Self-adaptive Traits in Collective Adaptive Systems

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Abstract. An adaptive system is currently on spot: collective adaptive system (CAS), which is inspired by the socio-technical systems. In CASs, highest degree of adaptation is self-adaptation consisting of *self-adaptive traits*. The overarching goal of CAS is to realize systems that are tightly entangled with humans and social structures. Meeting this grand challenge of CASs requires a fundamental approach to the notion of self-adaptive trait. To this end, taking advantage of the coinductive approach we construct self-adaptation monoid to shape series of self-adaptive traits in CASs and some significant relations.

Keywords: Adaptedness · Bisimulation · Coinduction · Collective adaptive system · Equivalence · Self-adaptation · Self-adaptive trait · Series

1 Introduction

The socio-technical structure of our community increasingly depends on systems, which are built as a collection of varied agents and are tightly coupled with humans and social interrelations. Their agents more and more need to be able to develop, cooperate and work all by themselves as a part of an artificial community. Hence, for such collective adaptive systems (CASs), one of major challenges is how to support self-adaptation in the face of changing interactions [5,6]. In other words, how does a CAS understand relevant interrelations and then self-adapt to become better able to live in its interactions?

Dealing with this grand challenge of CASs requires a well-founded modeling and in-depth analysis on the notion of *self-adaptive trait*. With this aim, we construct self-adaptation monoid to shape series of self-adaptive traits in CASs, then we justify the equivalence between two series of self-adaptive traits based on a powerful method so-called *proof principle of coinduction*.

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2 Outline

The paper is a reference material for readers who already have a basic understanding of CAS and are now ready to know the novel approach for constructing self-adaptive traits in CAS using coinduction [3].

Construction is presented in a straightforward fashion by discussing in detail the necessary components and briefly touching on the more advanced components. Several notes explaining how to use the notions, including justifications needed in order to achieve the particular results, are presented.

We attempt to make the presentation as self-contained as possible, although familiarity with the notion of self-adaptive trait in CAS is assumed. Acquaintance with the algebra and the associated notion of coinduction is useful for recognizing the results, but is almost everywhere not strictly necessary.

The rest of this paper is organized as follows: Sections 3 and 4 present the notions of collective adaptive systems (CASs) and self-adaptive trait, respectively. In section 5, self-adaptation monoid is constructed. In section 6, series of self-adaptive traits in CASs is developed in detail. Finally, a short summary is given in section 7.

3 Collective Adaptive Systems (CASs)

We define collective adaptive systems (CASs) as the following among various definitions that have been offered by different researchers:

Definition 1. CASs *are systems that consist of a collective of heterogeneous components, often called agents, that interact and adapt or learn.*

Hence, CASs are characterized by a high degree of adaptation, giving them resilience in the face of perturbations. We see that, in CASs, highest degree of adaptation is *self-adaptation* and we are interested in approaches to this characteristic of CASs.

This definition is concerned with three major factors of CAS:

- *A collective of heterogeneous agents* is large enough to build up systems that are tightly entangled with humans and social structures. Their agents increasingly need to be able to evolve, collaborate and function as a part of an artificial society. More importantly, the agents interact dynamically, and their interactions are either physical or involving the exchange of information.
- *Interactions* are rich, non-linear and primarily, but not exclusively, with immediate neighbors. They can be recurrent, i.e. any interaction can feed back onto itself directly or after a number of intervening stages. CASs are dynamic networks of interactions
- *Self-adaptation* is the self-evolutionary process whereby a CAS becomes better able to live in its interactions.

4 Self-adaptive Trait

An interesting aspect of CASs is that it makes distinction between self-adaptation (i.e. system-driven personalization and modifications) and self-adaptability (i.e. user-driven personalization and modifications). *Self-adaptedness* is the state of being self-adapted, i.e. the degree to which a CAS is able to live and reproduce in a given set of interactions. *Self-adaptive trait* is an aspect of the developmental pattern of the CAS which enables or enhances the probability of that CAS surviving and reproducing.

Hence, self-adaptation is a set of self-adaptive traits [4]. That is,

$$\text{self-adaptation} = \{\text{y} \mid \text{y is a self-adaptive trait}\} \tag{1}$$

Thus, each self-adaptive trait is an element in self-adaptation. In other words, using categorical language, this is written as $1 \xrightarrow{\text{self-adaptive trait}} \text{self-adaptation}$. CASs are self-adaptive in that the individual and collective behavior mutate and self-organize corresponding to interactions. Self-adaptation indicates that CAS is a mimicry of socio-technical systems.

5 Self-adaptation Monoid

In [4], self-adaptation is specified by the morphism $Self\text{-}A : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$, which defines the set $\{Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS\times\ Inter^{n\in T})\}$ of self-adaptive traits. Let $\mathbf{Self\text{-}A}^{n\in T}$ be the set of such self-adaptive traits, then

$$\mathbf{Self\text{-}A}^{n\in T} = \{Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS \times Inter^{n\in T})\} \tag{2}$$

Note that, in the case, we write $Self\text{-}A^{n\in T}_{i\in\mathbb{N}}$ to stand for $Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS\times\ Inter^{n\in T})$. Thus, we have

$$\mathbf{Self\text{-}A}^{n\in T} = \{Self\text{-}A^{n\in T}_{i\in\mathbb{N}}\} \tag{3}$$

This set with the composition operation ";" satisfies two following properties:

Composition of Self-adaptive Traits

Let f and g be members of $\mathbf{Self\text{-}A}^{n\in T}$, then the composition of self-adaptive traits $f;g : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$ is as $g : (f : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})) \longrightarrow (CAS \times Inter^{n\in T})$. In other words, let $f = Self\text{-}A^{n\in T}_{i\in\mathbb{N}}$ and $g = Self\text{-}A^{n\in T}_{j\in\mathbb{N}}$ then

$$(Self\text{-}A^{n\in T}_{i\in\mathbb{N}}\ ;\ Self\text{-}A^{n\in T}_{j\in\mathbb{N}}) = Self\text{-}A_{j\in\mathbb{N}}(Self\text{-}A^{n\in T}_{i\in\mathbb{N}}, CAS \times Inter^{n\in T}) \tag{4}$$

Identity of Self-adaptive Traits

There exist identities $1_{n\in T} : (CAS \times Inter^{n\in T}) \longrightarrow (CAS \times Inter^{n\in T})$ of self-adaptive traits in $\mathbf{Self\text{-}A}^{n\in T}$ such that, for every f in $\mathbf{Self\text{-}A}^{n\in T}$, $1_{n\in T}; f = f; 1_{n\in T} = f$ to be held. In other words, this can be specified by

$$\begin{aligned} Self\text{-}A_{i\in\mathbb{N}}^{n\in T} &= Self\text{-}A_{i\in\mathbb{N}}(1_{n\in T}, CAS \times Inter^{n\in T}) \\ &= Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, 1_{n\in T}) \\ &= Self\text{-}A_{i\in\mathbb{N}}(CAS \times Inter^{n\in T}, CAS \times Inter^{n\in T}) \end{aligned} \tag{5}$$

Thus, $\mathbf{Self\text{-}A}^{n\in T}$ with the composition operation ";" is called *self-adaptation monoid*. Moreover, the monoid $\mathbf{Self\text{-}A}^{n\in T}$ is also a monoid category including only one object to be the set $\{Self\text{-}A_{i\in\mathbb{N}}^{n\in T}\}$, each of whose members is a self-adaptive trait, and by the composition operation as a morphism, then the associativity and identity on the morphisms are completely satisfied.

6 Series of Self-adaptive Traits

A number of different notations are in use for denoting series of self-adaptive traits.

$$sf = (f_0, f_1, f_2, \ldots) \tag{6}$$

is a common notation which specifies a series of self-adaptive traits *sf* which is indexed by the natural numbers in $T(= \mathbb{N} \cup \{0\})$. We are also accustomed to

$$sf = (f_{t\in T}) \tag{7}$$

Informally, series of self-adaptive traits can be understood as a rope on which we hang up a sequence of self-adaptive traits for display. Hence it follows that

Definition 2 (Series of self-adaptive traits). *For morphisms* $1 \xrightarrow{t} T$ *and* $1 \xrightarrow{f_t} \mathbf{Self\text{-}A}^{n\in T}$*, there exists a unique morphism* $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ *such that the equation* $t; sf = f_t$ *holds. This is described by the following commutative diagram*

$$\begin{array}{ccc} 1 & \xrightarrow{\quad t \quad} & T \\ & \searrow^{f_t} & \downarrow{sf} \\ & & \mathbf{Self\text{-}A}^{n\in T} \end{array} \tag{8}$$

Morphism $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ *defines a series of self-adaptive traits.*

Note that morphism $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ is read as

$$\forall t[t \in T \Longrightarrow \exists!\, f_t[f_t \in \mathbf{Self\text{-}A}^{n\in T} \;\&\; sf(t) = f_t]]$$

In other words, $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ generates series of self-adaptive traits as an infinite sequence of $sf(0) = f_0$, $sf(1) = f_1$, ..., $sf(t) = f_t$, ... which is written as $(sf(0), sf(1), \ldots, sf(t), \ldots)$ or $(f_0, f_1, \ldots, f_t, \ldots)$

Definition 3 (Set of series of self-adaptive traits). *Given* $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ *then the set of series of self-adaptive traits, denoted by* $\mathbf{Self\text{-}A}_\omega^{n\in T}$*, is defined by*

$$\mathbf{Self\text{-}A}_\omega^{n\in T} = \{sf \mid T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}\} \tag{9}$$

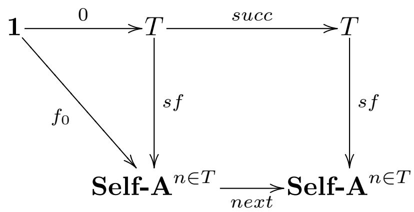
We obtain

Corollary 1. *If* $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ *then* $1 \xrightarrow{sf} \mathbf{Self\text{-}A}_\omega^{n\in T}$

Proof: This result stems immediately from definitions 2 and 3 Q.E.D.

This corollary means that for each morphism $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$, there is a morphism $1 \xrightarrow{sf} \mathbf{Self\text{-}A}_\omega^{n\in T}$ generating member in $\mathbf{Self\text{-}A}_\omega^{n\in T}$. That is, morphism $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ generates series of self-adaptive traits and $1 \xrightarrow{sf} \mathbf{Self\text{-}A}_\omega^{n\in T}$ constructs the set of series of self-adaptive traits.

For series of self-adaptive traits, we can define a mechanism to generate them. This mechanism consists of an object T equipping with structural morphisms $1 \xrightarrow{0} T \xrightarrow{succ} T$ with the property that for $\mathbf{Self\text{-}A}^{n\in T}$, any $1 \xrightarrow{f_0} \mathbf{Self\text{-}A}^{n\in T}$ and $\mathbf{Self\text{-}A}^{n\in T} \xrightarrow{next} \mathbf{Self\text{-}A}^{n\in T}$ then there exists a unique morphism $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ such that the following diagram commutes



(10)

Definition 4 (Construction of series of self-adaptive traits). *We define a construction morphism of series of self-adaptive traits, denoted by* ‡*, such that*

$$\mathbf{Self\text{-}A}^{n\in T} \times [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \xrightarrow{\ddagger} [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \tag{11}$$

This definition means that $\ddagger(A \times B \xrightarrow{f\times g} C \times D) = A \ddagger B \xrightarrow{f\ddagger g} C \ddagger D$.

It follows that any series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ can be represented in a format including two parts of *head* and *tail* to be connected by "‡" such that

$$T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T} \overset{equiv}{\equiv} \overbrace{1 \xrightarrow{0} T \xrightarrow{sf} \mathbf{Self\text{-}A}}^{f_0}{}^{n\in T} \ddagger \overbrace{1 \xrightarrow{t>0} T \xrightarrow{sf} \mathbf{Self\text{-}A}}^{f_{t>0}}{}^{n\in T} \tag{12}$$

where $\overbrace{1 \xrightarrow{0} T \xrightarrow{sf} \mathbf{Self\text{-}A}}^{f_0}{}^{n\in T} = sf(0)$ and $\overbrace{1 \xrightarrow{t>0} T \xrightarrow{sf} \mathbf{Self\text{-}A}}^{f_{t>0}}{}^{n\in T} = (sf(1), sf(2), \ldots)$ to be called head and tail, respectively.

Definition 5 (Head of series of self-adaptive traits). *We define a head construction morphism, denoted by* $1 \overset{0}{\Longrightarrow} (\_\ )$*, such that*

$$1 \overset{0}{\Longrightarrow} (\_\ ) : [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \longrightarrow \mathbf{Self\text{-}A}^{n\in T} \tag{13}$$

This definition states that $\forall(a \ddagger s)[(a \ddagger s) \in [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \Longrightarrow \exists!\ f_0[f_0 \in \mathbf{Self\text{-}A}^{n\in T}\ \ \&\ 1 \overset{0}{\Longrightarrow} (a \ddagger s) = a = f_0]]$

It follows that $1 \overset{0}{\Longrightarrow} (T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}) \overset{equiv}{\equiv} 1 \xrightarrow{0} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$.

Definition 6 (Tail of series of self-adaptive traits). *We define a tail construction morphism, denoted by* $(\_\ )'$*, such that*

$$(\_\ )' : [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \longrightarrow [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \tag{14}$$

This definition means that $\forall(a\ddagger s)[(a\ddagger s) \in [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}] \Longrightarrow \exists!(f_1, f_2, \ldots)$ $[(f_1, f_2, \ldots) \in [T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}]\ \&\ (a \ddagger s)' = s = (f_1, f_2, \ldots)]]$

As a convention, $(\_\ )^{\langle n\rangle}$ denotes applying recursively the $(\_\ )'$ n times. Thus, specifically, $(\_\ )^{\langle 2\rangle}, (\_\ )^{\langle 1\rangle}$ and $(\_\ )^{\langle 0\rangle}$ stand for $((\_\ )')', (\_\ )'$ and $(\_\ )$, respectively.

It follows that the first member of series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ is given by

$$1 \overset{0}{\Longrightarrow} ((T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})') \overset{equiv}{\equiv} 1 \xrightarrow{1} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T} \tag{15}$$

and, in general, for every $k \in T$ the k-th member of series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ is provided by

$$1 \overset{0}{\Longrightarrow} ((T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})^{\langle k\rangle}) \overset{equiv}{\equiv} 1 \xrightarrow{k} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T} \tag{16}$$

Series of self-adaptive traits to be an infinite sequence of all $f_{t\in T}$ is viewed and treated as single mathematical entity, so the derivative of series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ is given by $(T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})'$

Now using this notation for derivative of series of self-adaptive traits, we can specify series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ as in

Definition 7. *A series of self-adaptive traits* $T \xrightarrow{sf} \boldsymbol{Self\text{-}A}^{n\in T}$ *can be specified by*

- *Initial value:* $1 \xrightarrow{0} T \xrightarrow{sf} \boldsymbol{Self\text{-}A}^{n\in T}$ *and*
- *Differential equation:* $((T \xrightarrow{sf} \boldsymbol{Self\text{-}A}^{n\in T})^{\langle n\rangle})' = (T \xrightarrow{sf} \boldsymbol{Self\text{-}A}^{n\in T})^{\langle n+1\rangle}$

The initial value of $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ is defined as its first element $1 \xrightarrow{0} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$, and the derivative of series of self-adaptive traits, denoted by $(T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})'$, is defined by $((T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})^{\langle n\rangle})' = (T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})^{\langle n+1\rangle}$, for any integer n in T. In other words, the initial value and derivative equal the head and tail of $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$, respectively. The behavior of a series of self-adaptive traits $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ consists of two aspects: it allows for the observation of its initial value $1 \xrightarrow{0} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$; and it can make an evolution to the new series of self-adaptive traits $(T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})'$, consisting of the original series of self-adaptive traits from which the first element has been removed. The initial value of $(T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})'$, which is $1 \xRightarrow{0} ((T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})') = 1 \xrightarrow{1} T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ can in its turn be observed, but note that we have to move from $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T}$ to $(T \xrightarrow{sf} \mathbf{Self\text{-}A}^{n\in T})'$ first in order to do so. Now a behavioral differential equation defines a series of self-adaptive traits by specifying its initial value together with a description of its derivative, which tells us how to continue.

Note that every member $f_{t\in T}$ in $\mathbf{Self\text{-}A}^{n\in T}$ can be considered as a series of self-adaptive traits in the following manner. For every $f_{t\in T}$ in $\mathbf{Self\text{-}A}^{n\in T}$, a unique series of self-adaptive traits is defined by morphism f:

$$\overbrace{1 \xrightarrow{f_t} \mathbf{Self\text{-}A}^{n\in T} \xrightarrow{f} \mathbf{Self\text{-}A}_\omega^{n\in T}}^{(f_t,\circ,\circ,\ldots)} \tag{17}$$

such that the equation $f_t; f = (f_i, \circ, \circ, \ldots)$ holds, where $\circ$ denotes empty member (or null member) in $\mathbf{Self\text{-}A}^{n\in T}$. Thus $(f_t, \circ, \circ, \ldots)$ is in $\mathbf{Self\text{-}A}_\omega^{n\in T}$.

Definition 8 (Equivalence). *For any* $T \xrightarrow{sf1} \mathbf{Self\text{-}A}^{n\in T}$ *and* $T \xrightarrow{sf2} \mathbf{Self\text{-}A}^{n\in T}$, *sf1* $=$ *sf2 iff* $1 \xrightarrow{t} T \xrightarrow{sf1} \mathbf{Self\text{-}A}^{n\in T} = 1 \xrightarrow{t} T \xrightarrow{sf2} \mathbf{Self\text{-}A}^{n\in T}$ *with every* t *in* T.

Definition 9 (Bisimulation). *Bisimulation on* $\mathbf{Self\text{-}A}_\omega^{n\in T}$ *is a relation, denoted by* $\sim$, *between series of self-adaptive traits* $T \xrightarrow{sf1} \mathbf{Self\text{-}A}^{n\in T}$ *and*

$T \xrightarrow{sf2}$ **Self-A**$^{n \in T}$ *such that if* $sf1 \sim sf2$ *then* $1 \overset{0}{\Longrightarrow}(sf1) = 1 \overset{0}{\Longrightarrow}(sf2)$ *and* $(sf1)' \sim (sf2)'$.

Two series of self-adaptive traits are bisimular if, regarding their behaviors, each of the series "simulates" the other and vice-versa. In other words, each of the series cannot be distinguished from the other by the observation. Let us consider the following corollaries related to the bisimulation between series of self-adaptive traits.

Corollary 2. *Let* sf, $sf1$ *and* $sf2$ *be in* **Self-A**$_\omega^{n \in T}$. *If* $sf \sim sf1$ *and* $sf1 \sim sf2$ *then* $(sf \sim sf1) \circ (sf1 \sim sf2) = sf \sim sf2$, *where the symbol* $\circ$ *denotes a relational composition. For more descriptive notation, we can write this in the form*

$$\frac{sf \sim sf1, sf1 \sim sf2}{(sf \sim sf1) \circ (sf1 \sim sf2) = sf \sim sf2} \tag{18}$$

and conversely, if $sf \sim sf2$ *then there exists* $sf1$ *such that* $sf \sim sf1$ *and* $sf1 \sim sf2$. *This can be written as*

$$\frac{sf \sim sf2}{\exists sf1 : sf \sim sf1 \quad \text{and} \quad sf1 \sim sf2} \tag{19}$$

Proof: Proving (18) originates as the result of the truth that the relational composition between two bisimulations $L_1 \subseteq sf \times sf1$ and $L_2 \subseteq sf1 \times sf2$ is a bisimulation obtained by $L_1 \circ L_2 = \{\langle A, y \rangle \mid A \; L_1 \; z \text{ and } z \; L_2 \; y \text{ for some } z \in sf1\}$, where $A \in sf$, $z \in sf1$ and $y \in sf2$.

Proving (19) comes from the fact that there are always $sf1 = sf$ or $sf1 = sf2$ as simply as they can. Hence, (19) is always true in general. Q.E.D.

Corollary 3. *Let* $sf_i, \forall i \in \mathbb{N}$, *be in* **Self-A**$_\omega^{n \in T}$ *and* $\bigcup_{i \in \mathbb{N}}$ *be union of a family of sets. We have*

$$\frac{sf \sim sf_i \quad \text{with } i \in \mathbb{N}}{\bigcup_{i \in \mathbb{N}} (sf \sim sf_i) = sf \sim \bigcup_{i \in \mathbb{N}} sf_i} \tag{20}$$

and conversely,

$$\frac{sf \sim \bigcup_{i \in \mathbb{N}} sf_i}{\exists i \in \mathbb{N} : sf \sim sf_i} \tag{21}$$

Proof: Proving (20) stems straightforwardly from the fact that sf bisimulates sf_i (i.e., $sf \sim sf_i$) then, sf bisimulates each series in $\bigcup_{i \in \mathbb{N}} sf_i$.

Conversely, proving (21) develops as the result of the fact that for each $\langle A, y \rangle \in \bigcup_{i \in \mathbb{N}} (sf \times sf_i)$, there exists $i \in \mathbb{N}$ such that $\langle A, y \rangle \in sf \times sf_i$. In other words, it is formally denoted by $\bigcup_{i \in \mathbb{N}} (sf \times sf_i) = \{\langle A, y \rangle \mid \exists i \in \mathbb{N} : A \in sf \quad \text{and} \quad y \in sf_i\}$, where $A \in sf$ and $y \in sf_i$. Q.E.D.

The union of all bisimulations between sf and sf_i (i.e., $\bigcup_{i\in\mathbb{N}}(sf \sim sf_i)$) is the greatest bisimulation. The greatest bisimulation is called the *bisimulation equivalence* or *bisimilarity* [1,2] (again denoted by the notation $\sim$).

Corollary 4. *Bisimilarity $\sim$ on $\bigcup_{i\in\mathbb{N}}(sf \sim sf_i)$ is an equivalence relation.*

Proof: In fact, a bisimilarity $\sim$ on $\bigcup_{i\in\mathbb{N}}(sf \sim sf_i)$ is a binary relation $\sim$ on $\bigcup_{i\in\mathbb{N}}(sf \sim sf_i)$, which is reflexive, symmetric and transitive. In other words, the following properties hold for $\sim$

– Reflexivity:

$$\frac{\forall(a\sim b)\in\bigcup_{i\in\mathbb{N}}(sf\sim sf_i)}{(a\sim b)\sim(a\sim b)} \tag{22}$$

– Symmetry:

$$\frac{\forall(a\sim b),(c\sim d)\in\bigcup_{i\in\mathbb{N}}(sf\sim sf_i),\quad (a\sim b)\sim(c\sim d)}{(c\sim d)\sim(a\sim b)} \tag{23}$$

– Transitivity:

$$\frac{\forall(a\sim b),(c\sim d),(e\sim f)\in\bigcup_{i\in\mathbb{N}}(sf\sim sf_i),\quad ((a\sim b)\sim(c\sim d))\bigwedge((c\sim d)\sim(e\sim f))}{(a\sim b)\sim(e\sim f)} \tag{24}$$

to be an equivalence relation on $\bigcup_{i\in\mathbb{N}}(sf \sim sf_i)$. Q.E.D.

For some constraint α, if $sf1 \sim sf2$ then two series $sf1$ and $sf2$ have the following relation.

$$\frac{sf1 \models \alpha}{sf2 \models \alpha} \tag{25}$$

That is, if series $sf1$ satisfies constraint α then this constraint is still preserved on series $sf2$. Thus it is read as $sf1 \sim sf2$ in the constraint of α (and denoted by $sf1 \sim_\alpha sf2$).

For validating whether $sf1 = sf2$, a powerful method is so-called *proof principle of coinduction* [3] that states as follows:

Theorem 1 (Coinduction). *For any $T\xrightarrow{sf1}\textbf{Self-A}^{n\in T}$ and $T\xrightarrow{sf2}\textbf{Self-A}^{n\in T}$, if $sf1 \sim sf2$ then $sf1 = sf2$.*

Proof: In fact, for two series of self-adaptive traits $sf1$ and $sf2$ and a bisimulation $sf1 \sim sf2$. We see that by inductive bisimulation for $k \in T$, then $sf1^{\langle k\rangle} \sim sf2^{\langle k\rangle}$. Therefore, by definition 9, $1\overset{0}{\Longrightarrow}(sf1^{\langle k\rangle}) = 1\overset{0}{\Longrightarrow}(sf2^{\langle k\rangle})$. By the equivalence in (16), then $1\xrightarrow{k}sf1 = 1\xrightarrow{k}sf2$ with every $k \in T$. It follows that, by definition 8, we obtain $sf1 = sf2$ Q.E.D.

Hence in order to prove the equivalence between two series of self-adaptive traits $sf1$ and $sf2$, it is sufficient to establish the existence of a bisimulation relation $sf1 \sim sf2$. In other words, using coinduction we can justify the equivalence between two series of self-adaptive traits $sf1$ and $sf2$ in $\textbf{Self-A}_\omega^{n\in T}$.

Corollary 5 (Generating series of self-adaptive traits). *For every sf in* $\mathbf{Self\text{-}A}_{\omega}^{n \in T}$*, we have*

$$sf = 1 \xRightarrow{0} (sf) \ddagger (sf)' \tag{26}$$

Proof: This stems from the coinductive proof principle in theorem 1. In fact, it is easy to check the following bisimulation $sf \sim 1 \xRightarrow{0} (sf) \ddagger (sf)'$. It follows that $sf = 1 \xRightarrow{0} (sf) \ddagger (sf)'$ Q.E.D.

In (26), operation $\ddagger$ as a kind of series integration, the corollary states that series derivation and series integration are inverse operations. It gives a way to obtain sf from $(sf)'$ and the initial value $1 \xRightarrow{0} (sf)$. As a result, the corollary allows us to reach solution of differential equations in an algebraic manner.

7 Conclusions

In this paper, we have constructed self-adaptation monoid to establish series of self-adaptive traits in CASs based on coinductive approach.

We have started with defining CASs and self-adaptive traits in CASs. Then, $\mathbf{Self\text{-}A}^{i \in T}$ has been constructed as a self-adaptation monoid to shape series $T \xrightarrow{sf} \mathbf{Self\text{-}A}^{i \in T}$ of self-adaptive traits. In order to prove the equivalence between two series of self-adaptive traits, using coinduction, it is sufficient to establish the existence of their bisimulation relation. In other words, we can justify the equivalence between two series of self-adaptive traits in $\mathbf{Self\text{-}A}_{\omega}^{n \in T}$ based on a powerful method so-called *proof principle of coinduction.*

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