

Increasing the Quality of Medical Images Based on the Combination of Filters in Ridgelet Domain

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Abstract. In many fields, images become a tool that contains data such as medical images. However, the image not only has blur or noise, but also has blur and noise pair. The aim of deblurring and denoising image is to remove blur and noise detail but this process helps keep edges features and its information. In this paper, we have proposed a method for increasing the quality of medical images based on the combination of filters in ridgelet domain. The proposed method uses ridgelet transform combined with Bayesian thresholding for denoising process and uses Wiener filter for deblurring process in ridgelet domain. For demonstrating the superiority of the proposed method, we have compared the results with the other recent methods available in literature.

Keywords: Deblurring · Denoising · Bayesian thresholding · Wiener filter · Ridgelet transform

1 Introduction

Most of images not only have noise but also have blur. This problem reduces the quality of images and difficulty for viewers. Especially, for medical images, they have blur, noise or pair that influences diagnostic process of medical specialists because a small detail in a medical image is very useful for treatment process. The goal of denoising and deblurring is to remove noise and blur details from the corrupted image while maintaining edge features. In the past, many methods are proposed to increase the quality of images such as wavelet transform methods [1], Discrete Wavelet Transform (DWT) [2] method, etc. Although DWT is a powerful tool for this task, however, it has serious disadvantages such as lack information, shift-sensitivity [3] and poor directionality [5]. Several papers have proposed solutions for reducing these disadvantages. In case blur or noise visible in medical image, Wiener filter [8] has given good results in some cases. However, the area of image denoising is hard work and still a great challenge.

In this paper, we have proposed a method for increasing quality of medical images based on the combination of filters in ridgelet domain. The proposed method uses ridgelet transform combined with Bayesian thresholding for denoising process and uses Wiener filter for deblurring process in ridgelet domain. For demonstrating the superiority of the proposed method, we have compared the results with the other recent methods available in literature such as ridgelet transform [3] and Wiener filter [8]. For performance measure, we have used Peak Signal to Noise ratio (PSNR) and Mean Square Error (MSE) and it has shown that the present method yields far better results.

The rest of the paper is organized as follows: in section 2, we described the basic of ridgelet transform, and the principle of Bayesian thresholding and Wiener filter; details of the proposed method are given in section 3; the results of the proposed method are presented in section 4 and our conclusions in section 5.

2 Background

2.1 The Ridgelet Transform

Ridgelets have been recently applied in the image processing application [10], [11], [12]. The theory of ridgelets was developed by Candes [13]. In that work, Candes showed that one could develop a system of analysis based on the ridge functions:

$$\psi_{a,b,\theta}(x, y) = a^{-1/2} \psi((x \cos(\theta) + y \sin(\theta) - b) / a)$$

and the function is constant along the lines: $x \cos(\theta) + y \sin(\theta) = const.$ He introduced a continuous ridgelet transform:

$$R_f(a, b, \theta) = \langle \psi_{a,b,\theta}(x), f \rangle$$

with a reproducing formula and a Parseval relation. He showed the construction of frames, giving stable series expansions in terms of a special discrete collection of ridge functions. The approach was general, and gave ridgelet frames for functions in $L_2[0, 1]^d$ in all dimensions $d \geq 2$.

Let i be the triple (j, ℓ, k) where the indices run as follows:

$$i \in \ell := \{(j, \ell, k), j, k \in \mathbb{Z}, j \geq j_0, \ell \in \Lambda_j\}$$

and define the collection of discrete ridgelets $\psi_i(x)$ as

$$\psi_i(x) = 2^{j/2} \psi(2^j u_i^T x - k), \quad i \in \ell$$

where j is the ridge scale, k is the ridge location, i is the angular scale and ℓ is the angular location.

The range of the parameter ℓ is scale dependent as it depends on j . Ridgelets are directional and the interesting aspect is the discretization of the directional variable u ; this variable is sampled at increasing resolution so that at scale j .

The 2-D continuous ridgelet transform in R^2 can be defined as follows. First define a smooth wavelet function $\psi: R \rightarrow R$ satisfying the admissibility condition given by:

$$\int_{-\infty}^{\infty} \frac{|\psi(\varepsilon)|^2}{|\varepsilon|^2} d\varepsilon < \infty$$

where ψ is the Fourier transform of ψ .

The bivariate ridgelet $\psi_{a,b,\theta}: R^2 \rightarrow R^2$ is defined by $\psi_{a,b,\theta}(x, y)$ and the function is constant along the lines $x\cos(\theta) + y\sin(\theta) = \text{const}$. The ridgelet values for the continuous image $f(x, y)$ is given by:

$$Rf(a, b, \theta) = \iint f(x, y)\psi_{a,b,\theta}(x, y)dxdy$$

In short, the ridgelet transform is the application of a 1-D wavelet transform to the slice of the Radon transform where the angular variable θ is constant and t is varying [14]. This means the ridgelet coefficients $Rf(a, b, \theta)$ are given by the analysis of the Radon transform via,

$$Rf(t, \theta) = \int Rf(t, \theta)a^{-1/2}\psi((t-b)/a)dt$$

where $\Psi_{a,b}(t) = a^{-1/2}\Psi((t-b)/a)$ is a 1-D wavelet transform.

To make the ridgelet transform discrete, the Radon transform as well as the wavelet transform have to be discrete. The discrete wavelet transform is well defined but the same cannot be said about the discrete Radon transform. There are many ways to make the Radon transform discrete [15].

The ordinary ridgelet transform can be achieved as follows [16]:

- (i) Compute the 2D Fast Fourier Transform (FFT) of the image.
- (ii) Substitute the sampled values of Fourier transform obtained on the square lattice with sampled values on a polar lattice.
- (iii) Compute the 1D inverse FFT on each angular line.
- (iv) Perform the 1D scale wavelet transform on the resulting angular lines in order to obtain the ridgelet coefficients.

2.2 Bayesian Thresholding

Most of the existing thresholding procedures are essentially minimax. They do not take into account some specific properties of a concrete object in which we are interested. Now, we specify a prior distribution on the wavelet coefficients within a Bayesian framework [17]. Bayesian thresholding's idea is median of thresholdings.

The estimate noise variance σ and signal variance δ can be obtained by equation[19]:

$$\sigma = \left(\frac{\text{median}(|w_{i,j}|)}{0.6745} \right)^2$$

$$\delta^2 = \max \left(\frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N w_{i,j}^2 - \sigma^2, 0 \right)$$

where $w_{i,j}$ is the lowest frequency coefficient after the transformation, $M \times N$ is the sub-band's size.

There are two thresholdings: hard and soft thresholding. In hard thresholding, the important coefficients remain unchanged while the important coefficients are reduced by the absolute threshold value in the soft thresholding.

2.3 Wiener Filter

The filters may be summarized as follows [8]:

- (i) Mean-square value of the estimation error.
- (ii) Expectation of the absolute value of the estimation error.
- (iii) Expectation of third or higher powers of the absolutely value of the estimation error.

The Wiener deconvolution method has widespread uses in image deconvolution applications, as the frequency spectrum of most of visual images is fairly well behaved and may be estimated easily. One of the most widely used restoration techniques is the Wiener filter. Assuming white Gaussian noise, Wiener filter in the Fourier domain will be calculated by equation [8]:

$$W = \frac{R(\omega)^* S_{pp}(\omega)}{|R(\omega)|^2 S_{pp}(\omega) + \sigma n^2}$$

where $S_{pp}(\omega)$ is the power spectrum of the input projection and σn^2 is the variance of the Gaussian noise.

We compute the Wiener restoration filter and minimize issues associated with divisions by equation [8]:

$$G(k,1) = \frac{H^*(k,1)}{|H(k,1)|^2 + S_{-u}(k,1) / S_{-x}(k,1)}$$

$$G(k,1) = \frac{H^*(k,1) S_{-x}(k,1)}{|H(k,1)|^2 + S_{-x}(k,1) + S_{-u}(k,1)}$$

where S_{-u} is the signal power spectrum and S_{-x} is the noise power spectrum.

3 The Proposed Method

Deblurring medical images is very difficult for image processing. Special with medical images consist of blur and noise pair. In this section, we propose an approach for

medical image deblurring based on ridgelet transform using Bayesian thresholding combined with Wiener filter for medical images in case of blur combined with noise pair image.

In our proposed, we divide image processing with blur combined with noise pair into two processes: denoising and deblurring processes. The proposed method includes two periods: ridgelet coefficients computation with Bayesian thresholding for denoising, and Wiener filter for deblurring. The proposed method is used as figure 1:

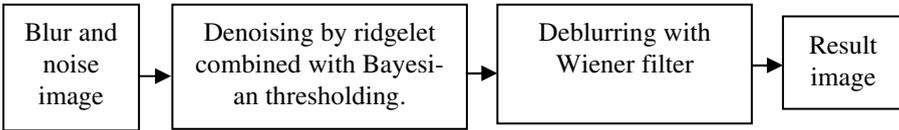


Fig. 1. The process of proposed method

Firstly, the input is the blur and noise image, we use ridgelet for image denoising. The process for image denoising is as follows:

- (i) Estimate noise variance.
- (ii) Calculate the ridgelet coefficients.
- (iii) Based on these coefficients to filter along rows with low and high sub-band, and columns with low sub-band.

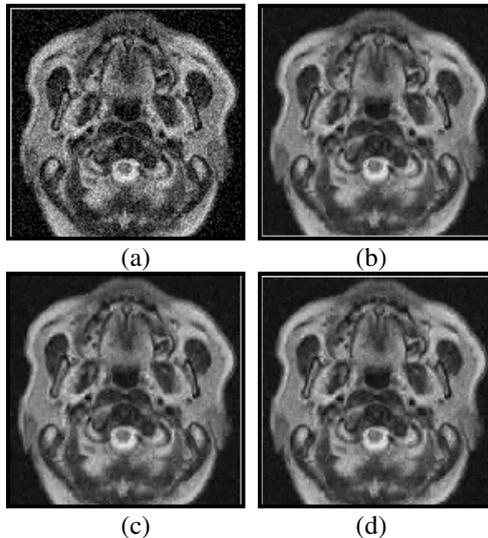


Fig. 2. Noise image with Gaussian noise and denoising images by different methods. (a) Noise image (PSNR = 21.5064 db). (b) Denoising image by Bayesian thresholding (PSNR = 27.9002 db). (c) Denoising image by ridgelet transform (PSNR = 28.1641 db). (d) Denoising image by ridgelet combined with Bayesian thresholding (PSNR = 28.3362 db).

Bayesian thresholding is also the composition in ridgelet transform. We calculate median thresholding and show the result based on new thresholding. With Bayesian thresholding, after calculating the thresholds based on sigma-hat, we continue to reconstruct the image.

If the value of pixel details coefficients is less than thresholding then the result is 0. Else, the result is array Y, where each element of Y is 1 if the corresponding element of pixel is greater than zero, 0 if the corresponding element of pixel equals zero, -1 if the corresponding element of pixel is less than zero. After this period, the input image has become image denoising.

Figure 2 shows the denoising of noise image in case Gaussian noise with Bayesian thresholding in ridgelet domain. From figure 2, we see that the result of the method – ridgelet combined with Bayesian thresholding - is better than the other methods such as Bayesian thresholding and ridgelet method. Therefore, this method gives the good result for denoising period.

Secondly, the blur in the image is not removed. In order to remove the blur, we use Wiener filter for the above image result in the previous period.

4 Experiments and Results

In this section, we apply the procedure described in section 3 and achieved superior performance in our deblurring experiments as demonstrated in this section. For performance evaluation, we compare the results of the proposed method based on ridgelet transform combined with Bayesian thresholding and Wiener filter with the other methods such as ridgelet transform and Wiener Filter. We test the result in medical image datasets, this dataset includes different image sizes: 256x256, 512x512.

Gaussian and Motion types are used to blur. In addition, Gaussian noise is added to these images. Hard thresholding is applied to the coefficients after decomposition in ridgelet domain. All of the above methods are done on the same images at similar scale.

The quality of the image is improved by comparison with the value of Mean Square Error (MSE) and Peak Signal-to-Noise Ratio (PSNR). The MSE defined as:

$$MSE = \sqrt{\frac{1}{N \times N} \sum_{i=1}^N \sum_{j=1}^N (x_{ij} - y_{ij})^2}$$

where x is the image which has blur and noise, y is the image result and N x N is the size of the image. PSNR is used as the measure of the quality of the reconstruction of image deblurring or denoising, defined as:

$$PSNR = 20 \log_{10} \left(\frac{MAX_1}{MSE} \right)$$

where MAX₁ is the maximum pixel value of the image. The proposed method will be compared with ridgelet transform and Wiener filter method by the MSE and PSNR values. The smaller the value of MSE is, the better it is. The higher the value of PSNR is, the better it is. We test so many medical images. In here, we show some test cases.

Figure 3 shows the deblurring of blur and noise image by Gaussian blur and Gaussian noise with our proposed method. Figure 4 shows the deblurring of blur and noise image by Motion blur and Gaussian noise with our proposed method.

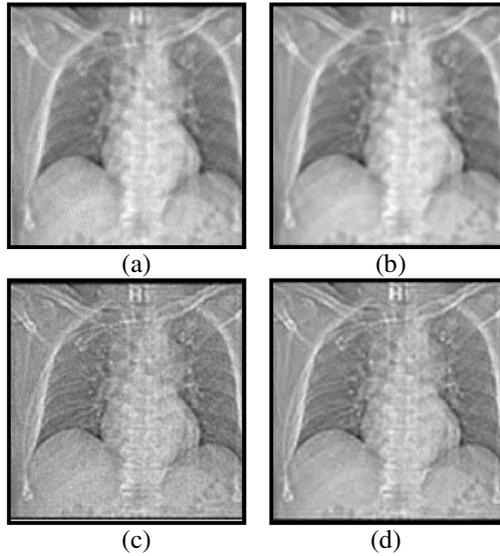


Fig. 3. Denoising and deblurring images by different methods in the case Gaussian blur combine with Gaussian noise in image. (a) Blur and noise image (PSNR = 26.4899 db). (b) Deblurred image by Ridgelet Transform (PSNR = 26.5783 db). (c) Deblurred image by Wiener filter (PSNR = 28.1759 db). (d) Deblurred image by RT-BT-WF (PSNR = 28.4001 db).

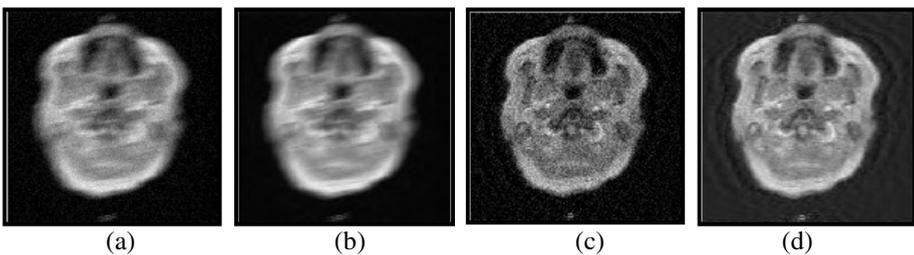


Fig. 4. Denoising and deblurring images by different methods in case Motion blur is combined with Gaussian noise in image. (a) Blur and noise image (PSNR = 21.5308 db). (b) Deblurred image by ridgelet transform (PSNR = 21.8387 db). (c) Deblurred image by Wiener filter (PSNR = 22.6385 db). (d) Deblurred image by proposed method (PSNR = 25.3566 db).

From figure 3 and figure 4, we see that the results of the proposed method are better than the other methods. Figure 5 and figure 6 show the plot of PSNR, MSE values of different image deblurring methods corrupted with Gaussian blur combined with Gaussian noise.

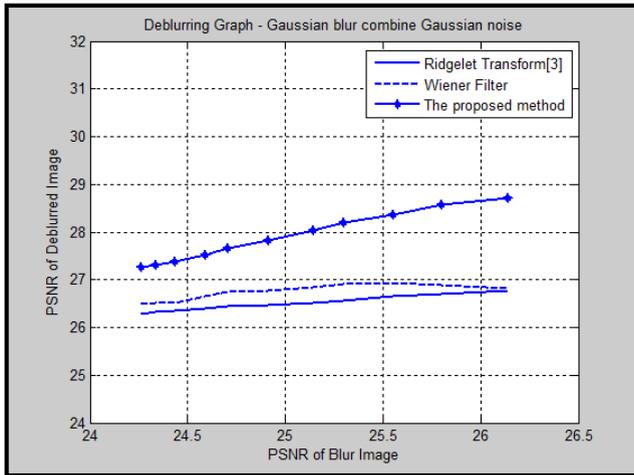


Fig. 5. Plot of PSNR values of deblurred images with Gaussian blur combined with Gaussian noise using different methods

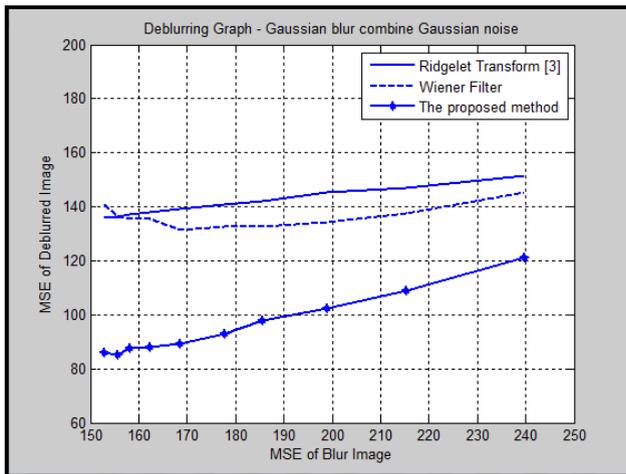


Fig. 6. Plot of MSE values of deblurred images with Gaussian blur combined with Gaussian noise using different methods

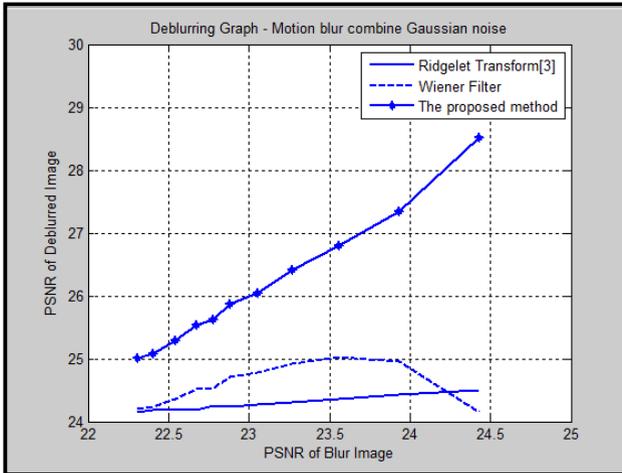


Fig. 7. Plot of PSNR values of deblurred images with Motion blur combined with Gaussian noise using different methods

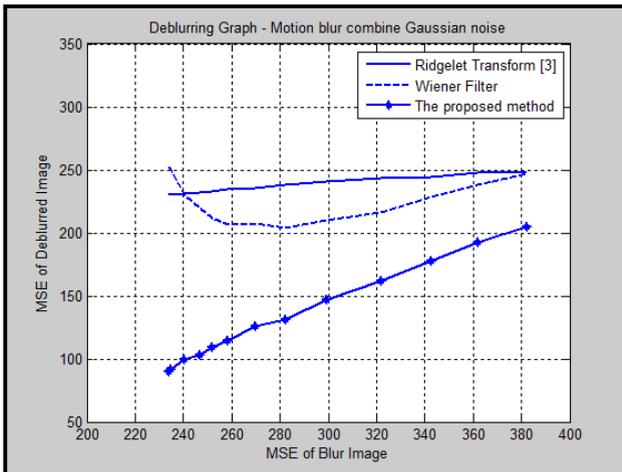


Fig. 8. Plot of MSE values of deblurred images with Motion blur combined with Gaussian noise using different methods

Figure 7 and figure 8 also show the plot of PSNR, MSE values of different image deblurring methods corrupted with Motion blur combined with Gaussian noise.

In these figures, it is well observed that the proposed method performs better than Wiener filter and ridgelet transform method.

5 Conclusions

In this paper, we propose a method for deblurring images for blur combined with noise image pair. The proposed method is divided into two processes: denoising and deblurring. Firstly, we denoise images based on combining ridgelet transform with Bayesian thresholding. Then, we apply Wiener filter for deblurring into denoising image. We test with Gaussian blur combined with Gaussian noise pair and Motion blur combined with Gaussian noise pair in medical image dataset. From the results of the above section, we conclude that the proposed method works well and better than the other recent methods available in literature. With this idea, we think the combination methods can upgrade the quality of image blurring and noising in case of denoising and deblurring.

Appendix

Table A1. PSNR Values (dB) of different denoised images using the image sizes with Gaussian blur combined with Gaussian noise

Test Image	Image Size	Blur & Noisy Image	Ridgelet Transform [3]	Wiener filter	Proposed Method
1	256 x 256	26.4899	26.5783	28.1759	28.4001
2		24.9402	25.0515	20.9087	25.1534
3		27.1148	27.2895	25.3263	28.4104
4		26.9914	27.2757	25.7539	28.6007
5		25.1912	25.3753	21.2920	25.6060
6		21.7783	21.8618	21.4116	23.0959
7		26.3854	26.6870	26.3591	28.9273
8		25.7983	26.6906	26.9004	28.5674
9		31.0315	33.3208	27.4723	33.6087
10		28.3717	29.4430	29.6220	31.3085
11	512 x 512	30.9153	33.4346	26.8820	33.7170
12		24.6946	25.2133	26.2104	27.4562
13		23.3097	23.7127	24.0278	26.1523
14		29.0223	31.3864	28.1622	32.9868
15		29.7922	31.9072	26.9748	32.9882
16		23.6980	23.9378	24.2840	27.1048
17		22.8516	24.2431	24.8694	26.1611
18		25.5965	27.7678	27.0068	30.1564
19		26.0585	29.2006	27.4821	30.1255
20		24.9059	30.2079	26.9002	30.3653

Table A2. PSNR Values (dB) of different denoising images using the image sizes with Motion blur combined with Gaussian noise

Test Image	Image Size	Blur & Noisy Image	Ridgelet Transform [3]	Wiener filter	Proposed Method
1	256 x 256	24.3540	24.9051	24.8830	25.0513
2		22.1404	22.4044	18.5093	23.3762
3		21.5308	21.8387	22.6385	25.3566
4		20.9264	21.7559	19.3466	23.6520
5		22.9106	24.8598	22.7247	25.3682
6		18.3682	18.9366	19.1854	20.6690
7		19.1375	19.9172	21.1649	21.9766
8		19.7133	20.7100	21.5687	22.5267
9		23.9735	27.1204	24.6441	28.1680
10		22.0956	24.0654	24.0284	24.7623
11	512 x 512	27.7823	28.9289	20.4775	31.3731
12		21.3390	21.5874	21.3583	24.9890
13		19.1648	19.3289	18.7874	23.3077
14		24.7470	25.5131	21.7559	29.4172
15		25.8007	26.9125	20.8364	30.0084
16		19.2751	19.5024	19.3169	23.5513
17		18.9320	19.6084	19.9875	22.3136
18		20.7517	22.9811	21.8471	24.3939
19		19.6183	24.4411	21.8352	22.9961
20		19.8857	26.1292	21.9116	23.8552

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