Network Coding Advantage over MDS Codes for Multimedia Transmission via Erasure Satellite Channels

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Abstract. In this paper, we focus on the performance analysis of packet-level Forward Error Correction (FEC) codes based on Systematic Random Linear Network Coding (SRNC) for multimedia transmission via erasure satellite channels. A performance comparison is presented against maximum distance separable (MDS) codes currently used in state-of-the-art satellite transmission air interfaces, specifically Reed Solomon (RS) codes. Firstly, a theoretical analysis is presented for which we first develop a matricial erasure channel model. The theoretical analysis shows that both the RS and SRNC have, as expected, similar error correction performance over different packet erasure lengths for commonly used size fields. Secondly, we present an on-the-fly progressive algorithm for SRNC, which takes advantage of the inherent randomness of SRNC encoding. Thirdly, a performance comparison is presented for two different satellite scenarios: 1) DVB-S2/RCS railway scenario and 2) Broadband Global Area Network (BGAN) mobile scenario. We use real channel parameters for the first scenario and channel traces of video streaming sessions for the second scenario. Our simulation results confirm that both the RS codes and SNRC have the same packet recovery capabilities. However, for low coding rates, SRNC is shown to achieve up to 71% delay gain as compared to RS codes.

Keywords: Network Coding, Erasure Channel, Forward Error Correction, MDS codes.

1 Introduction

The demands for satellite services are growing for wide range of applications such as land-mobile, emergency, disaster relief, etc. However, the performance over the satellite systems is usually limited by the erasures caused by fading events, blockage, congestion due to transmission over best effort channels etc. To recover from the erasure events, satellite services specifications like Digital Video Broadcasting (DVB) [1] – [3] has adopted an link layer (LL) LL-FEC. In general, the requirements for FEC codes are MDS encoding if possible and low latency. However, due to the advancement in the FEC codes, it is important to investigate their performance for multimedia transmission via erasure satellite channels. In this work we leave out complexity issues and focus on performance.

R. Dhaou et al. (Eds.): PSATS 2013, LNICST 123, pp. 199–210, 2013.

In this work, we focus on the FEC based on SRNC [4] – [6] for multimedia transmission. SRNC provides proactive random retransmissions without the prior knowledge of lost packets over the network. The decoding can be done progressively at the receiver due to the inherent random structure of the code. Hence, the delay in the delivery time of recover packets can be substantially reduced.

The remainder of this paper is organized as follows. In Section 2, we introduce the system model. In Section 3, we present the theoretical performance analysis of the SRNC codes. Section 4 presents the numerical results and Section 5 concludes the paper.

Notations: Let $\mathbb{F}_q$ be a finite field. We denote $\mathbb{F}_q^{a_1 \times a_2}$ the set of all $a_1 \times a_2$ matrices with entries in $\mathbb{F}_q$, and $\mathbb{F}_q^{a_1}$ as the set of all column vectors with a_1 entries in $\mathbb{F}_q$. We will use boldface uppercase letters to denote matrices and boldface letters to denote column vectors. $\mathbf{I}_a$ is used to denote $a \times a$ identity matrix. We use the notation $\cup \mathbf{I}_a^{a \times a_1}$ to represent the set that contains a_1 distinct columns of identity matrix $\mathbf{I}_a$.

2 System Model

We consider a satellite scenario for the multimedia transmission where the sender transmits data packets via satellite. The receiver of interest could be a mobile receiver either within the satellite network or in a terrestrial network in the case of a hybrid architecture.

Let us assume that the time is slotted and source injects K source packets at time slot t. We assume that the source input can be modeled as an input unit $\mathbf{S}(t) \in \mathbb{F}_q^{M \times K}$ where each packet is a column vector of M symbols. We denote the encoding function by $\mathcal{E}_t : \mathbb{F}_q^{M \times K} \to \mathbb{F}_q^{M \times N}$, that maps the K source packets to N encoded packets. The coding rate is given by $R = K/N$.

These N encoded packets are represented by $\mathbf{X}(t) \in \mathbb{F}_q^{M \times N}$ where each encoded packet is a column vector of M symbols. These encoded packets are the function of source packets, given by $\mathbf{X}(t) = \mathcal{E}\left(\mathbf{S}(t)\right)$. In our case, the encoding model is linear where $\mathbf{X}(t) = \mathcal{E}_t\left(\mathbf{S}(t)\right) = \mathbf{S}(t)\mathbf{G}(t)$ with generator matrix $\mathbf{G}(t) \in \mathbb{F}_q^{K \times N}$.

We consider an additive-multiplicative erasure-error channel model where the channel can 1) erase the transmitted packet or 2) introduce additive errors within the transmitted packet. Let us denote the channel function $\mathcal{H}_t : \mathbb{F}_q^{M \times N} \to \mathbb{F}_q^{M \times N_r(t)}$, that maps the N encoded packets to $N_r(t)$ received packets. We denote the received unit by the matrix $\mathbf{Y}(t) \in \mathbb{F}_q^{M \times N_r(t)}$ such that each received packet is a column vector of M symbols. In our case, the channel model is linear and we have,

$$\mathbf{Y}(t) = \mathcal{H}_t\left(\mathbf{X}(t)\right) = \mathbf{X}(t)\mathbf{H}(t) + \mathbf{Z}(t) = \mathbf{S}(t)\mathbf{G}(t)\mathbf{H}(t) + \mathbf{Z}(t) \tag{1}$$

with $\mathbf{H}(t) \in \cup \mathbf{I}_N^{N \times N_r(t)}$ and $\mathbf{Z}(t) \in \mathbb{F}_q^{M \times N_r(t)}$. The matrix $\mathbf{H}(t)$ is used to represent the erasure events, where $\mathbf{H}(t)$ consists of all the columns of $\mathbf{I}_N$ except the columns $i \in \{1, 2, , ..., N\}$ if the i^{th} column/packet is erased by the channel and the matrix $\mathbf{Z}(t)$ represents the additive errors.

In this work, we will focus only on the erasure events such that with (1) we have,

$$\mathbf{Y}(t) = \mathbf{X}(t)\mathbf{H}(t) = \mathbf{S}(t)\mathbf{G}(t)\mathbf{H}(t) \tag{2}$$

We also denote $E(t) = N - N_r(t)$ as the total number of packets erased by the channel for the time slot t.

In particular, our matricial erasure channel model can be used for channel encoding for any packet-level erasure channel. Specifically, in this work, we compare the SRNC and the RS codes.

For, the RS codes, generator matrix is given by $\mathbf{G}^{RS}(t) = \left[\mathbf{I}_K | \mathbf{C}^{RS}(t)\right]$ where $\mathbf{C}^{RS}(t) \in \mathbb{F}_q^{K \times N-K}$. An explicit formula for the $\mathbf{C}^{RS}$ can be found in [7]. Using (2), with RS codes, we have,

$$\mathbf{Y}^{RS}(t) = \mathbf{X}^{RS}(t)\mathbf{H}(t) = \mathbf{S}(t)\mathbf{G}^{RS}(t)\mathbf{H}(t) \tag{3}$$

For, the SRNC, generator matrix is given by $\mathbf{G}^{SRNC}(t) = \left[\mathbf{I}_K | \mathbf{C}^{SRNC}(t)\right]$ where $\mathbf{C}^{SRNC}(t) \in \mathbb{F}_q^{K \times N-K}$ and each symbol of $\mathbf{C}^{SRNC}(t)$ is chosen independently and equiprobably from $\mathbb{F}_q$. Using (2), with SRNC, we have,

$$\mathbf{Y}^{SRNC}(t) = \mathbf{X}^{SRNC}(t)\mathbf{H}(t) = \mathbf{S}(t)\mathbf{G}^{SRNC}(t)\mathbf{H}(t) \tag{4}$$

Usually, each column of K symbols from $\mathbf{G}^{SRNC}(t)$, also known as coding coefficients, is attached with the corresponding column/packet of $\mathbf{X}^{SRNC}(t)$. These coding coefficients are used at the receiver for decoding [4]. This introduces an extra overhead of $K \log(q)$ bits, therefore several other approaches [8]-[9] have been proposed to reduce such kind of overhead. However, this particular aspect is out of the scope of the paper and hence, we assume that the coding coefficients are known at the receiving ends. At the receiver side, we denote $\hat{\mathbf{G}}^{SRNC}(t) \in \mathbb{F}_q^{K \times N_r(t)}$ as the matrix with columns as coding coefficients (locally retrieved) corresponding to the received packets.

3 Theoretical Performance Analysis

In this Section, we compare the SRNC and the RS codes based on their theoretical erasure correction performance. We also discuss the delay in the delivery time for both of these schemes. For the simplicity in showing our results, we drop the index t in this section.

3.1 RS: Probability of Successful Decoding (p^{RS})

RS codes are the class of MDS codes that operate on $GF(q)$. In particular, $RS(N, K)$ codes can correct up to $d_{MDS} - 1 = N - K$ erasures where d_{MDS} is the minimum distance of the RS codes. Let us denote the probability of successful decoding of K source packets using RS codes by p^{RS}. Hence, if the number of packet erasures over the coding window of N packets are less than d_{MDS}; i.e., $E \leq d_{MDS} - 1$, then $p^{RS} = 1$ and if $E > d_{MDS}$, then $p^{RS} = 0$.

3.2 SRNC: Probability of Successful Decoding (p^{SRNC})

To evaluate the erasure correction performance of SRNC, let us first denote the probability of successful decoding of K source packets using $SRNC(N, K)$ codes by p^{SRNC}. Note that for the systematic coding, first K source packets are transmitted in the systematic phase and then $N-K$ encoded packets are transmitted in the non-systematic phase. To decode the K source packets successfully, the receiver should receive at least K independent packets out of N_r received packets, which means, that the rank of the locally retrieved coefficient matrix $\hat{\mathbf{G}}^{SRNC}(t)$ corresponding to the received packets should be K [4]. Therefore, to recover the K source packets successfully, following conditions should be satisfied:

1. L packets should be received from the first K transmissions of the systematic phase. The coding coefficients, corresponding to these L packets, are always independent as they belong to the columns of the identity matrix (4).
2. $J = N_r - L$ packets should be received from the next $N-K$ encoded packets of the non-systematic phase.
3. The coefficient matrix $\hat{\mathbf{G}}^{SRNC}$ of dimensions $K \times N_r$ should have full rank K given that L columns are independent.

Given these three conditions, we have

$$p^{SRNC} = \frac{1}{K} \sum_{L=1}^{K} p_{\hat{G}}(J, L, K) \tag{5}$$

where from [10] using urn model, we can obtain

$$p_{\hat{G}}(J, L, K) = \prod_{F=0}^{K-L-1} (1 - q^{F-J}) \tag{6}$$

where $p_{\hat{G}}(J, L, K)$ is the probability of condition 3 to be satisfied. In the Fig. 1-Fig. 3, we show the values of $1 - p^{SRNC}$ for different values of K, N and q. Firstly, these results show that the exact MDS like performance; i.e., the probability to correct exactly $d_{MDS} - 1 = N - K$ erasures is limited by the use of the field size. For example, for any combination of (N, K), we have $1 - p^{SRNC}$ equals to around 0.25, 10^{-2} and 10^{-3} for $q = 4$, $q = 64$ and $q = 256$ respectively. It shows that with the use of high finite field size, we can acheive very close to exact MDS like performance, for example, with $q = 256$ and $E = N - K$, we have $p^{SRNC} = 1 - 10^{-3}$ approaching to 1. Moreover, as the total number of erasures decreases, p^{SRNC} increases and $1 - p^{SRNC}$ decreases significantly. For e.g., in Fig. 3., for $RS(255, 127)$ and $SRNC(255, 127)$ codes, if there are $E = 61$ erasures in the coding window, RS will correct these erasures with $p^{RS} = 1$ $(N - K > E)$ and SRNC will correct these erasures with $p^{SRNC} = 1 - 10^{-10}$

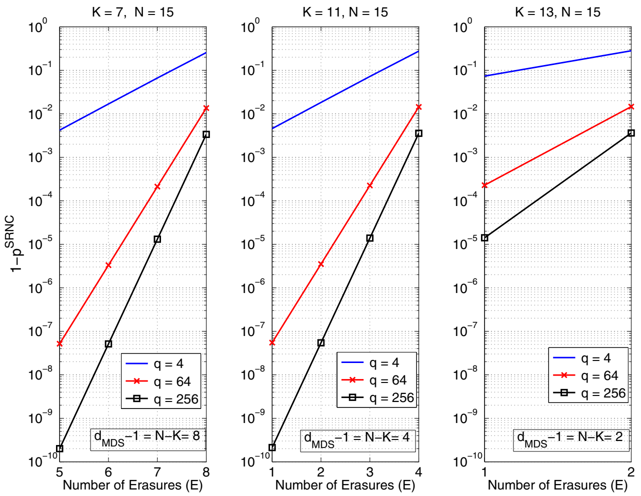

Fig. 1. $1 - p^{SRNC}$ for different K, E, q and $N = 15$

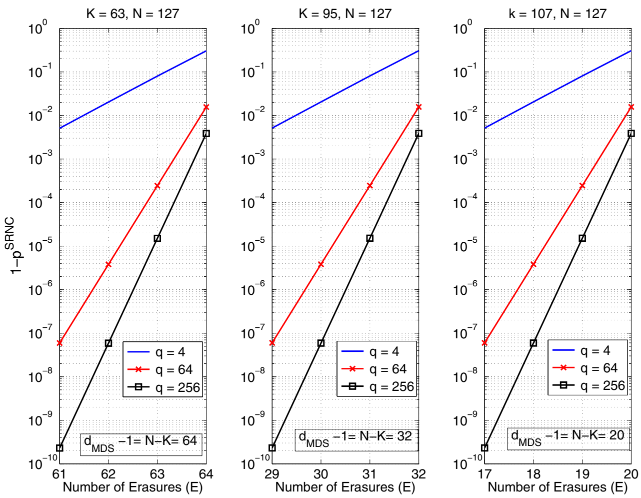

Fig. 2. $1 - p^{SRNC}$ for different K, E, q and $N = 127$

which is almost equal to 1. In fact, as the value of E decreases, the difference between p^{RS} and p^{SRNC} will approach to almost zero. So, we can conclude that both the RS and the SRNC have similar correction performance over the erasure channels.

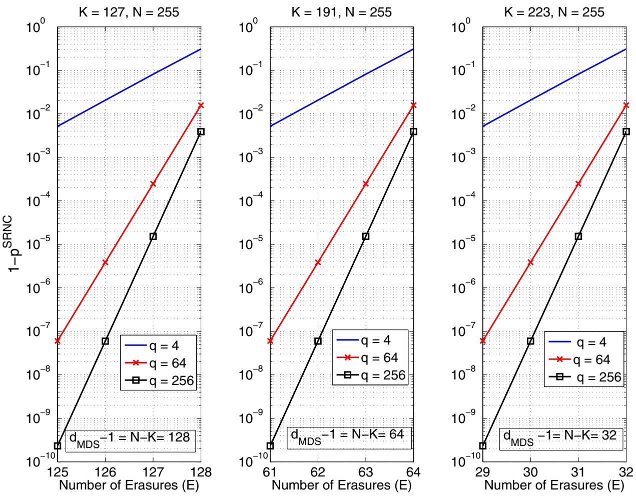

Fig. 3. $1 - p^{SRNC}$ for different K, E, q and $N = 255$

3.3 RS vs SRNC: Delay in the Delivery Time

Delivery time is the time that receiver has to spend until it is able to recover the packets. Note that transmission delay is assumed to be constant and same for both the schemes. Hence, in this work, we do not consider the transmission delay. For the RS systematic codes, first uncoded packets corresponding to systematic phase are transmitted and then the encoded packets are transmitted. The packets received by the receiver from the systematic phase are recovered without any delay. However, to recover the lost packets, receiver has to wait to receive all the unerased packets of the non-systematic phase. For $RS(N, K)$ with constant N, if coding rate R decreases which means when K decreases, delivery time increases due to the increase in size of the non-systematic phase $(N - K)$.

In SRNC, we perform decoding progressively using on-the-fly Gauss Jordan algorithm [11]. The decoding procedure is illustrated in Fig. 4. In the progressive decoding, receiver starts decoding as soon as it receives the first packet. Hence, it does not wait for all the packets to arrive. We show in the simulations that due to the progressive decoding, average delay in the delivery time for the packets obtained in the non-systematic phase is significantly reduced using SRNC.

4 Numerical Results

4.1 DVB-S2/RCS Railway Scenario

For the recovery from the erasure events, DVB has adopted an link layer (LL) LL-FEC. In particular, there can exist different frameworks for the LL-FEC mainly known as MPE-FEC, MPE-IFEC and extended MPE-FEC [12]. In this

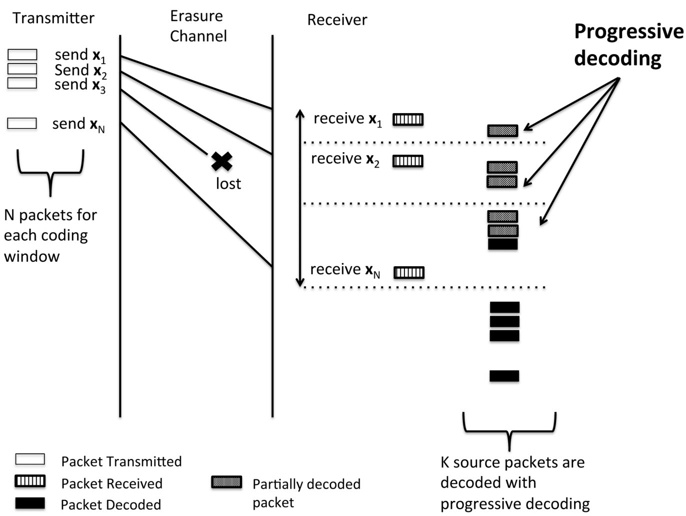

Fig. 4. Illustration of the progressive decoding over the erasure channel

work, we consider only the MPE-FEC framework. In particular, the size of the MPE-FEC frame is limited to 2 Mbits [13]. Following our system model, the encoded packets should form the columns of the MPE-FEC frame and each column of MPE-FEC frame should consists of M symbols. We keep field size $q = 256$ such that each symbol is equivalent to 1 byte. As the size of the MPE-FEC frame is limited to 2 Mbits, we have $N \times M \leq 2 \times 10^6$. With the packet size of $M = 1500 \times 8$ bits, we have the number of encoded packets limited to $N \leq 166$.

Now let us denote B_s as the physical layer symbol rate in baud/s, ς is the modulation constellation and r_{phy} is the physical coding rate where the bit rate is given by $B_s \varsigma r_{phy}$ bits/sec. For each packet we have $M = 1500 \times 8$ bits/packet, therefore we can define the packet rate as $B_p = \frac{B_s \varsigma r_{phy}}{M}$ packets/s. If more than $N - K$ packets are lost, then the decoding of K source packets is not possible. Hence, we are interested in the transmission time of $N - K$ packets, denoted as, $T_{N-K} = (N - K) \times \frac{1}{B_P}$.

Specifically, we analyze the performance over the railway scenario with line of sight, together with the effect of power arches. The presence of PAs in the railway environment can be modeled as a erasure channel where the packets are considered to be erased whenever there is a presence of PA. Let us denote l_{PA} as the width of the power arches (PA) and velocity of the train as v_{train}. During the time when the train passes through the PA, there is an erasure event. Let us denote this time as $T_{erasure} = l_{PA}/v_{train}$. We are able to obtain all the source K packets only when $T_{erasure} \leq T_{N-K}$.

In Fig. 5., we compare T_{N-K} and $T_{erasure}$ for different values of l_{PA} and v_{train}. Note that T_{N-K} does not have any dependency on l_{PA} and v_{train}. To vary T_{N-K} we consider two cases where $(N, K, R) = (166, 83, 0.5)$ and

$(N, K, R) = (166, 141, 0.85)$. We choose $N = 166$, as it is the maximum value availaible due to limited size of MPE-FEC frame. From [12], using $B_s = 27.5M\ bauds/s$, $\varsigma = 2$ and $r_{phy} = 1/2$, we get $T_{N-K} = .0362$ seconds for $(N, K, R) = (166, 83, 0.5)$ and $T_{N-K} = .012$ seconds for $(N, K, R) = (166, 141, 0.85)$.

From Fig. 5, we can see the MPE-FEC framework is only useful for $l_{PA} = 0.5$, when $l_{PA} \geq 0.5$, $T_{erasure}$ is always greater than T_{N-K}. This means that we will loose more than $N - K$ packets due to the PA and hence recovery of all the K source packets is not feasible. Therefore, we wil only focus for the case where $l_{PA} = 0.5$. For the cases where $l_{PA} \geq 0.5$, other frameworks should be followed, for which MDS-like performance of SRNC shall hold true as in the assessed framework. For the case $l_{PA} = 0.5$, $SRNC(166, 83)$ is sufficient to recover the source packets from the erasure events as from (5), p^{SRNC} approaches to 1.

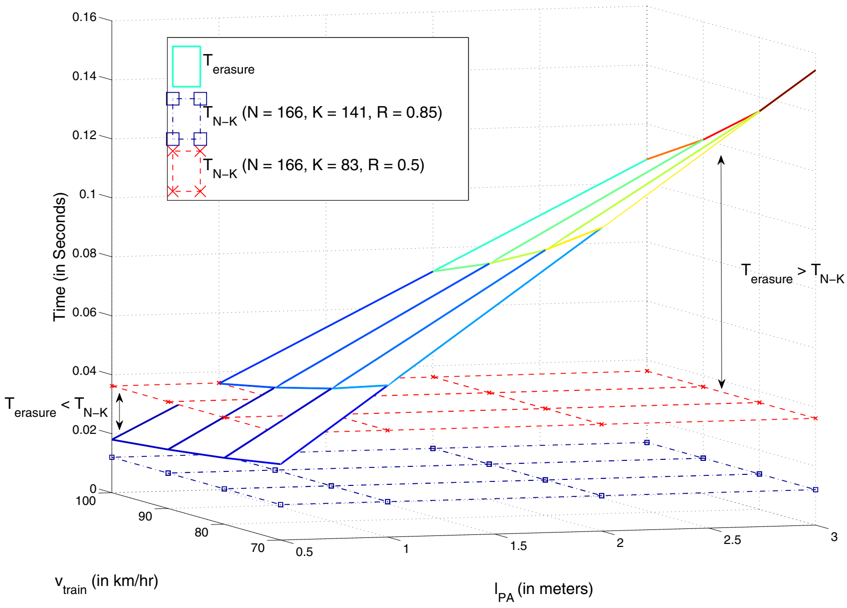

Fig. 5. 3-d mesh graph for comparison of $T_{erasure}$ and T_{N-K} over different values of l_{PA} and v_{train}

4.2 Broadband Global Area Network (BGAN) Mobile Scenario

For the second result, we consider real network traces over the BGAN mobile scenario. These traces are obtained from the video streaming sessions through a Inmarsat's Broadband Global Area Network (BGAN) mobile satellite network. The streaming session, which is over the best effort channel, suffers from the packet losses due to the network congestion. One way to control this congestion is by adapting the video codec rate [14]. However, adapting the video codec rate requires the cross layer feedback. Moreover, diminishing the video codec rate, may cause the degradation in Quality-of-Service (QoE). Hence, in this work, without adapting the video rate, we use the channel coding to counter these packet losses due to the congestion.

In the video streaming session, source/uncoded packets are transmitted and traces are recorded. First, let us denote the total number of transmitted encoded packets by $n_{encoded}$. For the given code (N, K), the encoding is performed over $|t| = \lceil \frac{n_{encoded}}{N} \rceil$ coding windows. Therefore, total number of source packets transmitted are given by $n_{source} = |t| \times K$. At the receiver, after recovery, if the total number of source packets lost is n_{lost}, we have the total source packets lost in percentage given by $p_{lost} = \frac{n_{lost}}{n_{source}} \times 100$. Using these parameters, we compare SRNC and RS codes with different encoding parameters for the recorded channel traces.

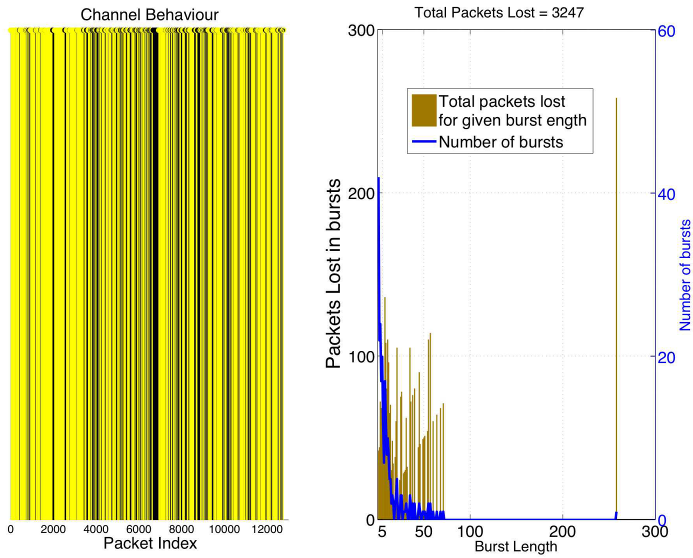

Fig. 6. Channel Traces: Channel behavior for the video transmission over the BGAN mobile satellite network. Lost packets are shown by black shades and received packets by yellow. In the left side, channel characteristics are shown with the number of bursty errors present in the channel.

The recorded channel traces is shown in Fig. 6. To obtain these traces, transmission is done without coding, and total $n_{encoded} = n_{source} = 12760$ source packets are transmitted where $n_{lost} = 3247$ source packets are lost such that $p_{lost} = 25.44\%$. In Fig. 6, channel behavior is shown where packets lost are represented by black shades and packets received by yellow corresponding to their packet index. In the right side of the figure, channel characteristic are shown in terms of burst lengths and the packets lost in those bursts. It is clear that the channel is bursty with different burst error lengths.

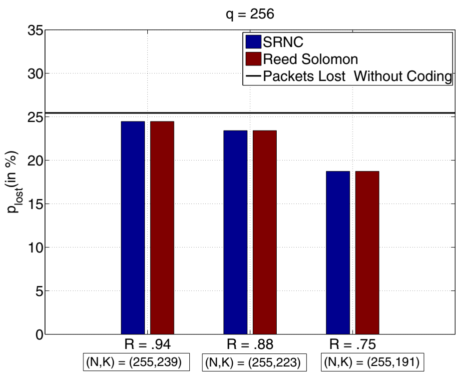

Fig. 7. Packets Lost for RS(N, K) and SRNC(N, K) for the traces shown in Fig. 6

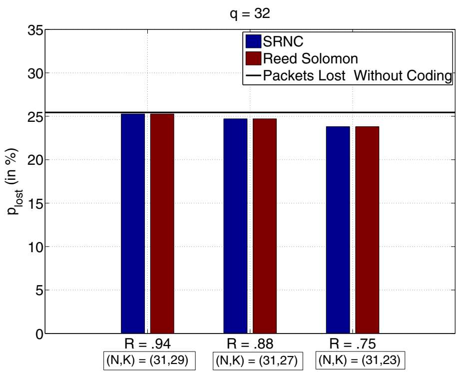

Fig. 8. Packets Lost for RS(N, K) and SRNC(N, K) for the traces shown in Fig. 6

As illustrated in Section 3, theoretically, RS codes and SRNC have similar erasure correction performance. This result is verified by simulations for recorded traces. Both $RS(N, K)$ and $SRNC(N, K)$ are compared with different values of K, N and q in Fig. 7. and Fig. 8. In all the cases, RS and SRNC are having the same erasure correction performance. In case of the RS codes, N is limited by $N < q$. This means that there is also an additional advantage while choosing SRNC because there is no limitation on N with respect to the field size for SRNC. However, in these simulations, to show the fair comparison we choose the same parameters for both the schemes. Due to the higher code length N, and higher redundancy $N - K$, p_{lost} decreases as the code length increases for the same coding rate R. Our results also illustrate that the field sizes ranging from $q = 256$ to $q = 32$ are sufficient for the SRNC, with the given channel traces, to give the same performance as the RS codes.

To illustrate the delay gain of SRNC, we consider delay in the per packet delivery time that receiver has to spend until is it able to recover the packet.

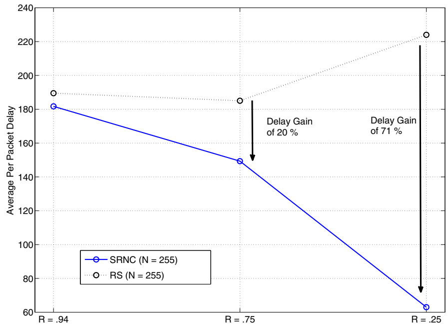

Fig. 9. Average per packet delay in the delivery time

As mentioned before, we are not considering delay due to the transmission time from the sender to the receiver as it should be constant and same for both the coding schemes. Note that the packets received from the systematic phase for both the schemes are recovered without any delay. Hence, we focus on the average per packet delay for the packets which are lost in the systematic phase and recover with the help of coded packets in the non-systematic phase. Fig. 9. shows the per packet delay performance for both the schemes. For the RS code, receiver has to wait for all the packets received from the non-systematic phase to recover the lost packets. In particular, for RS(N, K) with constant N, if code rate $R = K/N$ decreases, delivery time increases due to the increase in the size of the non-systematic phase. However, for SRNC, as progressive decoding is done, lost packets can be recovered progressively without waiting for rest of the packets. In fact, for the low coding gain, when the non-systematic phase is longer, the SRNC performs better as the average is done for all the packets recovered in the non-systematic phase. Specifically, as shown in the figure, up to 71% gain in the delivery time could be achieved with the use of SRNC coding.

5 Conclusions

In this work, we present the systematic random network coding for the multimedia transmission over the erasure satellite channels. We show the analytical analysis of the SRNC with the matricial erasure channel model. Theoretical expressions illustrate the similarity in the erasure correction performance of the SRNC and commonly used RS codes. We compare both these codes for the real traces obtained over the mobile satellite channel. Simulation results show that both the coding schemes have same packet recovery capabilities. Finally, we have illustrated the delay gain in the delivery time for the SRNC due to the early recovery of lost packets by progressive decoding.

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