

# Energy-Efficient Subcarrier Allocation for Downlink OFDMA Wireless Network

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**Abstract.** For the downlink of OFDMA network without Quality of Service (QoS) provision, it has been proved that the network energy efficiency (EE) achieved by best Channel Quality Indicator (CQI) subcarrier allocation scheme is in close proximity to the optimal EE. However, for the downlink of OFDMA network with the provision of QoS, the existing algorithms directly assigning subcarriers have the problem of high complexity. This paper proposed a new subcarrier allocation algorithm by readjusting the subcarrier allocation obtained via the allocation principle of best CQI. The proposed algorithm attempts to maximize the EE of network, and at the meanwhile reduce computational complexity. Simulation experiments indicate that the proposed algorithm significantly reduces computational complexity and achieves nearly the same EE as the optimal solution.

**Keywords:** OFDMA · QoS · Subcarrier allocation · Energy efficiency · Computational complexity

## 1 Introduction

With the explosive growth of high-data-rate wireless services, energy consumption has drawn more and more attention. It has been reported in [1] that for many mobile operators, the radio access part takes up more than 70% of the total energy consumption. For both the user equipment (UE) side and base station side, energy-efficient design is increasingly important and becomes an inevitable trend.

Orthogonal frequency division multiple access (OFDMA) has been widely used for the next generation wireless communication system due to its performance of anti-inter-symbol interference. Hence, resource allocation in OFDMA network recently has focused on maximizing energy efficiency [2-16] rather than maximizing throughput [17-20]. In [3], the tradeoff between EE and SE (spectrum efficiency) with certain rate constraints has been addressed. In the energy-efficient resource allocation, circuit power is also accounted in addition to the transmitted power. Circuit power is assumed to be static in [3-10], while in [11-12] circuit power includes static power and dynamic power decided by data rate. A water-filling based subcarrier allocation

algorithm in a single cellular downlink OFDMA is proposed in [4]. In [5], energy efficient resource allocation in uplink and downlink OFDMA wireless network has been addressed with the consideration of network QoS. The proposed suboptimal subcarrier allocation algorithm which is named MDSA (Maximizing-EE-lower-bound-based Downlink Subcarrier Allocation) achieves the network EE closed to the optimal solution, while the calculation of each user's circuit power factor greatly increases the computational complexity. The uplink energy-efficient transmission is studied in single-cell OFDMA systems in [6-7]. In [8], the convex optimization theory is utilized to obtain the optimal joint subcarrier and power allocation, but it's too complex to achieve the resolutions of transcendental equations. The fading channels are assumed to be flat in [9], however the fading channels across all subcarriers are frequency selective in reality world. In [9-10], the proposed iteration algorithms both use time-sharing technique to achieve high EE performance, but the complexity depends on the number of iterations. The work in [13-16] investigates multi-cell resource allocation in interference-limited scenarios to improve network energy efficiency. Base station closed strategies are studied in [13-14] while game theory based resource allocation algorithms are proposed in [15-16]. All these related work directly allocate the subcarriers by specific techniques, while the computational complexity of proposed algorithms doesn't seem satisfactory.

In this paper, we address the energy-efficient resource allocation in downlink OFDMA wireless network with QoS in frequency selective fading channels. We model the problem of energy-efficient resource allocation as the optimization problem of maximizing EE under certain constraints. To reduce computational complexity, the optimization problem is decomposed into two subproblems-subcarrier allocation and power allocation. In subcarrier allocation, we propose BCSA (Best Channel quality Subcarrier Adjustment) algorithm which readjusts the subcarrier allocation obtained via the allocation principle of best channel quality. Based on the subcarrier allocation obtained in BCSA, we will use BPA (Bisection-based Power Adaptation) algorithm proposed in [5] to finish power allocation. Compared with MDSA, BCSA significantly reduces computational time and achieves comparable network EE. This will be demonstrated in simulation results.

The rest of the paper is organized as follows. Section 2 describes the system model and formulates the energy-efficient resource allocation problem. Section 3 tackles subcarrier and power allocation problems and proposes BCSA algorithm. This is followed by the performance comparison in section 4, and the paper is concluded in section 5.

## 2 System Model

A typical downlink OFDMA wireless network with a single base station transmitting towards  $K$  users is shown in figure 1. As described in [2], the EE is defined as transmitted bits per consumed joule and can be expressed as:  $EE = \text{total data rate}/\text{total consumed power}$ .

According to [5], assume that the channel state information (CSI) of  $K$  users across  $N$  subcarriers is known to the scheduler of the base station and each subcarrier is only assigned to one user during one scheduling period. The energy-efficient resource allocation problem for the OFDMA downlink transmission can be formulated as

$$\hat{\eta}_{EE} = \max_{\rho \in \mathbf{P}, \mathbf{P} \in \mathbf{P}} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W \log_2 (1 + p_{k,n} \gamma_{k,n})}{\zeta P + P_c} \tag{1a}$$

subject to

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{k,n} = P \tag{1b}$$

$$\sum_{k \in \mathcal{K}} \rho_{k,n} = 1, \forall n \in \mathcal{N} \tag{1c}$$

$$\sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n} \geq R_{k,\min}, \forall k \in \mathcal{K} \tag{1d}$$

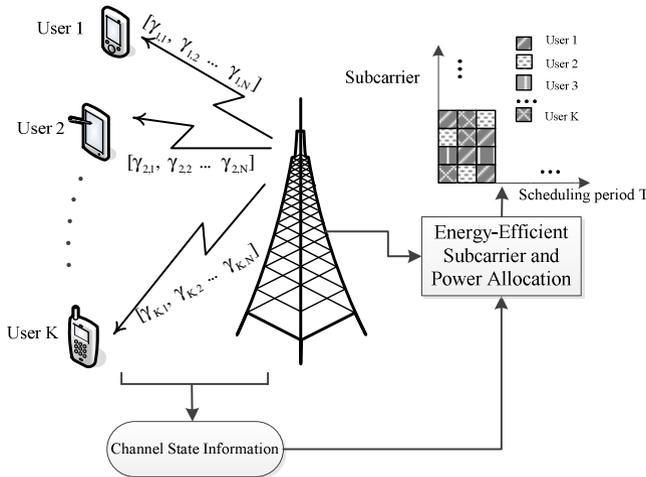


Fig. 1. A typical downlink OFDMA wireless network

Where  $\hat{\eta}_{EE}$  is the optimal downlink network energy efficiency, the bandwidth of subcarrier is  $W$ ,  $P_c$  is circuit power of the base station,  $P$  is the total transmission power, and  $\zeta$  is the reciprocal of drain efficiency of power amplifier.  $\gamma_{k,n}$  is channel-gain-to-noise ratio (CNR) of the  $k$ th UE on the  $n$ th subcarrier. The CSI of the network can be expressed as  $CNR = [\gamma_{k,n}]_{K \times N}$ .  $\mathcal{K} = \{1, 2, 3, \dots, K\}$  and  $\mathcal{N} = \{1, 2, 3, \dots, N\}$  denote the sets of  $K$  subcarriers and  $N$  UEs, respectively.  $\rho_{k,n} \in \{0, 1\}$  indicates whether the  $n$ th subcarrier is assigned to the  $k$ th UE. The subcarrier allocation matrix  $\mathbf{P}$  and power allocation matrix  $\mathbf{P}$  can be expressed as

$$\mathbf{p} \in \bar{\mathbf{p}} = \left\{ \left[ \mathbf{p}_{k,n} \right]_{K \times N} \left| \begin{array}{l} p_{k,n} \in \{0,1\}, \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ \sum_{k \in \mathcal{K}} p_{k,n} \leq 1, \forall n \in \mathcal{N} \end{array} \right. \right\} \quad (2)$$

$$\mathbf{p} \in \bar{\mathbf{p}} = \left\{ \left[ \mathbf{p}_{k,n} \right]_{K \times N} \left| \begin{array}{l} p_{k,n} \geq 0, \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{k,n} = P \end{array} \right. \right\} \quad (3)$$

Besides, to guarantee QoS for each UE, the scheduler provides minimum rate  $R_{k,\min}$  for every user.

### 3 Energy-Efficient Resource Allocation

The resource allocation Problem (1) is generally NP hard for the optimal solution. To reduce the computational complexity, we decompose the problem into two sub-problems. We first determine the subcarrier allocation and derive the power allocation by BPA algorithm based on water filling.

As to subcarrier allocation, a straightforward approach is to assign the subcarrier to the user which has the best channel quality. This allocation strategy is easy to implement and sometimes can lead satisfactory results, however it's not fair to the users whose channel quality are generally poor across all subcarriers. To balance high EE and computational complexity, [5] proposes MDSA which maximizes the minimum single-user EE. Numerical results show that the EE obtained by MDSA is close to that of the optimal solution. However, in MDSA, every user's circuit power factor needs to be solved, while the problem of solving circuit power factors is a multivariable and multi-constrained nonlinear optimization problem. The procedure increases the computational complexity and the circuit power factors need to be resolved once channel states change, which makes MDSA impractical in reality world.

For the network without guarantee of QoS, simulations have shown that the best CQI subcarrier allocation scheme can achieve the network EE which is very close to the optimal solution and the scheme takes very low complexity. In the situation with QoS, a promising solution is to readjust the subcarrier allocation obtained by best CQI, which can approach the optimal EE with low computational complexity. Based on this, we propose the BCSA algorithm which mainly readjusts the subcarriers between the highest EE UE and the lowest EE UE to achieve higher network EE. The EE of single UE is defined as

$$\eta_{EE,k} = \frac{R_k}{\zeta \sum_{n=1}^N p_{k,n} + \frac{\sum_{n=1}^N p_{k,n}}{N} P_c} \quad (4)$$

Where  $R_k$  is the total data rate for user k across the allocated subcarriers. The algorithm in detail is sketched in table 1. It starts by normalizing the CNR and determining the number of subcarriers allocated to each UE, which is shown from line 1 to

line 2. The original subcarrier allocation is determined by choosing the best quality channel, which is described from line 3 to line 10. We readjust the subcarrier allocation between the highest EE UE and the lowest EE UE to improve the total network EE, which is illustrated from line 11 to line 13.

**Table 1.** Best channel state subcarrier adjustment (bcsa) algorithm

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**Algorithm** BCSA

**Input:**  $\boldsymbol{\rho} = [\rho_{k,n}]_{K \times N} \leftarrow \mathbf{0}_{K \times N}$ ,  $\text{CNR} = [\gamma_{k,n}]_{K \times N}$ ,  $P_c$ ,  $\zeta$   
 $\{R_{k,\min} | \forall k \in \mathcal{K}\}$ ,  $M < N$

**Output:**  $\boldsymbol{\rho} = [\rho_{k,n}]_{K \times N}$

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1. To calculate average channel gain for each user and normalize the  $\gamma_{k,n}$   
 $\bar{\gamma}_k \leftarrow \sum_{n=1}^N \gamma_{k,n} / N$ ;  $\gamma_{k,n}^* \leftarrow \gamma_{k,n} / \bar{\gamma}_k$ ;
2. To determine the number of subcarriers assigned to each user  

$$N_k \leftarrow \left[ \left( R_{k,\min} / \sum_{k=1}^K R_{k,\min} \right) * N \right]$$
;
3. **While**  $\mathcal{N} \neq \emptyset$
4.  $k^*, n^* \leftarrow \underset{\forall k \in \mathcal{K}, \forall n \in \mathcal{N}}{\text{argmax}} (\gamma_{k,n}^*)$ ;
5. **If**  $\sum_{n=1}^N \rho_{k^*,n} < N_{k^*}$  &&  $\sum_{k=1}^K \rho_{k,n^*} = 0$
6.  $\rho_{k^*,n^*} \leftarrow 1$ ;  $\gamma_{k^*,n^*}^* \leftarrow 0$ ;  $\mathcal{N} \setminus \{n^*\}$ ;
7. **Else**
8.  $\gamma_{k^*,n^*}^* = 0$ ;
9. **End**
10. **End**
11. Using single-user water filling [5] to obtain power allocation  $P_{k,n}$  and data rate  $R_k$  for user  $k$ . According to (1a) and (4) to get the network EE  $\eta_{EE}$ , and the EE for user  $k$   $\eta_{EE,k}$ .
12. **Flag**  $\leftarrow 0$ ;  $\mathcal{N} \leftarrow \{1, 2, 3, \dots, N\}$ ;
13. **Repeat**  
 $k_1 \leftarrow \underset{\forall k \in \mathcal{K}}{\text{arg max}} (\eta_{EE,k})$ ;  $k_2 \leftarrow \underset{\forall k \in \mathcal{K}}{\text{arg min}} (\eta_{EE,k})$ ;  
 $[\alpha_n]_{1 \leq n \leq N} \leftarrow [\text{Inf}]_{1 \leq n \leq N}$ ; For the  $n$ th subcarrier assigned to user  $k_1$ ,  
 $\alpha_n \leftarrow \gamma_{k_1,n} / \gamma_{k_2,n}$ ;  
**If** **Flag**  $\neq 0$   
 $\text{Flag} \leftarrow 0$ ;  $\alpha_{\text{Flag}} \leftarrow \text{Inf}$ ;  
**End**  
 $n^* \leftarrow \underset{\forall n \in \mathcal{N}}{\text{arg min}} (\alpha_n)$ ;  $\rho_{k_1,n^*} \leftarrow 0$ ;  $\rho_{k_2,n^*} \leftarrow 1$ ;

**Table 1.** (continued)

Using single-user water filling to obtain power allocation  $P_{k_1,n}^*$ ,  $P_{k_2,n}^*$  and data rate  $R_{k_1}^*$ ,  $R_{k_2}^*$  for user  $k_1$  and  $k_2$ , respectively. According to (1a) and (4) to get new EE,  $\eta_{EE,k_1}^*$ ,  $\eta_{EE,k_2}^*$  and  $\eta_{EE}^*$ .

**If**  $\eta_{EE}^* > \eta_{EE}$

$$\eta_{EE,k_1} \leftarrow \eta_{EE,k_1}^*; \eta_{EE,k_2} \leftarrow \eta_{EE,k_2}^*; \eta_{EE} \leftarrow \eta_{EE}^*;$$

$$P_{k_1,n} \leftarrow P_{k_1,n}^*; P_{k_2,n} \leftarrow P_{k_2,n}^*; R_{k_1} \leftarrow R_{k_1}^*; R_{k_2} \leftarrow R_{k_2}^*;$$

**Else**

$$\rho_{k_1,n^*} \leftarrow 1; \rho_{k_2,n^*} \leftarrow 0; \text{Flag} \leftarrow n^*;$$

**End**

$$M \leftarrow M - 1;$$

**Until**  $M = 0$ ;

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Compared with MDSA algorithm, we don't calculate the power factor  $\alpha_k$  [5] for each UE in BCSA algorithm. The complexity of the MDSA algorithm is roughly  $\mathcal{O}(N_{OL}K/\delta^2 + N_{OL}N)$  times of water-filling, while  $\mathcal{O}(N_{OL}K/\delta^2)$  is for the calculation of  $\alpha_k$  and  $\mathcal{O}(N_{OL}N)$  is for the subcarrier allocation. The complexity of BCSA algorithm is roughly  $\mathcal{O}(N_{OL}M)$  where  $M$  is less than  $N$  and determined as a quarter of  $N$  in the following simulation. Obviously the complexity of BCSA is greatly less than that of MDSA.

After the subcarrier allocation is determined, we tackle the power allocation. As proved about problem (1) in [5], when  $\mathbf{p}$  is fixed,  $\eta_{EE}$  is continuously differentiable and strictly quasiconcave in  $P$ , and can be easily obtained by the proposed algorithm which is named bisection-based power adaptation (BPA). Finally, the resource allocation is finished.

## 4 Performance Comparing

In this section, we use MATLAB to present simulation results to verify the benefit of BCSA compared with MDSA. We also simulate the PFB (Proportional Fairness Scheduling) algorithm which similarly determines the number of subcarriers allocated to each user and assigns every subcarrier to the user of best channel quality. In our simulation, the bandwidth of each subcarrier is 15kHz and the circuit power is 10W. For the downlink transmission, there are four UEs each with the same minimum rate requirement of 100 kbps. We assume that the drain efficiency of power amplifier is 38%.

The model of the wireless channel includes the distance dependent path loss, shadowing fading and small scale fading. The simulation parameters in detail are listed in Table 2.

**Table 2.** Simulation Parameter

Parameter	Setting
Number of users	4
Circuit power	10W
Number of subcarriers	<72
Bandwidth of subcarrier	15kHz
Shadowing standard deviation	7dB
Small scale fading distribution	Rayleigh distribution
Thermal noise spectral density	-174 dB/Hz
Minimum rate requirement	100kbps
Drain efficiency of power	0.38

To verify when  $\rho$  is fixed,  $\eta_{EE}$  is strictly quasiconcave in  $P$ , We determine  $\rho$  by allocating the subcarrier to the user whose channel quality is the best. Figure 2 shows the relationship between  $\eta_{EE}$  and total consumed power  $P$  with different circuit power. From it, we can see that  $\eta_{EE}$  is strictly quasiconcave in  $P$  even the circuit power is different and the network EE increases as the circuit power decreases.

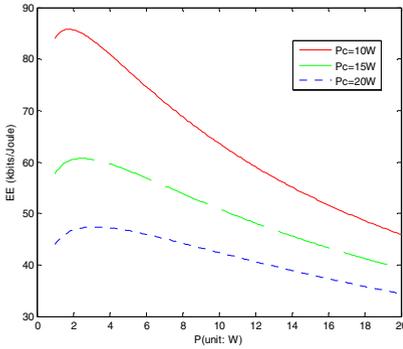
Similarly, Figure 3 shows the relationship between  $\eta_{EE}$  and total consumed power  $P$  with different number of total subcarriers. It shows that  $\eta_{EE}$  is strictly quasiconcave in  $P$  even the number of subcarriers is different and the network EE increases as the number of total subcarriers increases.

Figure 4 compares the EE of algorithm MDSA, PFB and BCSA. From it, we can see that the EE of BCSA is higher than that of PFB, which is obvious as PFB is just similar with part of BCSA without the procedure of readjusting the subcarrier allocation. The EE of BCSA is a little lower than that of MDSA, sometimes even higher when the number of subcarriers is small.

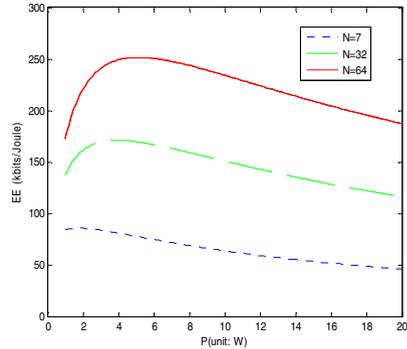
Figure 5 plots the throughput corresponding to the EE in figure 4. We can find that the throughput of BCSA and MDSA are higher than that of PFB, while the throughput of BCSA is nearly close to that of MDSA. When the number of subcarriers is not large, the throughput of BCSA is higher than that of MDSA.

Like figure 5, Figure 6 plots the transmitted power corresponding to the EE in figure 4. It indicates that PFB consumes more power than MDSA and BCSA, while MDSA consumes less power than BCSA.

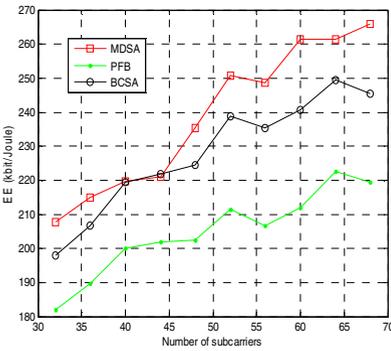
Figure 7 compares the CPU running time of the three algorithms. Compared with the left graph in the figure, in the right graph the CPU running time of MDSA doesn't include the time to calculate each user's circuit power factor. We can see that MDSA takes greatly more time than the other two algorithms so that it's impractical to implement in reality world. MDSA consumes the most time in the three algorithms even the time used to calculate factors is removed. It is easy to understand that PFB is less than BCSA as PFB is similar with part of BCSA. Compared with MDSA, BCSA consumes greatly less CPU running time.



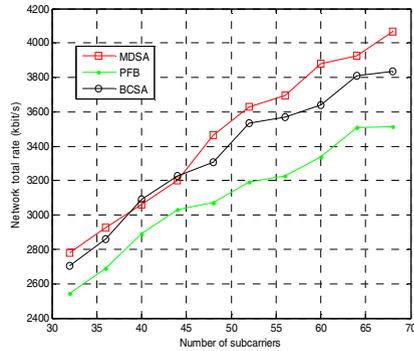
**Fig. 2.** Energy Efficiency-total translated Power relationship with  $P_c=10W$ ,  $P_c=15W$ ,  $P_c=20W$ , respectively. The number of subcarriers is 7.



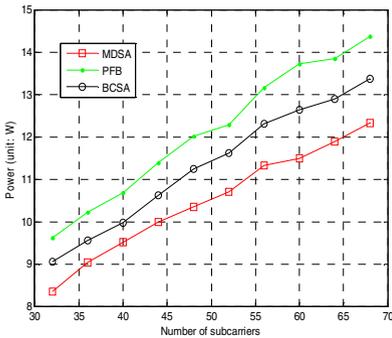
**Fig. 3.** Energy Efficiency-total translated Power relationship with  $N=7$ ,  $N=32$ ,  $N=64$ , respectively. The circuit power is 10W.



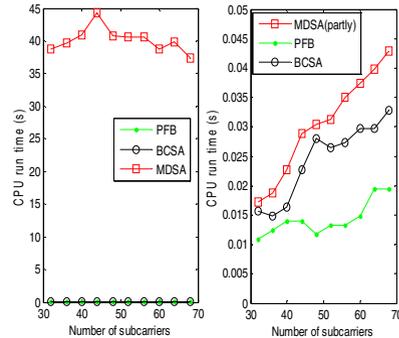
**Fig. 4.** Comparing of the EE for three algorithms as the number of subcarriers changes



**Fig. 5.** Comparing of the network total rate for three algorithms as the number of subcarriers changes



**Fig. 6.** Comparing of the transmitted power for three algorithms as the number of subcarriers changes



**Fig. 7.** Comparing of the CPU running time for three algorithms. The CPU is Intel(R) Core(TM)2 Duo CPU E7500 @2.94G.

## 5 Conclusion

This paper studies the energy-efficient resource allocation in downlink OFDMA wireless network. Although the subcarrier allocation MDSA algorithm achieves the close performance to the optimal EE of the network, but it takes impractical computational time to operate. To reduce the computational complexity, we propose the subcarrier allocation algorithm BCSA, which readjusts the subcarrier allocation in terms of the best channel quality. Simulation results show that the proposed algorithm with low complexity can achieve significant improvement on CPU execution time as well as the comparable EE of network with respect to the MDSA scheme.

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