Comparative Study between CFO and SCFO in OFDM Systems

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Abstract. OFDM has emerged at the beginning of the second half of last century. Its applications are diverse and extend to different wireless and optical communication systems. It is well known that OFDM is very sensitive to frequency shifts. The imperfections of the local oscillators of the transmitter and receiver affects the carrier, while the Doppler-effect affects the carrier, also the sub-carriers of the OFDM signal. This gives rise to inter-carrier interference (ICI) responsible for the degradation of system performance. In this paper we have studied OFDM in a radio mobile channel. We proposed a new model that includes the offsets generated by the Doppler-effect at different sub-carriers. We compared it with the existing model which considers only the carrier frequency offset and we calculated the error between the two mathematical models.

Keywords: Orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), sub carriers frequency offset (SCFO), Doppler-effect, Inter carriers Interferences (ICI), carrier to interferences ratio (CIR).

1 Introduction

The pair FFT / IFFT was the missing piece in digital communications. Its use has allowed the emergence of OFDM [1]. The OFDM has earned a special place in various wireless networks like HIPERLAN/2 and IEEE 802.11a. Recently, she found its applications in the optical domain [2]-[3]. The use of the OFDM is limited in part by its great sensitivity to frequency offsets. They destroy the orthogonality and gives rise to inter-carrier interference. These discrepancies are mainly due to imperfections of the local oscillators of the transmitter and receiver [4]. The Doppler effect present in the radio mobile channel accentuates this shifts [5]. The offset due to the Doppler-effect affects the carrier. In a recent study [7], a new model was proposed. It includes all the offset of the subcarriers of the OFDM signal. In the model proposed in [6], the Doppler-effect is supposed to affect the carrier, it then generates the carrier frequency offset CFO. In the model proposed in [7], the Doppler effect affects not only the carrier but also the subcarriers of the OFDM signal. It then generates the subcarriers frequency offset SCFO. Our work consists in compare and deduce the resulting error between the two models. We gave the analytical expression for this error and then we

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make a number of simulation to clarify the results. This work is structured as follows, in section 2, we study the carrier frequency offset (CFO) and the subcarriers frequency offset (SCFO). In section 3, we analysis the inter-carrier interference generated by the two models. Section 4 compare performances of both models and calculate the error introduced on the CIR. Finally, section 5 concludes the paper.

2 Analysis of Frequency Shifts

The frequency shifts in an OFDM signal is caused by two different parameters, the Doppler-effect and the imperfections of local oscillators. The total normalized offset is then [7]:

$$\varepsilon_t = \varepsilon(1+\beta) \pm k\frac{v}{c}cos\alpha = \varepsilon_p + k\beta \;\; with \;\; \left(\beta = \pm\frac{v}{c}cos\alpha\right) \tag{1}$$

The normalized value of the carrier frequency offset created by the imperfections and Doppler-effect is noted ε_p, while ε_t represents the normalized value of total offset. The relative velocity between the transmitter and receiver is represented by v, α represents the angle formed by the velocity vector and the direction of electromagnetic wave, c is the speed of light and k the subcarrier index. If we neglect the offset created by the Doppler-effect on all subcarrier, we introduce an error which is equal to:

$$Error = \pm k\frac{v}{c}cos\alpha = k\beta \tag{2}$$

This error is proportional to the subcarrier index k.

3 Analysis of Inter-carrier Interference

The OFDM signal affected by a frequency offset is written as[7]:

$$y(n) = \sum_{k=0}^{N-1} X(k)\exp\left(j2\pi\frac{n}{N}(k+\varepsilon_t)\right) \tag{3}$$

Where $X(k)$is the transmitted signal. By replacing ε_t by its value, the equation becomes:

$$y(n) = \exp\left(j2\pi\frac{n\varepsilon_p}{N}\right) \times \sum_{k=0}^{N-1} X(k)\exp\left((j2\pi\frac{nk}{N}\right)(1+\beta) \tag{4}$$

The carrier frequency offset creates a phase shifts of the signal $y(n)$, while the Doppler-effect expand it (or compress it). At the output of the FFT, we obtain the symbols $Y(k)$, its expression is given by the relation [7]:

$$Y(k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} X(k)\exp\left(j2\pi\frac{n}{N}\left(k-m+k\beta+\varepsilon_p\right)\right) \tag{5}$$

$$= \sum_{k=0}^{N-1} X(k)S_{km}(k-m) \tag{6}$$

After some manipulations $S_{km}(k-m)$is then written [7]:

$$S_{km}(k-m) = \frac{\sin\pi\left(k-m+k\beta+\varepsilon_p\right)}{N\sin\frac{\pi}{N}\left(k-m+k\beta+\varepsilon_p\right)}\exp\left(j\pi\left(k-m+k\beta+\varepsilon_p\right)\right. \tag{7}$$

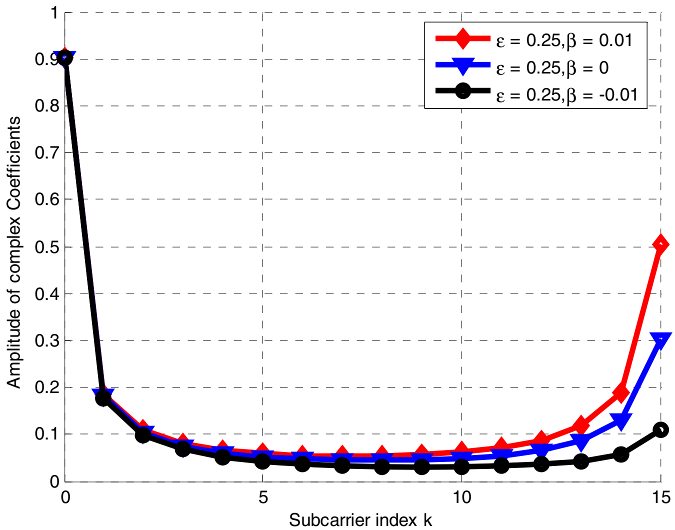

Fig. 1. Amplitudes of complex coefficients depending on the subcarrier index k for different values of the relative speed

The amplitudes of the complex coefficients depend on the difference $(k-m)$, the subcarrier index k and the relative speed β. In the model where we only consider the carrier frequency offset, the complex coefficients have the value [6]:

$$S(k-m) = \frac{\sin\pi\left(k-m+\varepsilon_p\right)}{N\sin\frac{\pi}{N}\left(k-m+\varepsilon_p\right)}\exp j\pi\left(1-\frac{1}{N}\right)\left(k-m+\varepsilon_p\right) \tag{8}$$

La figure 1 plot the complex coefficients for different values of relative speed. The error is given by the relation:

$$Error = \left|\frac{sin\pi(k - m + k\beta + \varepsilon_p)}{Nsin\frac{\pi}{N}(k - m + k\beta + \varepsilon_p)}\right| - \left|\frac{\sin\pi(k - m + \varepsilon_p)}{Nsin\frac{\pi}{N}(k - m + \varepsilon_p)}\right| \quad (9)$$

Note that this error increased when the subcarrier index increases.

4 Errors Analysis

Performance is measured in terms of CIR. The useful signal is calculated for $k = m$, while the interference is computed for $k \neq m$. For the model with CFO, CIR is expressed [6]:

$$CIR = \frac{|S(0)|^2}{\sum_{\substack{k=0 \\ k\neq m}}^{N-1} |S(k - m)|^2} \quad (10)$$

It is the same for any sub-carrier. If we consider the Doppler-effect affects all sub-carriers, the CIR is written[7]:

$$CIR = \frac{|S_{kk}(0)|^2}{\sum_{\substack{k=0 \\ k\neq m}}^{N-1} |S_{km}(k - m)|^2} \quad (11)$$

The CIR is not the same for differents sub-carriers[7].

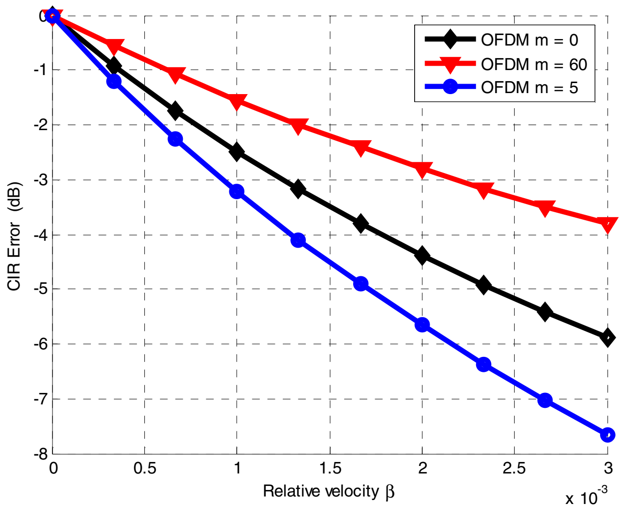

Fig. 2. Plot of the error on the CIR as a function of relative speed

We derive the error on the CIR, it is equal to:

$$Error = 10log_{10}\left(\frac{|S_{ll}(0)|^2}{\sum_{\substack{k=0\\k\neq m}}^{N-1}|S_{lk}(k-m)|^2}\right) - 10log_{10}\left(\frac{|S(0)|^2}{\sum_{\substack{k=0\\k\neq m}}^{N-1}|S(k-m)|^2}\right) \quad (12)$$

This error is plotted in figure 2, its value varies from 3.8 to 7.65 dB.

5 Conclusion

In this paper, we studied the OFDM in a mobile radio channel. The frequency shifts were studied in more detail. In fact, we took into account the offset of the different subcarriers of the OFDM signal created by the Doppler-effect. A new mathematical model is considered. Then, we compared the proposed model with the existing model which considers only the carrier frequency offset as a source of inter-carrier interference. This comparison has mounted that there is a difference between the two models. In terms of CIR, this difference varies between 3.8 and 7.65 dB.

References

1. Weinstein, S., Ebert, P.: Data transmission by frequency-division multiplexing using the discrete Fourier transform. IEEE Trans. Commun. 19, 628–634 (1971)
2. Armstrong, J.: OFDM for Optical communications. Journal of Lightwave Technology 27(3) (February 1, 2009)
3. Yang, Q., He, Z., Yang, Z., Yu, S., Yi, X., Shieh, W.: Coherent optical DFT-Spread OFDM transmission using orthogonal band multiplexing. OPTICS EXPRESS 20(3), 2379–2385 (2012)
4. Sathananthan, K., Tellambura, C.: Performance analysis of an OFDM system with carrier frequency offset and phase noise. In: IEEE Vehicular Technology Conference, VTC 2001 Fall, Atlantic City, NJ, USA, October 7-11, vol. 4, pp. 2329–2332 (2001)
5. Capoglu, R., Li, Y., Swami, A.: Effect of Doppler Spread in OFDM-Based UWB Systems. IEEE Transactions on Wireless Communications 4(5) (September 2005)
6. Zhao, Y., Haggman, S.-G.: Intercarrier interference self-cancellation scheme for OFDM mobile communication systems. IEEE Trans. Commun. 49, 1185–1191 (2001)
7. Tayebi, M., Bouziani, M.: Performance of OFDM in radio mobile channel. In: Elmoataz, A., Mammass, D., Lezoray, O., Nouboud, F., Aboutajdine, D. (eds.) ICISP 2012. LNCS, vol. 7340, pp. 142–148. Springer, Heidelberg (2012)