Mathematical Model of Hit Phenomena as a Theory for Human Interaction in the Society

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Abstract. A mathematical model for the hit phenomenon in entertainment within a society is presented as a stochastic process of interactions of human dynamics. The model uses only the time distribution of advertisement budget as an input, and word-of-mouth (WOM) represented by posts on social network systems is used as data to compare with the calculated results. The unit of time is a day. The calculations for the Japanese motion picture market based on to the mathematical model agree very well with the actual residue distribution in time.

Keywords: Hit phenomena, stochastic process, movie.

1 Introduction

Human interaction in real society can be considered in the sense of "many body" theory where each person can be treated as atoms or molecules in the ordinary many body theory. With the popularization of social network systems (SNS) like blogs, Twitter, Facebook, Google+, and other similar services around the world, interactions between accounts can be stocked as digital data. Though the SNS society is not the same as real society, we can assume that communication in the SNS society is very similar to that in real society. Thus, we can use the huge stock of digital data of human communication as observation data of real society [1-4]. Using this observed huge data (so -called "Big Data"), we can apply the method of statistical physics to social sciences. Since word-of-mouth (WOM) is very significant in marketing science [5-8], such analysis and prediction of the digital WOM in the sense of statistical physics become very important today.

Recently, we present a mathematical theory for hit phenomena where effect of advertisement and propagation of reputation and rumors due to human communications are included as the statistics physics of human dynamics [9]. This theory has also been applied to the analysis of the local events in Japan successfully [10]. Our model was originally designed to predict how word-of-mouth communication spread over social networks or in the real society, applying it to conversations about movies in particular, which was a success. Moreover, we also found that when they overlapped their predictions with the actual revenue of the films, they were very similar.

K. Glass et al. (Eds.): COMPLEX 2012, LNICST 126, pp. 159–164, 2013.

In the model [9], the key factors to affect the mind of the consumers are three: advertisement effects, the word-of-mouth (WOM) effects and the rumour effects. Recognising that WOM communication, as well as advertising, has a profound effect on whether a person goes to see a movie or not, whether this is talking about it to friends (direct communication or WOM) or overhearing a conversation about it in a café (indirect communication or the rumour), we accounted for this in our calculations. The difference between our theory and the previously presented researches [11-26] are discussed in ref.9.

We found that the effects of advertisements and WOM are included incompletely and the rumor effect is not included in the previous works [11-26]. Therefore, from the point of view of statistical physics, we present in our previous paper a model to include these three effects: the advertisement effect, the WOM effect, and the rumor effect. The presented model has been applied to the motion picture business in the Japanese market, and we have compared our calculation with the reported revenue and observed number of blog postings for each film.

In this paper, the responses in social media are observed using the social media listening platform presented by Hottolink. Using the data set presented by M Data Co.Ltd monitors the exposure of each films.

2 Model

We start the modeling from the viewpoint of the individual consumer. We define the purchase intention of the individual consumer, labeled i, at time t as $I_i(t)$. We assume that the number of products adopted until time t can be written as

$$Y(t) = p\int_{t_0}^{t}\sum_{i=1}^{N} I_i(t)\,dt \quad , \tag{1}$$

where N is the maximum number of adopted persons, p is the price of the product and t_0 is the release date of the product. Thus, our problem is to define the equation of the purchase intention of each consumer $I_i(t)$.. We consider the modeling of the effects of advertisements, WOM, and rumor for the purchase intention in the following subsections.

The advertisement effect through mass media like TV, newspapers, magazines, the Web, Facebook, or Twitter is modeled as an external force for the equation of the purchase intention of the individual consumer:

$$\frac{dI_i(t)}{dt} = c_i A(t) \quad , \tag{2}$$

where A(t) is the time distribution of the effective advertisement effect per unit time and the coefficient describes the impression of the advertisement for consumer i. The external force A(t) can be considered trends in the world or political pressure on the market.

Usually, a film's success spreads through WOM. Such WOM sometimes has a very significant effect on the success of the movie. Thus, the WOM effect should be included in our theory. The WOM effect should be distinguished into two types: WOM direct from friends, and indirect WOM as rumors. We name the WOM effect between friends *direct communication,* because customers obtain information directly from their friends. In the previous paper[9], we include also the communication between non-adopters. It is very significant for movie entertainment, especially before the opening of the movie. Let us consider that person i hears information from person j. The probability per unit of time for the information to affect the purchase intention of person i can be described as $D_{ij}I_j(t)$, where $I_j(t)$ is the purchase intention of person j and D_{ij} is the coefficient of the direct communication. Thus, we can write the effect of the direct communication as follows:

$$\sum_{j=1}^{N} D_{ij} I_j(t) , \tag{3}$$

where the summation is done without $j = i$.

In this paper, rumor effect is named *indirect communication.* In this form of communication, a person hears a rumor while chatting on the street, overhearing a conversation from the next table in a restaurant or on a train, or finds the rumor in blogs or on Twitter. To construct the theory using mathematics, we focus on one person who listens to a conversation happening around him/her. Let us consider that person I overhears the conversation between person j and person k. The strength of the effect of the conversation can be described as $D_{jk}I_j(t)I_k(t)$. The probability per unit time for the conversation to affect the purchase intention of person i is defined as $Q_{ijk}D_{jk}I_j(t)I_k(t)$, where Q_{ijk} is the coefficient. Thus, the indirect communication coefficient can be defined as $P_{ijk} = Q_{ijk}D_{jk}$.

Therefore, direct communication is two-body interaction and indirect communication is three-body interaction. Thus, our theory for the hit phenomenon like hit movies can be described as the equation of the purchase intention of person i with two-body interaction and three-body interaction terms.

According to the ref.9, we write down the equation of purchase intention at the individual level as

$$\frac{dI_i(t)}{dt} = -aI_i(t) + \sum_j d_{ij} I_j(t) + \sum_j \sum_k h_{ijk} d_{jk} I_j(t) I_k(t) + f_i(t) \tag{4}$$

where d_{ij}, h_{ijk}, and $f_i(t)$ are the coefficient of the direct communication, the coefficient of the indirect communication, and the random effect for person i, respectively. We consider the above equation for every consumer so that i = 1, …, N_p.

Taking the effect of direct communication, indirect communication, and the decline of audience into account, we obtain the above equation for the mathematical model for the hit phenomenon. The advertisement and publicity effect for each person can be described as the random effect $f_i(t)$.

Eq. (12) is the equation for all individual persons, but it is not convenient for analysis. Thus, we consider here the ensemble average of the purchase intention of individual persons as follows:

$$\langle I(t) \rangle = \frac{1}{N} \sum_i I_i(t) \tag{5}$$

Taking the ensemble average of eq. (12), we obtain for the left-hand side:

$$\left\langle \frac{dI_i(t)}{dt} \right\rangle = \frac{1}{N}\sum_i \frac{dI_i(t)}{dt} = \frac{d}{dt}\left(\frac{1}{N}\sum_i I_i(t)\right) = \frac{d\langle I\rangle}{dt} \tag{6}$$

For the right-hand side, the ensemble average of the first, second, and third is as follows:

$$\langle -aI_i\rangle = -a\frac{1}{N}\sum_i I_i(t) = -a\langle I(t)\rangle \tag{7}$$

$$\left\langle \sum_j d_{ij} I_j(t) \right\rangle = \left\langle \sum_j dI_j(t) \right\rangle = \frac{1}{N}\sum_i \sum_j dI_j(t) = \sum_i d\frac{1}{N}\sum_j I_j(t) = Nd\langle I(t)\rangle \tag{8}$$

$$\begin{aligned}\left\langle \sum_j \sum_k p_{ijk} I_j(t) I_k(t) \right\rangle &= \left\langle p\sum_j \sum_k I_j(t) I_k(t) \right\rangle \\ &= \frac{1}{N}\sum_i p\sum_j \sum_k I_j(t) I_k(t) \\ &= \sum_i p\frac{1}{N}\sum_j \sum_k I_j(t) I_k(t) \\ &= Np\sum_i \frac{1}{N}\sum_j I_j(t)\frac{1}{N}\sum_k I_k(t) \\ &= N^2 p\langle I(t)\rangle^2\end{aligned} \tag{9}$$

where we assume that the coefficient of the direct and indirect communications can be approximated to be

$$d_{ij} \cong d$$
$$h_{ijk} d_{jk} = p_{ijk} \cong p$$

under the ensemble average.

For the fourth term, the random effect term, we consider that the random effect can be divided into two parts: the collective effect and the individual effect:

$$f_i(t) = \langle f(t)\rangle + \Delta f_i(t) \tag{10}$$

$$\langle f_i(t)\rangle = \frac{1}{N}\sum_i f_i(t) = \langle f(t)\rangle \tag{11}$$

where $\Delta f_i(t)$ means the deviation of the individual external effects from the collective effect, $\langle f_i(t)\rangle$. Thus, we consider here that the collective external effect term $\langle f_i(t)\rangle$ corresponds to advertisements and publicity to persons in the society. The deviation term $\Delta f_i(t)$ corresponds to the deviation effect from the collective advertisement and publicity effect for individual persons, which we can assume to be

$$\left\langle \Delta f_i(t) \right\rangle = \frac{1}{N} \sum_i \Delta f_i(t) = 0 \tag{12}$$

Therefore, we obtain the equation for the ensemble-averaged purchase intention in the following manner as shown in ref.9:

$$\frac{d\left\langle I(t) \right\rangle}{dt} = -a\left\langle I(t) \right\rangle + D\left\langle I(t) \right\rangle + P\left\langle I(t) \right\rangle^2 + \left\langle f(t) \right\rangle \tag{13}$$

where

$$Nd = D$$
$$N^2 p = P \tag{14}$$

Eq. (13) can be applied to the purchase intention in the real market.

3 Calculation

Here, we show the results for three movies in figs.3 4 and 5. In fig.3, we show the observed daily number of blog postings and calculation for the Japanese movie "Unfear the answer". The calculation shows good agreement with the observed blog postings.

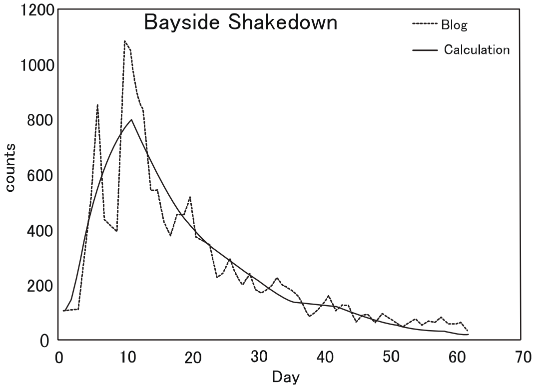

Fig. 3. Our calculation and the corresponding blog posting for the Japanese move, *Bayside Shakedown (Odoru Daisousasen).* The dashed curve is the observed blog posting counts and the solid curve is our calculation.

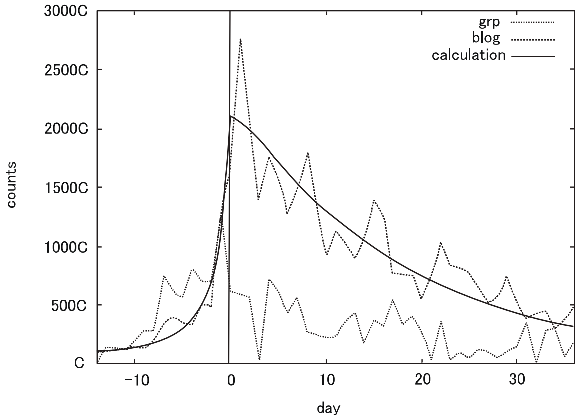

Fig. 4. Our calculation and the corresponding blog posting for the Japanese move, *Once in a Blue Moon.* The dotted curve is the observed blog posting counts, the gray curve is the gross rate point and the red curve is our calculaion.

In fig.4, we show the observed daily number of blog postings, the advertisement cost curve (gross rate point) and calculation for the Japanese movie "Once in a Blue Moon". The calculation shows good agreement with the observed blog postings.

We find that our calculations agree well with the observed data of the daily number of blog postings. It means that the exposure data from the data of M Data works as well as the daily advertisement cost used in our previous paper of ref.9. It means that the application of this model is very easy for many people, because everyone can buy the exposure data of each movie or something from M Data. The daily advertisement cost is very difficult to get even for scientific research in Japan and probably all over the world.

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