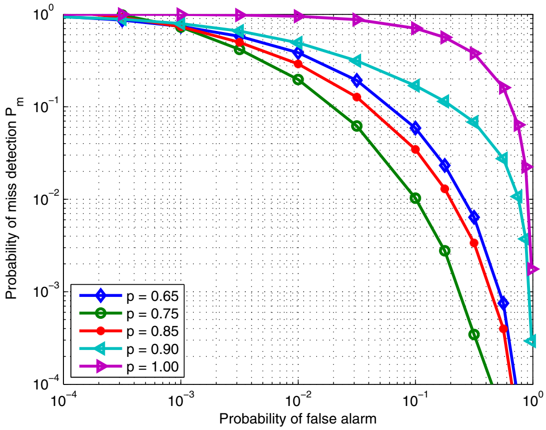

Fig. 2. The complementary ROC of proposed detector for average $SNR = -3dB$.

the values are normalized with respect to means difference value in $p = 1$ which is used in cyclostationary detectors. As can be seen in Fig. 1, for example, the difference of means increases about 0.75 percent in $p = 0.75$ for $\sigma_w^2 = 1$ and also for other value of noise variance, we can found specific p that improves the detector performance.

7 Simulation Results

In this section, we provide simulation results of cyclostationary-based detectors performance in fractional order of observations Monte Carlo simulation and we compared it with other detectors. For this purpose, we assume a linear modulation for PU signals which its pulse width for outgoing data is $1ms$. This signal has Gaussian distribution with unit variance which has been sampled in receiver. To detect these signals that affected by environmental additive Gaussian noise, we have used cyclostationary detector in fractional order of observations. Also, we assume the number of lag times is 16.

In Fig. 4, performance of this detector has been investigated in orders of $p = 0.65, 0.75, 0.85, 0.9$ and 1, with the probability of detection P_d versus SNR with assumption $\sigma_w^2 = 1$ and fixed probability of false alarm 0.01. As can be seen, by changing the fractional orders, the detector performance will changes and when the value get close to 0.75, detector performance improves approximately $3dB$ compared to $p = 1$ has been used used in previous detectors. This change and improvement is due to an increase in mean difference of observations under the two hypotheses.

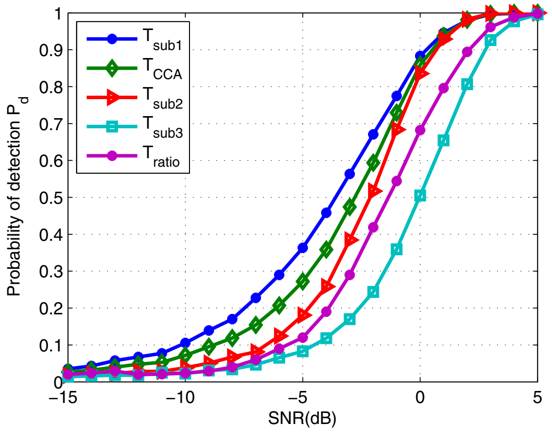

Fig. 3. The probability of detection of different detectors versus SNR for $P_{\mathrm{fa}} = 0.01$.

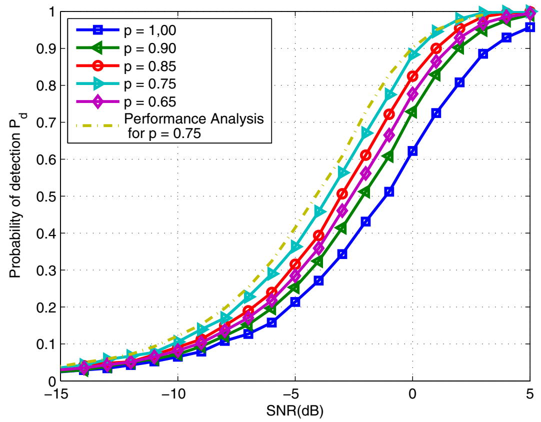

Fig. 4. The probability of detection of T_{sub1} versus SNR for $P_{\mathrm{fa}} = 0.01$ and various fractional moment with assumption $\sigma_w^2 = 1$.

Fig. 2 depicts the receiver operating characteristics (ROC) curve of proposed cyclostationary detector for different fractional order of observations. This figure reveals of the detector behavior for different values of the false alarm probability P_{fa}.

In Fig. 3, performance of detectors has been investigated by the probability of detection P_{d} versus SNR with assumption $\sigma_w^2 = 1$ and fixed probability of false alarm 0.01. This figure compares performance of obtained GLR-based detectors with detectors that are mentioned in [15,16]. In [15], the ratio of CAF absolute value in cyclic frequency and another amount has been proposed as detector, $\mathrm{T}_{ratio} = \left|\frac{R^{\alpha}_{xx^*}(\tau)}{R^{\alpha+\delta}_{xx^*}(\tau_i)}\right|$, where δ is a frequency shift. In [16], authors by using canonical correlation analysis to detect presence of PU signal for M antennas SU. If λ_m is mth eigenvalue of canonical correlation analysis result, statistic is defined as, $\mathrm{T}_{CCA} = \sum_{m=1}^{M} \ln(1-\lambda_m^2)$. As we expected, when noise and signal variance are known, the best performance of the detector can be achieved.

8 Conclusion

In this paper, we investigated the problem of cyclostationary spectrum sensing in cognitive radio networks based on cyclic properties of linear modulated signal. First, we derived GLR detector for the situation in which SU has knowledge of cyclic frequency of signal. Then, we found the optimum value for fractional moment of observations in additive Gaussian noise and the exact performance of the GLR detector is evaluated analytically. Finally, we simulated and derived the GLR detector performance for various values of fractional moment of observations. We revealed that GLR detector performance improves for Gaussian noise if we use fractional moment of observation for any value of noise variance. We found the optimum value for the fractional moment, p. Our results have been confirmed by simulation.

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Appendix

Covariance Matrices Estimation

According to [14], in order to calculate of correlation between two lag times mth and nth of CAF, we need,

$$S_{x_{\tau_m}x_{\tau_n}}(2\alpha,\alpha) = \frac{1}{T}\sum_{s=-\frac{T-1}{2}}^{\frac{T-1}{2}} W(s)F_{\tau_n}(\alpha - \frac{2\pi s}{N})F_{\tau_m}(\alpha + \frac{2\pi s}{N}), \tag{A-1}$$

$$S^*_{x_{\tau_m}x_{\tau_n}}(0,-\alpha) = \frac{1}{T}\sum_{s=-\frac{T-1}{2}}^{\frac{T-1}{2}} W(s)F^*_{\tau_n}(\alpha + \frac{2\pi s}{N})F_{\tau_m}(\alpha + \frac{2\pi s}{N}). \tag{A-2}$$

Where $S_{x_{\tau_m}x_{\tau_n}}(2\alpha,\alpha)$ and $S^*_{x_{\tau_m}x_{\tau_n}}(0,-\alpha)$, respectively are unconjugated and conjugated cyclic-spectrum of observations and

$$F_\tau(\omega) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x^p(n)x^{*p}(n+\tau)e^{-j\omega n}. \tag{A-3}$$

Thus, covariance matrix estimation of vector $\mathbf{r}_{xx^*}^{\alpha}$ can be calculated as,

$$[\mathbf{\Sigma}]_{i,j} = Re\{\frac{S_{x_{\tau_i}x_{\tau_j}}(2\alpha,\alpha) + S^*_{x_{\tau_i}x_{\tau_j}}(0,-\alpha)}{2}\}, i,j = 1,2,...,M. \quad \text{(A-4)}$$

*p*th Moment of Gaussian Random Variable

Suppose N is a Gaussian random variable with mean μ and variance σ_n^2. Thus,

$$\mathbb{E}[N^p] = \frac{2^{\frac{p}{2}}\sigma_n^p e^{-\frac{\mu^2}{2\sigma_n^2}}}{\sqrt{\pi}j^p}\int (jt)^p e^{\left(-t^2 - j\frac{\sqrt{2}\mu j}{\sigma_n}t\right)}dt. \quad \text{(B-1)}$$

By assumption of $\beta^2 = 1$ and $q = \frac{\sqrt{2}\mu j}{\sigma_n}$ in section 3.462 of [16], (B-1) has been calculated for $p > -1$ as follows,

$$\mathbb{E}[N^p] = \frac{2^{\frac{p}{2}}\sigma_n^p e^{-\frac{\mu^2}{2\sigma_n^2}}}{\sqrt{\pi}j^p}\left[2^{-\frac{p}{2}}\sqrt{\pi}e^{\frac{\mu^2}{4\sigma_n^2}}D_p\left(\frac{j\mu}{\sigma_n}\right)\right] = \frac{\sigma_n^p e^{-\frac{\mu^2}{4\sigma_n^2}}}{j^p}D_p\left(\frac{j\mu}{\sigma_n}\right), \quad \text{(B-2)}$$

where $D_p(.)$ is parabolic cylinder function,

$$D_p(z) = 2^{\frac{p}{2}}e^{\frac{-z^2}{4}}\left[\frac{\sqrt{\pi}}{\Gamma\left(\frac{1-p}{2}\right)}\Phi\left(-\frac{p}{2},\frac{1}{2};\frac{z^2}{2}\right) - \frac{\sqrt{2\pi}z}{\Gamma\left(-\frac{p}{2}\right)}\Phi\left(\frac{1-p}{2},\frac{3}{2};\frac{z^2}{2}\right)\right], \quad \text{(B-3)}$$

and also $\Phi(.,.;.)$ is Kummer confluent hypergeometric function, $\Phi(a,b;c) = \sum_{k=0}^{\infty}\frac{a^{\overline{k}}}{b^{\overline{k}}}\frac{c^k}{k!}$.Where, $a^{\overline{k}}$ is rising factorial function, $a^{\overline{k}} = \frac{\Gamma(a+k)}{\Gamma(a)}$.

Mean and Variance of (12)

Mean of (12) under two hypotheses is,

$$\mu_{\mathrm{T}_{sub3}|\mathcal{H}_\nu} = \frac{1}{M}\sum_{m=1}^{M}\mu_\nu(m), \nu = 0,1, \quad \text{(C-1)}$$

and variance of (12) can be calculated as follows,

$$\sigma^2_{\mathrm{T}_{sub3}|\mathcal{H}_\nu} = \frac{1}{M^2}\sum_{m_1=1}^{M}\sum_{m_2=1}^{M}\mathbb{E}[r_{xx^*}^{\alpha}(m_1)r_{xx^*}^{\alpha}(m_2)|\mathcal{H}_\nu] - \mu_\nu(m_1)\mu_\nu(m_2). \quad \text{(C-2)}$$

Therefore, variance of (12) is sum of (A-4) entries.

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