A New Fuzzy Associative Memory

Pham Viet Binh, Nong Thi Hoa(✉), Vu Duc Thai,
and Quach Xuan Truong

Thainguyen University of Information Technology and Communication,
Thainguyen, Vietnam
nongthihoa@gmail.com

Abstract. Fuzzy Associative Memory (FAM) is a neural network that stores associations of patterns. The most important advantage of FAM is recalling stored patterns from noisy inputs (noise tolerance). Some FAMs only show associations or content of pattern separately. Therefore, we propose a model of FAM that shows both associations and content of patterns effectively. In learning process, each pair of pattern is learned by the minimum of input and output pattern. Then, all pairs of pattern are generalized by mean of the learning results of each pair. In recalling process, a new threshold is added to improve noise tolerance. We have conducted experiments in pattern recognition to prove the effectiveness of our FAM. Experiment results show that our model tolerates noise better than previous FAMs in two types of noise.

Keywords: Fuzzy associative memory · Associative memory · Artificial neural network · Noise tolerance · Pattern recognition

1 Introduction

FAM is an artificial neural network storing pattern associations and retrieves the desired output pattern from a noisy input pattern by operators of fuzzy logic and mathematical morphology (MM) [7, 8]. FAM consists of two processes, namely, learning process and recalling process. Learning process learns and stores associations of patterns. Recalling process retrieves a stored pattern from a input pattern. FAMs have three important advantages including noise tolerance, unlimited storage, and one pass convergence in which noise tolerance is the most important advantage. It means that FAM recalls stored patterns from noisy inputs by an output function. Thus, FAM has been widely applied in many fields such as image processing, prediction, inference, and estimation.

Design of FAM has been studied to improve the ability of recalling. First, Kosko proposed the first FAM which uses the minimum of patterns in learning process and recalling process [6]. Junbo et al. used fuzzy implication operator to learn pairs of pattern and inherited output function of Kosko [5]. Fulai and Tong inherited learning process of Junbo and used a t-norm for the output function [4]. Xiao et al. proposed a new equation for learning process by using the minimum and the maximum of the input pattern and the output pattern, and applied the multiplication operator for the output function [3]. Wang et al. proposed a new model, which learned patterns by

P.C. Vinh et al. (Eds.): ICCASA 2013, LNICST 128, pp. 219–227, 2014.
DOI: 10.1007/978-3-319-05939-6_22, © Springer International Publishing Switzerland 2014

the division operator and applied the addition for the output function [2]. Sussner and Valle created a family of FAM, which are called Implicative FAM [1]. Patterns are stored by a fuzzy implication operator and computed the output function through an s-norm operator. However, previous FAMs only show content of patterns [1–5] or associations of patterns [1, 6]. Meaning that, no FAM shows both content and association of patterns.

In this paper, we propose a model of FAM presenting more effectively both associations and content of patterns. In learning process, our FAM learns and stores both content and associations of patterns. In recalling process, we improve output patterns of FAM by adding a new threshold. Experiments are conducted in pattern recognition of grey-scale images with two types of noise. Results from experiments show that proposed FAM recalls better than previous FAMs.

The rest of the paper is organized as follows. In Sect. 2, background is presented. Section 3 show the design of our novel model. The next section is experiment results to show the advantages of the proposed FAM.

2 Over View of Fuzzy Logic, Mathematical Morphology and Associative Memory

2.1 Operators of Fuzzy Logic

Let A and B are fuzzy subsets of a non-empty set X.

The **intersection** of A and B is defined as

$$\mu_{A \wedge B}(x) = \min\{\mu_A(x), \mu_B(x)\} \tag{1}$$

for all $x \in X$.

The **union** of A and B is defined as

$$\mu_{A \vee B}(x) = \max\{\mu_A(x), \mu_B(x)\} \tag{2}$$

for all $x \in X$.

The **fuzzy conjunction** of A and B is an increasing mapping C: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies

$C(0, 0) = C(0, 1) = C(1, 0) = 0$ and $C(1, 1) = 1$.

For example, the minimum operator and the product are typical examples.

A fuzzy conjunction T: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies

$T(x, 1) = x$ for $x \in [0, 1]$ is called triangular norm or t-norm.

The fuzzy conjunctions C_M, C_P, and C_L are examples of t-norms.

$$C_M(x, y) = x \wedge y \tag{3}$$

$$C_P(x, y) = x.y \tag{4}$$

$$C_L(x, y) = 0 \vee (x + y - 1) \tag{5}$$

A **fuzzy disjunction** is an increasing mapping D: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies

D(0, 0) = 0 and D(0, 1)= D(1, 0) = D(1, 1) = 1.
A fuzzy disjunction S: [0, 1] × [0, 1] → [0, 1] that satisfies
S(1, x) = x for every x ∈ [0, 1] is called triangular co-norm or short s-norm.
The following operators represent s-norms:

$$D_M(x, y) = x \vee y \tag{6}$$

$$D_P(x, y) = x + y - x.y \tag{7}$$

$$D_L(x, y) = 1 \wedge (x + y) \tag{8}$$

An operator I: [0, 1] × [0, 1] → [0, 1] is called a **fuzzy implication** if I extends the usual crisp implication on [0, 1] × [0, 1] with I(0, 0) = I(0, 1) = I(1, 1) = 1 and I(1, 0) = 0.

Some particular fuzzy implications:

$$I_M(x, y) = \begin{cases} 1, x \leq y \\ y, x > y \end{cases} \tag{9}$$

$$I_P(x, y) = \begin{cases} 1, x \leq y \\ \frac{y}{x}, x > y \end{cases} \tag{10}$$

$$I_L(x, y) = 1 \wedge (y - x + 1) \tag{11}$$

2.2 Basic Operators of Mathematical Morphology

A complete lattice is defined as a partially ordered set L in which every (finite or infinite) subset has an infimum and a supremum in L. For any Y ⊆ L, the infimum of Y is denoted by the $\wedge Y$ and the supremum is denoted by the $\vee Y$. The class of fuzzy sets inherits the complete lattice structure of the unit interval [0, 1].

Erosion and dilation are two basic operators of MM. A erosion is a mapping ε from a complete lattice L to a complete lattice M that satisfies the following equation:

$$\varepsilon\left(\bigwedge Y\right) = \bigwedge_{y \in Y} \varepsilon(y) \tag{12}$$

Similarly, an operator δ: L → M is called dilation if it satisfies the following equation:

$$\varepsilon\left(\bigvee Y\right) = \bigvee_{y \in Y} \varepsilon(y) \tag{13}$$

2.3 Associative Memory

Associative memory (AM) stores pattern associations and retrieves the desired output pattern upon presentation of a noisy input pattern [9]. The associative memory is defined as follows:

Given a finite set of desired associations $(\mathrm{A}^k, \mathrm{B}^k), \mathrm{k} = 1, \ldots, \mathrm{p}$, determine a mapping G such that $\mathrm{G}(\mathrm{A}^k) = \mathrm{B}^k$ for all $\mathrm{k} = 1, \ldots, \mathrm{p}$. Moreover, the mapping G need have the ability of noise tolerance. Meaning, $G(A'^k)$ should be equal to B^k for noisy or incomplete version A'^k of A^k.

The set of associations $(A^k, B^k), k = 1, \ldots, p$ is called *fundamental memory set* and each association (A^k, B^k) in this set is called a *fundamental memory*. An auto-associative memory is the fundamental memory set with form $(A^k, A^k), k = 1, \ldots, p$. The memory is said to be hetero-associative if the output B^k is different from the input A^k.

The process of determining G is called learning process and the mapping G is called *associative mapping*. A neural associative memory when the associative mapping G is described by an artificial neural network. In particular, we have a FAM if the associative mapping G is given by a fuzzy neural network and the patterns A^k and B^k are fuzzy sets for every $k = 1, \ldots, p$.

3 Proposed a Novel Fuzzy Associative Memory

We propose a novel FAM that shows both the associations and content of pairs of pattern. In learning process, we use the minimum of input pattern and output pattern in a pair of patterns to store the association of two patterns. Then, the mean of every association shows content of patterns. Recalling process improves unwanted outputs based on adding a new threshold in the output function.

Assuming FAM stores **p** pairs of pattern (A^k, B^k) where $A^k = \left(A_1^k, \ldots, A_m^k\right)$and $B^k = \left(B_1^k, \ldots, B_n^k\right)$. The design of our novel FAM is presented as follow:

Step 1: Learning process consists of following steps:

- Learn the association of the pattern pair (A^k, B^k) which is stored in weight matrix W^k.

$$W_{ij}^k = \min\left\{A_i^k, B_j^k\right\} \tag{14}$$

- Generalize the associations of pattern pairs by mean of associations and store in general weight matrix W.

$$W_{ij} = \frac{1}{p}\sum_{k=1}^{p} W_{ij}^k \tag{15}$$

Step 2: Recalling process is executed as follow:

- Output Y is computed from an input X by the following equation:

$$Y_j = \bigvee_{i=1}^{m} X_i.W_{ij} \vee \theta_j \tag{16}$$

- θ_{j} is computed by:

$$\theta_j = \frac{1}{p} \bigwedge_{k=1}^{p} B_j^k \tag{17}$$

Our novel FAM has three advantages of FAM including noise tolerance, unlimited storage, and one pass convergence. Moreover, we improve both learning and recalling process. Therefore, proposed FAM can improve noise tolerance significantly.

4 Experiment Results

We conduct two experiments for two sets of patterns selecting from faces database and grey-scale image database. Moreover, distorted noise and "pepper & salt" noise are tested to measure noise tolerance of FAMs in both two associative modes (respectively, auto-associative mode and hetero-associative mode).

Our FAM is compared to standard FAMs [1–6]. We choose results of the best models from each study to compare. We measure noise tolerance of FAMs by the peak signal-to-noise ratio (PSNR). PSNR is used to measure quality between a training pattern and a output pattern. The higher the PSNR, the better the quality of the output image. PSNR is computed by the following equation:

$$PSNR = 40 log_{10} \frac{R^2}{MSE} \tag{18}$$

where R is the maximum fluctuation in data type of input image. Working with grey-scale images, value of R is 255. MSE represents the cumulative squared error between the training pattern and a output pattern. MSE is formulated as follow:

$$MSE = \frac{\sum_{M,N} (I_1(m,n) - I_2(m,n))^2}{M * N} \tag{19}$$

Fig. 1. Training patterns and noisy inputs in Experiment 1. The first image of each row is the training image and nine next images are noisy inputs.

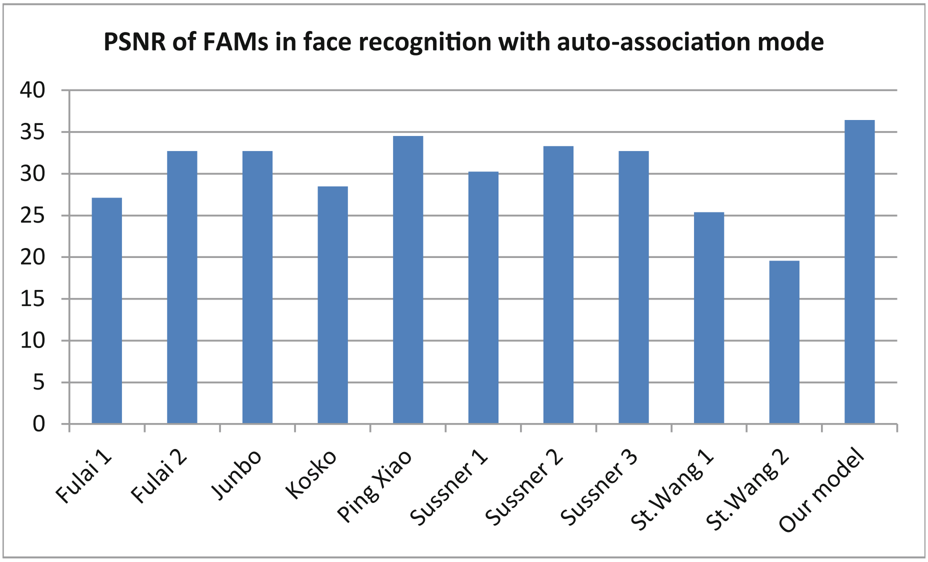

Fig. 2. In auto-association mode, PSNR of models in face recognition from distorted inputs in Experiment 1.

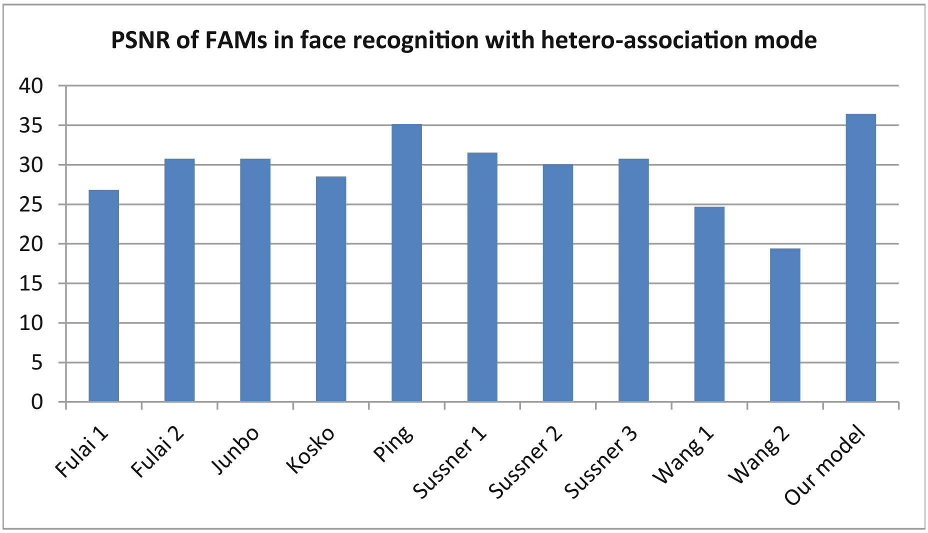

Fig. 3. In hetero-association mode, PSNR of models in face recognition from distorted inputs in Experiment 1.

Fig. 4. Training patterns in Experiment 2.

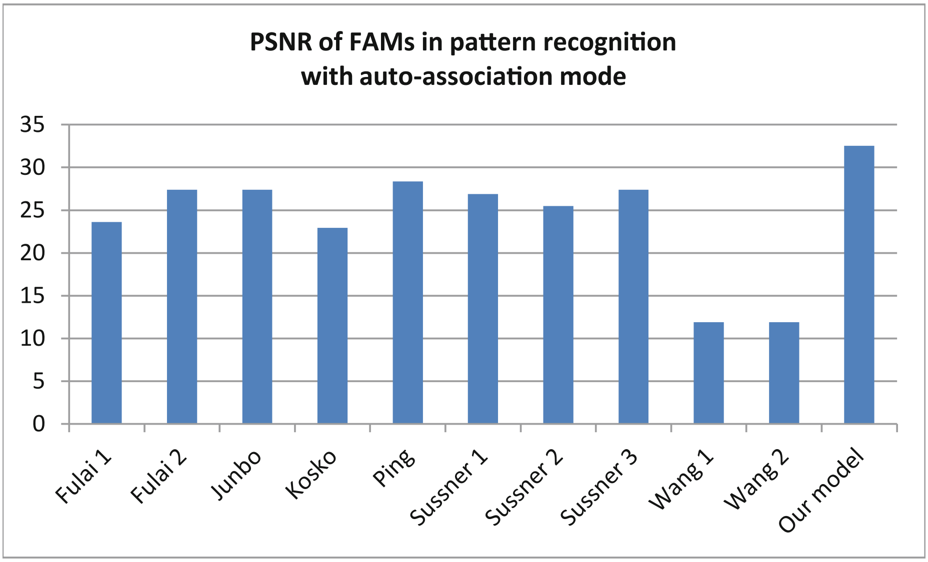

Fig. 5. In auto-association mode, PSNR of models in pattern recognition from distorted inputs in Experiment 2.

4.1 Experiment 1: Face Recognition from Distorted Inputs

We select 4 images from the faces database of AT & T of Laboratories Cambridge. Each person has 10 images, including one normal image and nine distorted images. Normal images are training patterns and nine distorted images are noisy inputs. Figure 1 shows patterns of this experiment.

In auto-associative mode, Peak Signal-to-Noise Ratio of FAMs is shown in Fig. 2. In hetero-associative mode, performance of FAMs is presented in Fig. 3. Data from Figs. 2 and 3 show that our FAM recall better than standard FAM. Our model recalls higher than the best second model about 7 %.

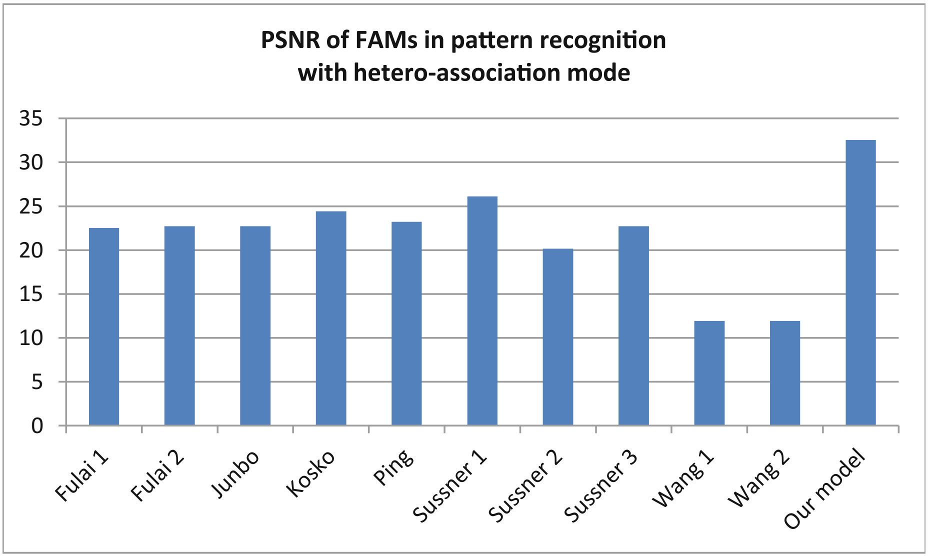

Fig. 6. In hetero-association mode, PSNR of models in pattern recognition from distorted inputs in Experiment 2.

4.2 Experiment 2: Pattern Recognition from Incomplete Inputs

We select 10 images from the grey-scale image database[1] of the Computer Vision Group, University of Granada, Spain. Normal images are training patterns and nine noisy images are inputs for experiments. Noisy inputs are made from the training images by deleting a part of image with many different shapes. Figure 4 shows training images of Experiment 2.

Figure 5 and Fig. 6 show Peak Signal-to-Noise Ratio of FAMs in auto-associative mode and hetero-associative mode. Results from Experiment 2 show that the ability of recalling of our FAM is the highest. Our model greatly improves noise tolerance greatly comparing to best second model (respectively, 28 % in auto-associative mode and 48 % in hetero-associative mode).

5 Conclusion

In this paper, we have proposed a new FAM showing both the content as well as the associations of patterns. We conduct experiments for pattern recognition with two types of noise including distorted noise and incomplete noise. Experiment results show that noise tolerance of our FAM is better than other FAMs in both auto-associative mode and hetero-associative mode.

[1] http://decsai.ugr.es/cvg/wellcome.html

In auto-associative mode, our model works effectively in pattern recognition with a small number of training patterns. In hetero-associative mode, proposed model is applied for inference applications such as predicting prices of the stock and designing controller. We will investigate developing a new learning rule of FAM to increase noise tolerance with many types of noise in the future.

References

1. Sussner, P., Valle, M.E.: Implicative fuzzy associative memories. IEEE Trans. Fuzzy Syst. **14**(6), 793–807 (2006)
2. Wang, S.T., Lu, H.J.: On new fuzzy morphological associative memories. IEEE Trans. Fuzzy Syst. **12**(3), 316–323 (2004)
3. Xiao, P., Yang, F., Yu, Y.: Max-min encoding learning algorithm for fuzzy max-multiplication associative memory networks. In: Proceedings of 1997 IEEE International Conference on Systems, Man, and Cybernetics (1997)
4. Chung, F., Lee, T.: Towards a high capacity fuzzy associative memory model. In: Proceedings of 1994 IEEE International Conference on Neural Networks (1994)
5. Junbo, F., Fan, J., Yan, S.: A learning rule for FAM. In: Proceedings of 1994 IEEE International Conference on Neural Networks, pp. 4273–4277 (1994)
6. Kosko, B.: Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence. Prentice Hall, Englewood Cliffs (1992)
7. Zadeh, L.A.: Fuzzy sets and information granularity. In: Gupta, M., Ragade, R., Yager, R. (eds.) Advances in Fuzzy Set Theory and Applications Book. North Holland Publishing, Amsterdam (1979)
8. Serra, J.: Image Analysis and Mathematical Morphology. Academic Press, London (1982)
9. Sussner, P., Valle, M.E.: Fuzzy associative memories and their relationship to mathematical morphology. In: Pedrycz, W., Skowron, A., Kreinovich, V. (eds.) Handbook of Granular Computing, pp. 1–41. Wiley-Interscience, New York (2008)