# Sensor Deployment in Bayesian Compressive Sensing Based Environmental Monitoring

Chao Wu(✉), Di Wu, Shulin Yan, and Yike Guo

Imperial College London, London SW7 2AZ, UK
{chao.wu, di.wu11, shu.yan09, y.guo}@imperial.ac.uk

**Abstract.** Sensor networks play crucial roles in the environmental monitoring. So far, the large amount of resource consumption in traditional sensor networks has been a huge challenge for environmental monitoring. Compressive sensing (CS) provides us a method to significantly decrease the number of sensors needed and Bayesian compressive sensing (BCS) makes it possible to deploy sensors selectively rather than randomly. By deploying sensors to the most informative places, we expect to reduce the reconstruction errors further compared with random sensor deployment. In this paper we employ multiple sensor deployment algorithms and BCS based signal recovery algorithm to build novel environmental monitoring systems, in which the environmental signals can be recovered accurately with undersampled measurements. Besides, we apply these environmental monitoring models to ozone data experiments to evaluate them and compare their performance. The results show a significant improvement in the recovery accuracy from random sensor deployment to selective sensor deployment. With 100 measurements for 16641 data points, the reconstruction error of one of the sensor deployment approaches was 40 % less than that of random sensor deployment, with 3.52 % and 6.08 % respectively.

**Keywords:** Environmental monitoring · Compressive sensing · Sensor networks · Sparse Bayesian learning

## 1 Introduction

Since the ozone depletion in Antarctic was found in 1985, environmental issues have been drawing global attentions. Environmental monitoring can be used to estimate the future environmental impacts and evaluate the performance of strategies designed to mitigate environmental damage.

As the main approaches to monitor environment signals, sensor networks play important roles in signal sampling phases. However, the enormous cost for the sensors has been hampering the development of environmental monitoring in many countries. Even a single sensor station could be very expensive for some specific environmental signals [7]. In such cases, compressive sensing, which allows sensing at rates much smaller than the Nyquist-Shannon limit and reconstructing the signal without much loss [1, 2, 15], can be applied to decrease the number of the sensors needed and recover the monitored signal accurately.

Where to deploy the sensors and how to recover the environmental signals based on the measurements are two fundamental but crucial tasks in environmental monitoring.


I. Stojmenovic et al. (Eds.): MOBIQUITOUS 2013, LNICST 131, pp. 37–51, 2014.
DOI: 10.1007/978-3-319-11569-6_4

In our previous work, we applied Bayesian compressive sensing (BCS) techniques to environmental monitoring and built a novel environmental monitoring system with random sensor deployments [3]. The reconstruction error was less than 5 % with the number of used sensors no more than 1 % of all possible sensor places in the ozone monitoring experiments presented in [3]. In this paper, we attempt to decrease the reconstruction errors further by deploying sensors to the most informative places. To be specific, we combine different sensor deployment algorithms with Bayesian compressive sensing based signal recovery algorithm to build the environmental monitoring systems and evaluate their performance by ozone monitoring experiments.

Both open-loop and closed-loop sensor deployment algorithms are involved in this paper. Open-loop means that the deployment of the sensors and sampling phase are separated, while closed-loop (adaptive) means that the sampling phase and the deployment of sensors are implemented simultaneously, i.e. the two processes are interrelated. The experiment results show significant improvements in reconstruction accuracy using adaptive BCS sensor deployment algorithms.

The remainder of the paper is organised as follows. In Sect. 2, we consider the environmental monitoring problem as a linear regression problem and introduce the Bayesian learning algorithm to solve it. This is also the algorithm to reconstruct environmental signals based on sensor measurements. In Sect. 3, we introduce open-loop sensor deployment algorithms with different criteria, such as entropy criterion and mutual information criterion, and how they can be used in environmental monitoring. An adaptive compressive sensing approach to deploy sensors is also presented in Sect. 3. In Sect. 4, we evaluate how well these sensor deployment algorithms perform with respect to their reconstruction errors in ozone monitoring experiments. Conclusions and future work are presented in Sect. 5.

## 2 Bayesian Compressive Sensing Based Environmental Monitoring

In this section we will briefly introduce the method to reconstruct environmental signals based on the sensor measurements. This signal recovery algorithm is also the one used in the environmental monitoring system presented in [3].

We consider the monitoring of ozone as an example. Given all the ozone data in a monitored region, these values can be aggregated into an n by 1 vector $X_r$. $X_r$ represents the instant ozone distribution in this region. The goal in environmental monitoring is to recover $X_r$ with the measurements of a limited number of sensors. The reconstruction of the signal $X_r$ can be summed up as solving a linear regression problem as follow:

$$Y_{m\times 1} = \varnothing X_r + e = \Theta_{m\times n} w_{n\times 1} + e \quad (1)$$

where $Y_{m\times 1}$ is an m by 1 vector ($m \ll n$) stands for the sensor measurements, $\Theta = \varnothing B$ is the projection matrix, B is a fixed Basis matrix, $\varnothing$ is the sampling matrix, w is the sparse weights to be estimated, and e are the zero-mean Gaussian distributed noises in the measurements. The measurement/sampling matrix $\phi$ represents the sensor locations to observe the ozone signal. Each row in the sampling matrix $\varnothing$ is exactly a unit vector with only one non-zero element in it. In this way, $\varnothing X_r$ is an m by 1 vector composed of the observed values of the sensors.

Compressive sensing is a technique for estimating sparse solutions to underdetermined linear regression. Only when w is sparse (whose elements are mostly zeros) will the estimate of w be feasible and accurate [15]. It is reasonable to assume that most environmental signals are sparse under Gaussian Kernel basis B. Thus, the algorithm is executed by decomposing the original environmental signal $X_r$ as $X_r = Bw$, and the reconstructed signal X can be recovered by multiplying the basis matrix B and w. The Gaussian basis matrix $B_{n \times n}$ is defined as follow:

$$B = [\Psi(X_{r1})\Psi(X_{r2})\cdots\Psi(X_{rn})]^T \tag{2}$$

wherein $\Psi(X_{ri}) = [K(X_{ri}, X_{r1})\cdots K(X_{ri}, X_{rn})]$ and $K(X_{ri}, X_{rj})$ is Gaussian Kernel function

$$K(X_{ri}, X_{rj}) = \exp\left\{-\eta_1(X_{ri1}, -X_{rj1})^2 - \eta_2(X_{ri2} - X_{rj2})^2\right\} \tag{3}$$

where $\eta_1$ and $\eta_2$ are hyper-parameters of the kernel function, and the coordinates of $X_{ri}$ is $(X_{ri1}, X_{ri2})$.

Generally speaking, we have $l_p$ minimization [5], greedy/iterative algorithms and some other algorithms, such as the model based CS [6] and Bayesian compressive sensing, to reconstruct the signal in CS. Bayesian compressive sensing is proposed by Shihao Ji in 2008 [10] to estimate the sparse vector $w_{n \times 1}$ in (1), in which Bayesian models are applied to maximise the posterior probability of $w_{n \times 1}$.

BCS recovery algorithm combines hierarchical sparseness priors for $w_{n \times 1}$ and e [10] with Relevance Vector Machine (RVM) based Bayesian CS inversion [13] to estimate $w_{n \times 1}$. Given $Y_{m \times 1}$ and $\Theta_{m \times n}$, we estimate $\alpha$ and $\sigma_0^2$ that are the hyper-parameters in Gaussian priors for $w_{n \times 1}$ and e [10] by maximising $P(w|y, \alpha, \sigma_0^2)$, and then the sparse vector w can be determined. Moreover, the BCS recovery algorithm used in this paper employs a fast sparse Bayesian learning algorithm to improve the computational speed. The detailed processes in this fast algorithm can be referred in [14]. Compared with other recovery algorithms in compressive sensing, BCS provides the posterior density function for $w_{n \times 1}$ instead of a point estimate of w. This property enables us to indicate the measure of confidence of the reconstructed signal with the "error bars" provided by BCS. Furthermore, the construction of the sampling matrix $\varnothing$ can be diversified in BCS, which means different sensor deployment strategies can be employed in BCS rather than deploying the sensors randomly.

## 3 Sensor Deployment

In this section, we will present one of the crucial tasks in environmental monitoring, i.e. how to deploy the sensors to the most informative places. The following subsections identify two open-loop sensor deployment algorithms that deploy sensors before the sampling phases and one adaptive sensor deployment algorithm that deploys sensors during the sampling phases.

### 3.1 Open-Loop Entropy Approach

Given some sensors that have been deployed in the monitored region, we consider the case that we want to deploy another sensor and hope to achieve the best recovery accuracy with the BCS recovery algorithm. Intuitively, we can deploy the sensor to the place with the highest uncertainty.

"Entropy" is a widely used measure of the uncertainty in the information theory and many other areas. In this open-loop entropy approach, the "entropies" of all possible sensor places are calculated and we deploy the next sensor to the place with the highest entropy. We denote y as one of all possible sensor places, A as the set of the places that have been sensed and $X_y$ as the sensor measurement at place y. The entropy in this approach is defined as follow:

$$H\left(X_y|X_A\right)=\frac{1}{2}\log\left(2\pi e\sigma^2_{X_y|X_A}\right) \tag{4}$$

where $\sigma^2_{X_y|X_A} = \Sigma_{yy} - \Sigma_{yA}\Sigma^{-1}_{AA}\Sigma_{Ay}$ [11] is the variance of $X_y$ given $X_A$ and $\Sigma_{ij}$ is a measure of the information redundancy between place i and place j.

There are many methods to define $\Sigma_{ij}$ so far [11]. The most commonly used method is to define $\Sigma_{ij}$ as a Gaussian function that decreases exponentially with the distance between place i and place j. As the distance between y and A increases so does the value of $H(X_y|X_A)$. Thus, far apart places tend to give high entropies. In this way, the sensor deployments can be estimated by maximising $\sigma^2_{X_y|X_A}$ repeatedly.

### 3.2 Open-Loop Mutual Information Approach

The entropy criterion presented in Sect. 3.1 tends to place sensors along the boundary of the monitored region [11]. Thus, a sensor on the boundary cannot detect the signals out of the region and may waste sensed information. The phenomenon was noticed by Ramakrishnan in 2005 [8].

Andreas Krause presented in [9] that the mutual information (MI) criterion can be applied to solve the problem. The mutual information of a possible sensor place y is defined as follow:

$$MI(y) = H\left(X_y|X_A\right) - H\left(X_y|X_{V\backslash A}\right) = \frac{1}{2}\log\left(\frac{\sigma^2_{X_y|X_A}}{\sigma^2_{X_y|X_{V\backslash A}}}\right) \tag{5}$$

where A is the set of the places that have been sensed and V is the set of all possible sensor places.

In this approach, we maximise the mutual information criterion shown in Eq. (5) to estimate the optimal sensor places. The place y with the highest MI(y) is the optimal place to deploy the next sensor based on the set of former sensor places A. This equation is sub modular style and it avoids the problem deploying sensors along the boundaries by subtracting the uncertainty of place y given V\A from the entropy $H(X_y|X_A)$. We will evaluate its performance in Sect. 4.

### 3.3 Adaptive BCS Approach

Bayesian compressive sensing (BCS) provides us the “error bars” to measure the uncertainty of the reconstructed signal [10]. This property enables us to adaptively estimate the optimal next projection to be added into the measurement matrix. In this way, our measurement matrix is designed based on former measurements and the recovery accuracy could be improved compared with other methods.

**Selecting Projections Adaptively.** The sparse weights vector w is actually a multi-variate Gaussian distribution with the mean μ and covariance matrix $\sum$ [11]. In [10], Shihao Ji proposed to design the projection matrix Θ to minimise the differential entropy [12] $h(X) = -\int P(X) \log P(X)dX$ for the reconstructed signal X = Bw. To deploy a new sensor is equivalent to adding a new row on the projection matrix. If we add a new projection τ on Θ, where $\tau^T$ is a new row, and we want to minimise the h(X), it has been proven in [10] that the goal is equivalent to maximising the $\tau^T\Sigma\tau$.

$$\tau^T\Sigma\tau = \tau^T\text{Covariance}(w)\tau \cong \text{Variance}(Y) \tag{6}$$

We can conclude from (6) that the $\tau^T$ to be added into Θ represents the most informative measurement. $\tau^T\Sigma\tau$ is equivalent to a measure of the “information gain” in our case.

Given the environmental monitoring problem shown as Eq. (1), the projection matrix Θ = ϕB is actually choosing rows from basis matrix B and we aim to choose the optimal row from B one by one to build the Θ and minimise $\tau^T\Sigma\tau$.

In this case, $\tau^T$ is a row in B. The measure of how informative $\tau^T$ is can be then be described as follows:

$$\text{next}_{\text{score}}(i) = \tau^T\Sigma\tau = a^T B\Sigma B a = B_a^T \Sigma B_a \tag{7}$$

where $a^T$ is a 1 by n unit vector in which the i-th element is one and $B_a^T$ is the i-th row of the basis matrix B.

This adaptive sensor deployment algorithm uses the measurements of deployed sensors as the feedbacks to help guide the deployment of the next sensor. Thus, the information Moreover, the approach greedily takes the current estimated variances as the criterion to optimising sensor deployments. Thus, the results of this method may not be absolutely optimal.

**The Computational Model.** Compared with non-adaptive BCS approach, the main difference is that our adaptive BCS approach first builds a random small projection matrix to do initial measurements, and then we add measurements gradually based on the feedback of former measurements. With the same number of measurements, the adaptive method improves the recovery accuracy by designing the projection matrix selectively instead of randomly. Figure 1 shows the work flow of the adaptive BCS.

There are three phases in the adaptive BCS environmental monitoring model, which are sampling, recovery and reconstruction. The three phases are detailed as follows:

1. Sampling phase: We sample the environmental signal with the sampling/sensing matrix, in which sensor deployment information is contained. An initial sensor deployment consisted of few random sensor places is generated to start the work flow shown in Fig. 1.
2. Recovery phase: We estimate the sparse vector w and its covariance matrix with the BCS technique. Then we estimate the next sensor location with the method described in this section and revise the sampling matrix. We run the sampling-recovery loop until termination condition is met.
3. Reconstruction phase: The monitored signal can be reconstructed via a simple matrix multiplication X = Bw.

In the adaptive BCS approach, the sensor deployment phase and sampling phase are implemented simultaneously. This property of the adaptive BCS environmental monitoring model may bring difficulties to practical engineering. Thus, we propose to train the sensor locations with history data. This history data based adaptive BCS approach will be discussed in Sect. 4.3.

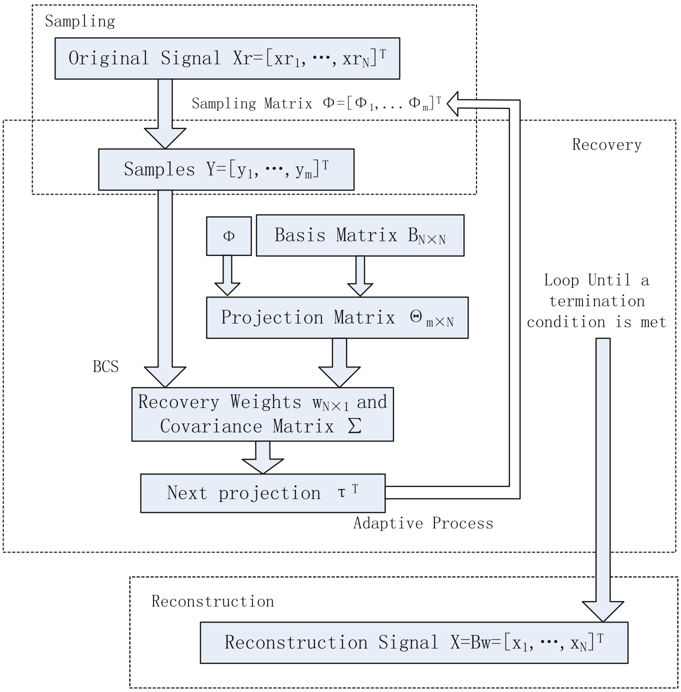


**Fig. 1.** Work flow of the adaptive BCS

# 4 Experiment and Results

With the open-loop sensor deployment algorithms and the adaptive BCS approach presented in Sect. 3, the sensors are deployed selectively rather than randomly in the monitoring systems. Thus, we expect to achieve better recovery accuracy than the environmental monitoring system proposed in [3].

To evaluate these monitoring models, we applied different sensor deployment algorithms to ozone monitoring experiments and compare their performance. We mainly introduce the environmental monitoring tests on two different sizes of ozone signals and analyse the performance of the history data based adaptive BCS approach. The ozone distribution data can be retrieved from the database of NASA and is available at ftp://toms.gsfc.nasa.gov/pub/eptoms/data/monthly_averages/ozone. The ozone data sets used in our experiments are monthly averages data sets that are merely the daily ozone values for an entire month divided by the number of days. The discussion of the experiment results is shown in Sect. 4.4.

## 4.1 30 by 30 Ozone Distribution Monitoring

To compare the performance of the sensor deployment algorithms presented in Sect. 3, we apply these algorithms to a 30 by 30 ozone distribution monitoring experiment in this subsection. The original ozone distribution data we need to recover is a part of the global ozone distribution map available from NASA. It is a 30 by 30 ozone distribution in Feb 2005 (Latitudes 36.5 North to 65.5 North with 1 degree step and Longitudes 179.375 West to 143.125 West with 1.25 degree steps).

The proposed adaptive Bayesian compressive sensing algorithm in the ozone data experiment is as follow:

1. Randomly choose 30 rows from B to build the projection matrix $\Theta$ and run the BCS recovery algorithm [13, 14].
2. Calculate the next_scores shown in Eq. (7) for all the rows in B that are not in $\Theta$ so far. Add the row that corresponds with the largest next_score to $\Theta$.
3. Run the BCS recovery algorithm with the new $\Theta$.
4. If the termination condition is met, otherwise goto 2.
5. Reconstruct the signal with Basis matrix B and the sparse weights vector w.

In Fig. 2(a) and (b) show the deployment of 60 sensors with entropy criterion and mutual information criterion respectively in this experiment.

It can be seen from Fig. 2 that the number of the boundary sensors in (b) is significantly less than that in (a). The mutual information criterion solves the problem that entropy criterion tends to deploy sensors along boundaries very well.

The reconstruction errors of different approaches can be seen in Fig. 3. The adaptive BCS approach and the open-loop entropy approach perform the best in this experiment. It can also be seen that the mutual information criterion performs no better than the entropy criterion.

Both entropy criterion and mutual information criterion are open-loop approaches. Thus, the whole sensor deployment tasks in these two approaches are completed before

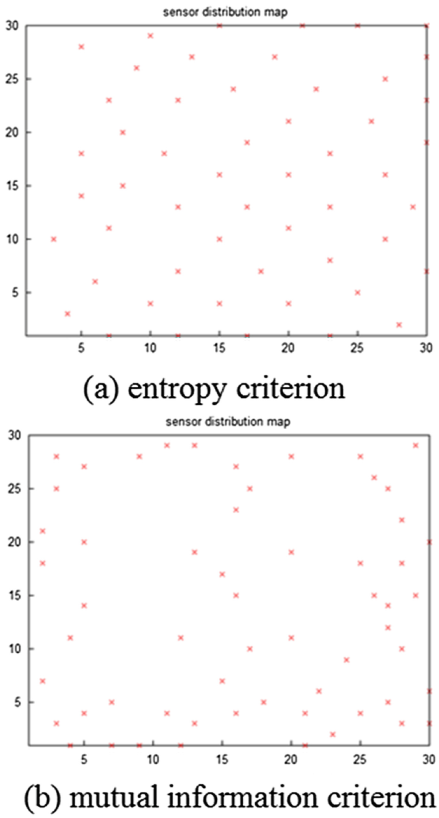


**Fig. 2.** The sensor distribution map of entropy criterion and mutual information criterion (60 sensors)

measurements. Although mutual information criterion prevents deploying too many sensors on the boundary, we cannot say one criterion is dominating the other one in terms of their recovery accuracies.

### 4.2 129 by 129 Ozone Distribution Monitoring

In order to adequately bear out the recovery accuracies brought by the sensor deployment algorithms, we apply these algorithms to a 129 by 129 ozone distribution monitoring experiment in this subsection. Compared with the experiment in Sect. 4.1, this experiment is implemented upon an ozone dataset with much larger resolution. The performance of these environmental monitoring models when dealing with massive environmental signals can be evaluated through this experiment typically. Figure 4 shows the original ozone distribution map that we need to recover.

A comparison of the reconstructed signals with the random sensor deployment approach and the adaptive BCS approach can be seen in Fig. 5 (100 sensors).

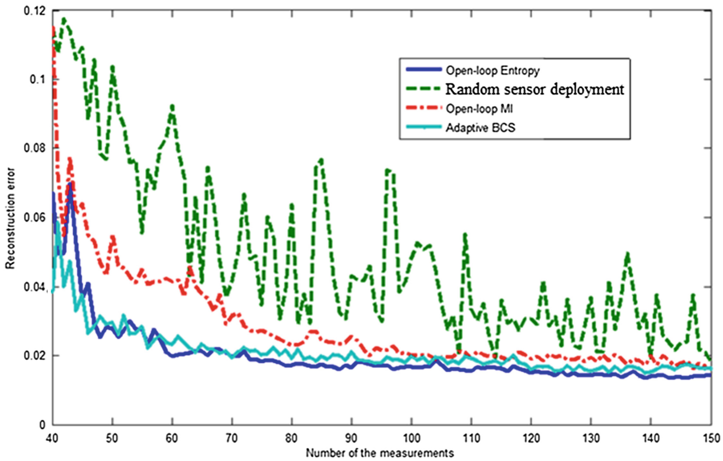


**Fig. 3.** Comparison of the reconstruction errors of different approaches

It can be seen from Fig. 5 that the reconstructed signal with adaptive sensor deployment is much closer to the original one compared with random sensor deployment. The adaptive BCS approach can improve the signal recovery accuracy of the environmental monitoring system significantly.

A comparison of the reconstruction errors with different approaches can be seen in Fig. 6.

It is shown in Fig. 6 that the adaptive BCS performs the best among the three approaches. Its properties are especially suitable for monitoring environmental signals with few sensors.

It is worthwhile to point out that the open-loop approaches depend heavily on the definitions of $\sigma^2_{X_y|X_A}$ shown in Eqs. (4) and (5). The hyper-parameters in $\sigma^2_{X_y|X_A}$ will greatly affect the performance of the algorithms. With same hyper-parameters, the

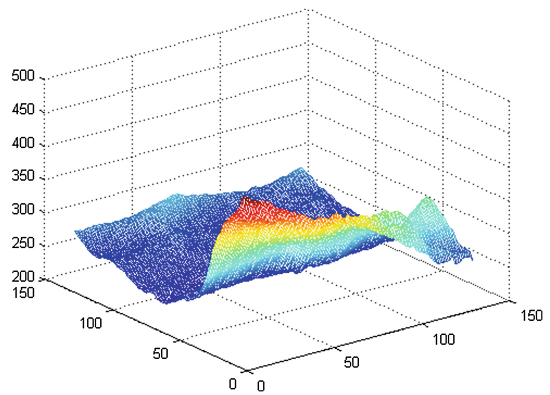


**Fig. 4.** A 129 by 129 ozone distribution in Feb 2005 (Latitudes 62.5 South to 65.5 North with 1 degree step and Longitudes 179.375 West to 19.375 West with 1.25 degree steps)

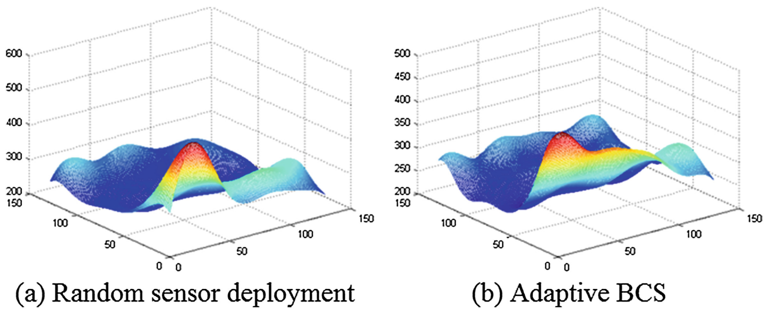


**Fig. 5.** The reconstructed signals with conventional BCS and adaptive BCS (100 sensors used, based on Fig. 4)

open-loop entropy approach performs well in the 30 by 30 ozone distribution monitoring experiment, but not that well in the 129 by 129 ozone distribution monitoring experiment.

Furthermore, the computational complexities for open-loop approaches are generally very high, especially when the set of all possible sensor places V is very large. This is also the reason that we do not have the reconstruction error curve for the open-loop mutual information approach in Fig. 6.

### 4.3 Ozone Monitoring with History Data

Adaptive BCS approach has achieved good performance in our experiments. It is a closed-loop approach that bases on real observed values. That means the sensor locations are closely related to the measurements. The sensor places will be changed if the monitored environmental signal changes. In the real phenomena monitoring industry, the environmental signals are always changing and this will bring difficulties to the adaptive BCS approach.

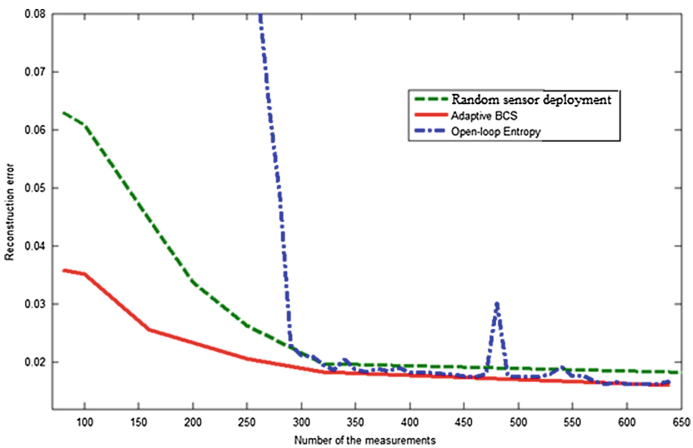


**Fig. 6.** Comparison of the reconstruction errors with three different approaches (Based on Fig. 4)

On the other hand, it is impossible that a fixed sensor deployment works very well for all environmental signals. If the sensor deployment cannot be changed adaptively, what we can do is to find a fixed sensor deployment that generally works well or deploy more sensors.

If the history dataset is similar to the test dataset, the sensor deployment trained by the history data would work also very well for the test data in general. In the real world, a signal usually changes little if the time does not change a lot. Thus, applying the sensor deployment trained by the latest data to the environmental monitoring tasks will work, especially for the environmental signals such as ozone signals that change slowly.

In this subsection, we train the sensor placements with the adaptive BCS approach based on an ozone data of February 2005 and test it on the ozone data of March 2005. Moreover, we will compare its performance with other approaches.

The training data and test data is shown in Fig. 7.

Figure 8(a) shows the reconstructed ozone signal with the sensor deployment trained by Fig. 7(a), and Fig. 8(b) shows the reconstructed ozone signal with the sensor deployment trained by Fig. 7 (b).

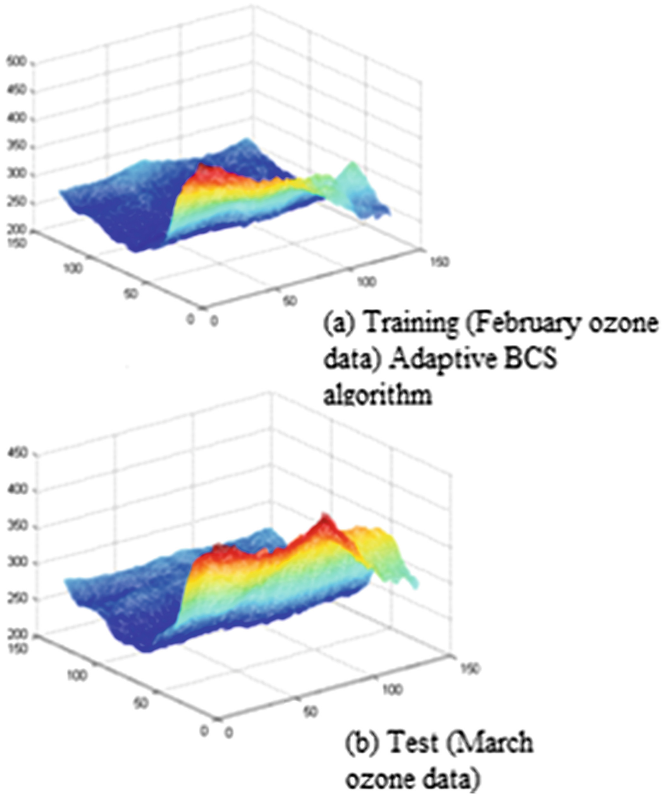


**Fig. 7.** Training ozone data and test ozone data

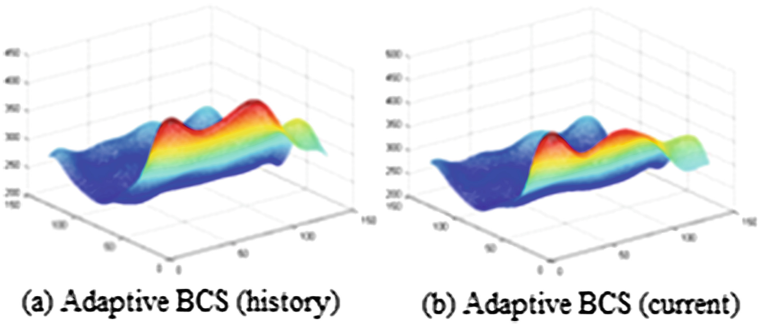


**Fig. 8.** The reconstructed ozone distributions with different approaches (150 sensors used)

It can be seen from Fig. 8 that the adaptive BCS (history) approach performs even better than adaptive BCS (current) approach in terms of the recovery accuracies.

The curves of the reconstruction errors with three different sensor deployments are illustrated in Fig. 9.

It is shown in Fig. 9 that the sensor deployment trained by February ozone data also works very well for the March ozone data. The performance of adaptive BCS (history) is nearly the same as that of adaptive BCS (current). The experiment shows that our strategy to train the sensor deployment with history data is feasible if the locations of the sensors cannot be changed adaptively.

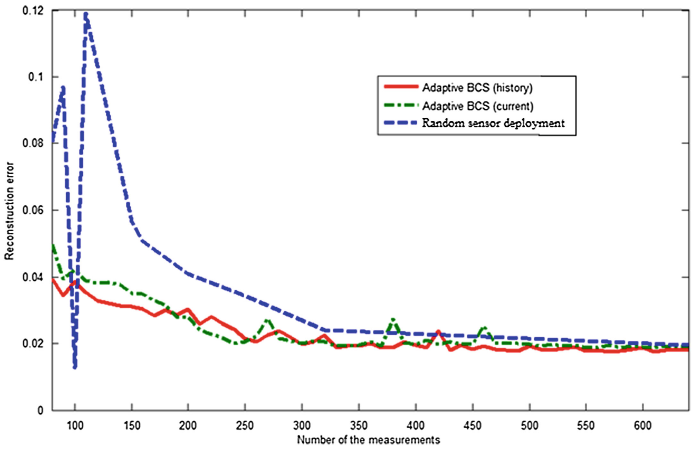


**Fig. 9.** A comparison of the reconstruction errors with three different approaches

## 4.4 Discussion of the Experiment Results

Based on the results of the ozone monitoring experiments, we have the following discussion:

**Open-loop entropy and mutual information**: In our experiments, open-loop approaches perform well with proper hyper-parameters. We used the same hyper-parameters in Sects. 4.1 and 4.2. Based on our experiment results, we believe that adjusting the hyper-parameters carefully in Sect. 4.2 will improve the performance of the open-loop entropy approach further. Adaptive algorithms for adjusting the parameters can be investigated in the future.

Mutual information can solve the problem that entropy criterion tends to place sensors along the boundaries and avoid the "information wasting" of the sensors in many cases. However, mutual information does not perform better than entropy criterion in terms of the reconstruction error.

The computational complexities of the open-loop approaches are generally very high. We built a truncated algorithm by ignoring the influence between far apart sensors (removing the small elements in the $\Sigma$ presented in Sect. 3.1) [11] and applied it to our experiments. We also did not calculate Eqs. (4) and (5) for all possible sensor places, but for selected places. Although these approaches had been used in our experiments, the computational complexities were still very high. This problem is particularly acute in dealing with massive environmental signals.

**Adaptive compressive sensing approach**: Adaptive BCS environmental monitoring model generally performs the best in our experiments. With this environmental monitoring model, we do not need to worry about the hyper-parameters and computational complexities. Thus, adaptive BCS based sensor deployment algorithm is more applicable compared with other sensor deployment algorithm.

Sometimes the sensors cannot be placed adaptively for practical purposes. We proposed to use the adaptive BCS approach to train the sensor placements with the latest history data. The results of our experiments validated the feasibility of this approach.

## 5 Conclusions and Future Work

The environmental monitoring task can be generalised as a problem to solve the equation $Y_{m\times 1} = \Theta_{m\times n} w_{n\times 1} + e = \varnothing_{m\times n} B_{n\times n} w_{n\times 1} + e$. We applied different sensor deployment algorithms and BCS based signal recovery algorithm to environmental monitoring and compare their properties. Their performance was tested under different ozone data resolutions.

It can be seen that compression degree was even lower for larger environmental signals. Generally speaking, compressive sensing is particularly suitable for decreasing the number of the samples needed in the monitoring of massive environmental signals.

The performance of open-loop design methods can be very well with proper hyper-parameters. Thus, scholars have been trying to design better definitions for the variances in the open-loop sensor deployment algorithms to improve their performance, such as nonstationary covariance matrices proposed by Nott [4]. However, the high complexities of the open-loop approaches presented in this paper are hampering these algorithms. Truncated algorithms can be applied to reduce the complexity of open-loop design approaches to some extent, but the problem has not been resolved satisfactorily so far. How to adjust the hyper-parameters adaptively is also a research direction remained to be developed in the future.

Given the measurements of placed sensors, the mechanism of the adaptive Bayesian compressive sensing approach can be explained as deploying the next sensor to the place with the highest estimated variance. With 100 measurements for 16641 global ozone distribution data points, the reconstruction error of adaptive sensor deployment approach was 40 % less than that of random sensor deployment, with 3.52 % and 6.08 % respectively. We then presented the feasibility to train the sensor placements by the adaptive BCS approach with history data and evaluated its performance with experiments. This approach works well especially for steady environmental signals such as ozone.

The adaptive BCS approach generally performed very well in our experiments. It is a greedy algorithm to choose the next sensor location based on current measurements. Sensors are deployed one by one with this algorithm. Thus, the final sensor deployment of this approach may not be global optimal solution. Inspired by the history data based adaptive BCS approach, we may combine the history data to restrict the "score" in the Eq. (7) to improve its performance further. To be specific, we may ignore the places that have been proved to contribute less in the monitoring of history data.

We will apply our environmental monitoring system to real environmental monitoring industries in the future. A lot of unveiled problems in our experiments will appear in environmental monitoring industries. For example, there will be a lot of locations that are not physically reachable in the real world. Furthermore, the world is a multiple dimensions world rather than two dimensions in the experiments, thus where to localise the sensors in the real world becomes a problem. The accuracy of the GPS system will also affect the accuracies of sensor placements.

**Acknowledgments.** This work is partially supported by the Guangdong Innovation Group Project from Guangdong Government of China.

## References

1. Candès, E.J., Romberg, J., Tao, T.: Stable signal recovery from incomplete and inaccurate measurements. Comm. Pure Appl. Math. **59**, 1207–1223 (2006)
2. Donoho, D.L.: Compressed sensing. IEEE Trans. Inf. Theor. **52**(4), 1289–1306 (2006)
3. Yan, S., Wu, C., Dai, W., Ghanem, M., Guo, Y.: Environmental monitoring via compressive sensing. In: Proceedings of the SensorKDD'12 Sixth International Workshop on Knowledge Discovery from Sensor Data, 12–16 August 2012, Beijing, China, pp. 61–68 (2012)
4. Nott, D.J., Dunsmuir, W.T.M.: Estimation of nonstationary spatial covariance structure. Biometrika **89**, 819–829 (2002)
5. Candès, E.J., Braun, N., Wakin, M.B.: Sparse signal and image recovery from compressive samples. ISBI **2007**, 976–979 (2007)
6. Baraniuk, R., Cevher, V., Duarte, M.F., Hegde, C.: Model-based compressive sensing. IEEE Trans. Inf. Theor. **56**(4), 1982–2001 (2010)
7. Ingelrest, F., Barrenetxea, G., Schaefer, G., Vetterli, M., Couach, O., Parlange, M.: SensorScope: application-specific sensor network for environmental monitoring. TOSN **6**(2) (2010)

8. Ramakrishnan, N., Bailey-Kellogg, C., Tadepalli, S., Pandey, V.N.: Gaussian processes for active data mining of spatial aggregates. In: SIAM Data Mining (2005)
9. Caselton, W.F., Zidek, J.V.: Optimal monitoring network designs. Stat. Probab. Lett. **2**(4), 223–227 (1984)
10. Ji, S., Xue, Y., Carin, L.: Bayesian compressive sensing. IEEE Trans. Signal Process. **56**(6), 2346–2356 (2008)
11. Krause, A., Singh, A.P., Guestrin, C.: Near-optimal sensor placements in gaussian processes: theory, efficient algorithms and empirical studies. J. Mach. Learn. Res. **9**, 235–284 (2008)
12. Cover, T.M., Thomas, J.A.: Elements of Information Theory. Wiley, New York (1991)
13. Tipping, M.E.: Sparse Bayesian learning and the relevance vector machine. J. Mach. Learn. Res. **1**, 211–244 (2001)
14. Tipping, M.E., Faul, A.C.: Fast marginal likelihood maximisation for sparse Bayesian models. In: Bishop, C.M., Frey, B.J. (eds.) Proceedings of the Ninth International Workshop on Artificial Intelligence and Statistics, Key West, FL, 3–6 January 2003
15. Davenport, M.A., Duarte, M.F., Eldar, Y.C., Kutyniok, G.: Introduction to Compressed Sensing. In: Eldar, Y., Kutyniok, G. (eds.) Compressed Sensing: Theory and Applications. Cambridge University Press, Cambridge (2011)