Effect of Finite Wordlength on the Performance of an Adaptive Network

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Abstract. In this paper we consider the performance of incremental least mean square (ILMS) adaptive network when it is implemented in finite-precision arithmetic. We show that unlike the infinite-precision case, the steady-state curve, described in terms of mean square deviation (MSD) is not always a monotonic increasing function of step-size parameter. More precisely, when the quantization level is small, reducing the step-size may increase the steady-state MSD.

Keywords: adaptive networks, distributed estimation, least mean-square (LMS), quantization.

1 Introduction

An adaptive network is a collection of nodes that interact with each other, and function as a single adaptive entity that is able to respond to data in real-time and also track variations in their statistical properties. [1]. Although adaptive networks were initially proposed in the literature to perform decentralized information processing and inference tasks, they are also well-suited to model complex and self-organized behavior encountered in biological systems, such as fish joining together in schools and birds flying in formation [2-4].

Depending on the manner by which the nodes communicate with each other, they may be referred to as incremental algorithms [5-9] or diffusion algorithms [10-13]. Incremental strategies rely on the use of a cyclic path through the network. In general, determining a cyclic path that covers all nodes is an NP-hard problem. The given algorithms in [10–13] use different adaptive filter in their structure, such as LMS, recursive least-squares (RLS), and affine projection. In comparison, in adaptive diffusion implementations, information is processed locally at the nodes and then diffused in real-time across the network and no cyclic path is required.

In the original incremental LMS (ILMS) adaptive network [5], it is assumed that the infinite-precision weights (local estimates) are exchanged among the nodes

V.V. Das and P.M. El-Kafrawy (Eds.): SPIT 2012, LNICST 117, pp. 115–120, 2014.

through ideal links. More precisely, in [5] some theoretical relations which explain the steady-state performance of ILMS algorithm (in terms of mean-square deviation (MSD), excess mean-square error (EMSE), and mean-square error (MSE)) are derived. In [14, 15] we have studied the performance of ILMS estimation algorithm when it is implemented in finite-precision arithmetic. The importance of such a study arises from the fact that the performance of adaptive networks (like ILMS) can vary significantly when they are implemented in finite-precision arithmetic.

In this paper, our objective is to go beyond these earlier works in [14, 15] to show that the steady-state behavior of quantized ILMS adaptive network is different form its unquantized version. More precisely, unlike the infinite-precision case, the MSD curve is not a monotonically increasing function of step-size parameter. We use the derived results in [14, 15] to explain the mentioned result.

Throughout the paper, we adopt boldface letters for random quantities and normal font for nonrandom (deterministic) quantities. The $*$ symbol is used for both complex conjugations for scalars and Hermitian transpose for matrices.

2 Incremental LMS Algorithm

Consider a network composed of N nodes which are used to estimate an unknown vector $w^o \in R^M$ from measurements collected at N nodes in a network. Each node k has access to time-realizations $\{d_k(i), u_{k,i}\}$ of zero-mean spatial data $\{d_k, u_k\}$ where each d_k is a scalar measurement and each u_k is a $1\times M$ row regression vector. In [2] the ILMS adaptive network has been proposed to estimate w^o. The update equation in ILMS is given by

$$\psi_{k,i} = \psi_{k-1,i} - \mu u^*_{k,i}\left(e_k(i)\right) \tag{1}$$

Where $e_k(i) = d_k(i) - u_{k,i}\psi_{k-1,i}$ and μ is the step-size. In (1) the $M\times 1$ vector $\psi_{k,i}$ denotes the local estimate of w^o at node k at time i. Due to incremental cooperation, the calculated estimates are sequentially circulated from node to node (see Fig. 1).

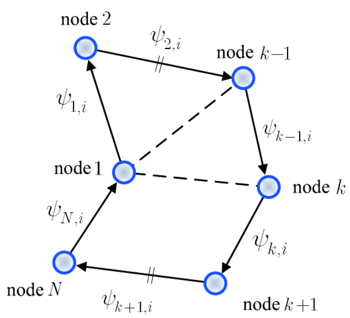

Fig. 1. A schematic of ILMS adaptive network

3 Quantized Incremental LMS, (Q-ILMS)

The ILMS algorithm can be implemented in finite-precision at every node k as shown in Fig. 2. In the finite-precision case the update equation (1) changes to [7, 8]

$$\psi^q_{k,i} = \psi^q_{k-1,i} + \mu u^*_{k,i} e^q_k(i) - p_{k,i} \tag{2}$$

where e^q_k and $\psi^q_{k,i}$ are the quantized values of e_k and $\psi_{k,i}$ respectively. Moreover, $p_{k,i}$ stands for the effect of quantization errors in evaluation of $\psi^q_{k,i}$. Its covariance matrix is given by [7]

$$R_{p,k} = E\{p_{k,i} p^*_{k,i}\} = 2\sigma^2_r I_M + \mu^2 \sigma^2_r E\{| e^q_k(i) |^2\} \tag{3}$$

In (3) σ^2_r is the variance of quantization error which is given by

$$\sigma^2_r = \frac{1}{12}\frac{L^2_r}{2^{n_r}} \; \sigma^2_r \; = \frac{1}{12}\frac{L^2_r}{2^{n_r}} \tag{4}$$

where n_r and L_r, denote the number of bits and the saturation level of quantization.

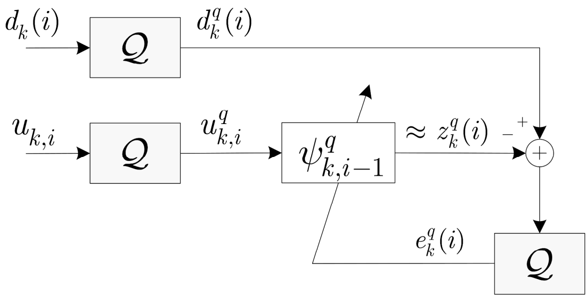

Fig. 2. A block diagram representation of quantized implementation of ILMS algorithm at node k

The steady-state performance of adaptive networks can be expressed in terms of MSD at every node k which is defined as

$$\eta_k = E\{\left\| w^o - \psi_{k-1,\infty} \right\|^2\} \tag{5}$$

As we have shown in [7], for Q-ILMS with Gaussian data, the MSD at every node k can be approximated as

$$\eta_k \approx (\mu^2_1 \sigma^2_{\bar{v},1} \lambda^T_1 + b^T_1 + \cdots + \mu^2_N \sigma^2_{\bar{v},N} \lambda^T_k + b^T_N)\Omega^{-1} c \tag{6}$$

where $\sigma^2_{\bar{v},k}$ variance of modified noise variable defined in [7]. The other symbols are Λ_k is a diagonal matrix with the eigenvalues of $R_{u,k}$, Γ_k is a diagonal matrix with the eigenvalues of $R_{p,k}$, $\lambda_k = \text{diag}\{\Lambda_k\}$ (a $M\times 1$ vector), $b_k = \text{diag}\{\Gamma_k\}$ (a $M\times 1$ vector), $c = \text{diag}\{I_M\}$ (a $M\times 1$ vector), and also

$$\Omega = 2(\mu_1\Lambda_1 + \mu_2\Lambda_2 + \cdots \mu_N\Lambda_N) \tag{7}$$

To show the non-monotonic dependence of the MSD with respect to the step-size in finite-precision case we assume that for all nodes we have $R_{u,k} = \lambda I_M$, $\mu_k = \mu$, $\sigma^2_{\bar{v},k} = \sigma^2_{\bar{v}}$ and $b_k = \gamma\text{diag}\{I_M\}$. Using these assumptions we have

$$\eta_k = \frac{M(\mu^2\sigma^2_{\bar{v}} + \gamma)}{2\mu} \tag{8}$$

which clearly is not a monotonic increasing function of μ. We can also easily see that as the number of bits (i.e. n_r) increases, we have $\sigma^2_r \to 0$ and $(b_k) \to 0$. As a result, η_k approaches the MSD of a ILMS adaptive networks which is a monotonic increasing function of μ.

To explain this behavior we consider again the update equation (2). For small μ, the channel noise term say $p_{k,i}$ is dominant term in update equation, so as $\mu \to 0$, the steady state performance deteriorates. As μ increases, the effect of channel noise term decreases and finally as μ becomes larger the steady state performance deteriorates again like any adaptive algorithm.

4 Simulation Results

In this section we present the simulation results to clarify the discussions. To this aim, we consider a network with $N = 15$ nodes with independent Gaussian regressors where their eigenvalue spread is 1. We assume that unknown vector $w^o = [1111]^T$ relates to the $\{d_k(i), u_{k,i}\}$ via $d_k(i) = u_{k,i}w^o + v_k(i)$ where $v_k(i)$ is white noise term with variance $\sigma^2_{v,k} \in (0, 10^{-1})$. To implement the Q-ILMS, we set $L_r = 1$. The steady-state curves are generated by running the network learning process for 2000 iterations. The MSD curve is obtained by averaging the last 200 samples. Each curve is obtained by averaging over 100 independent experiments.

Fig. 3 shows the global MSD (which is defined as $1/N\sum_{k=1}^{N}\eta_k$) for different values of μ and n_r (including sign bit). As it is clear from Fig. 3, there the steady-state curve is not a monotonic increasing function of step-size. Moreover, for sufficiently large number of bits, the MSD curve becomes a monotonic increasing function of μ.

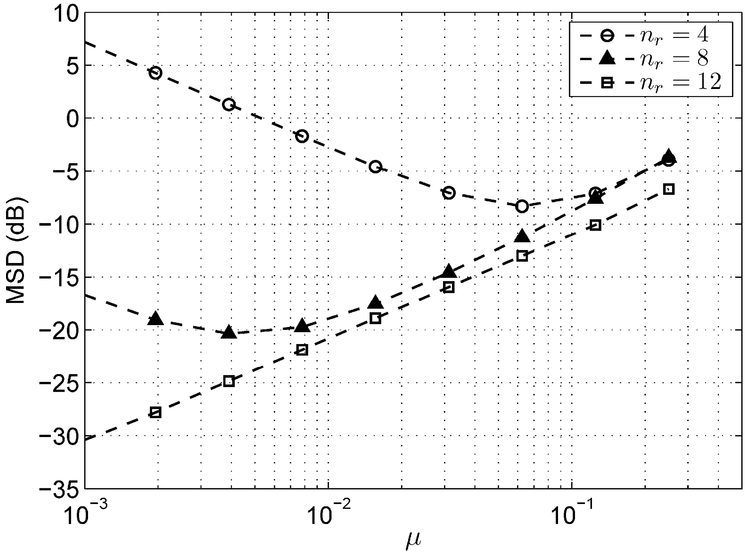

Fig. 3. The steady-state MSD (in dB) curve as a function of μ and for different number of bits n_r

5 Conclusions

In this paper, we considered the steady-state evaluation of the finite-precision DILMS algorithm. Using the results derived in [7] and [8] it was shown that unlike the infinite-precision case, in the quantized case the steady-state MSD curve is not always a monotonic increasing function of step-size parameter. Specifically, when the quantization level is small, reducing the step-size may increase the steady-state MSD. This behavior of adaptive networks has also been observed when the links between the nodes in the network are noisy (see [9, 10]).

References

1. Sayed, A.H., Cattivelli, F.: Distributed adaptive learning mechanisms. In: Haykin, S., Ray Liu, K.J. (eds.) Handbook on Array Processing and Sensor Networks, pp. 695–722. Wiley (2009)
2. Tu, S.Y., Sayed, A.H.: Mobile adaptive networks. IEEE J. Selected Topics on Signal Processing 5(4), 649–664 (2011)
3. Chen, J., Sayed, A.H.: Bio-inspired cooperative optimization with application to bacteria motility. In: Proc. ICASSP, Prague, Czech Republic, pp. 5788–5791 (2011)
4. Li, J., Sayed, A.H.: Modeling bee swarming behavior through diffusion adaptation with asymmetric information sharing. EURASIP Journal on Advances in Signal Processing 18 (2012), doi:10.1186/1687-6180-2012-18

5. Lopes, C.G., Sayed, A.H.: Incremental adaptive strategies over distributed networks. IEEE Trans. on Signal Process. 55(8), 4064–4077 (2007)
6. Ram, S.S., Nedic, A., Veeravalli, V.V.: Stochastic incremental gradient descent for estimation in sensor networks. In: Proc. Asilomar Conf. Signals, Systems, Computers, Pacific Grove, CA, pp. 582–586 (2007)
7. Sayed, A.H., Lopes, C.G.: Distributed recursive least-squares strategies over adaptive networks. In: Proc. Asilomar Conf. Signals, Systems, Computers, Monterey, CA, pp. 233–237 (2006)
8. Li, L., Chambers, J.A.: A new incremental affine projection based adaptive learning scheme for distributed networks. Signal Process 88(10), 2599–2603 (2008)
9. Lopes, C.G., Sayed, A.H.: Randomized incremental protocols over adaptive networks. In: Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing (ICASSP), Dallas, TX, pp. 3514–3517 (2010)
10. Lopes, C.G., Sayed, A.H.: Diffusion least-mean squares over adaptive networks: Formulation and performance analysis. IEEE Trans. Signal Process. 56(7), 3122–3136 (2008)
11. Cattivelli, F.S., Sayed, A.H.: Diffusion LMS strategies for distributed estimation. IEEE Trans. Signal Process. 58(3), 1035–1048 (2010)
12. Cattivelli, F.S., Sayed, A.H.: Diffusion strategies for distributed Kalman filtering and smoothing. IEEE Trans. Autom. Control 55(9), 2069–2084 (2010)
13. Cattivelli, F.S., Lopes, C.G., Sayed, A.H.: Diffusion recursive least-squares for distributed estimation over adaptive networks. IEEE Trans. Signal Process. 56(5), 1865–1877 (2008)
14. Rastegarnia, A., Tinati, M.A., Khalili, A.: Performance analysis of quantized incremental LMS algorithm for distributed adaptive estimation. Signal Process 90(8), 2621–2627 (2010)
15. Rastegarnia, A., Tinati, M.A., Khalili, A.: Steady-state analysis of quantized distributed incremental LMS algorithm without Gaussian restriction. Signal, Image and Video Process (to appear)
16. Khalili, A., Tinati, M.A., Rastegarnia, A.: Steady-state analysis of incremental LMS adaptive networks with noisy links. IEEE Trans. Signal Processing 56(5), 2416–2421 (2011)
17. Khalili, A., Tinati, M.A., Rastegarnia, A., Chambers, J.A.: Steady-state analysis of diffusion LMS adaptive networks with noisy links. IEEE Trans. Signal Processing 60(2), 974–979 (2012)