HFSWR Polarization DOA Estimation Based on Quaternion

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Abstract. This paper examines the direction of arrival (DOA) estimation for polarized signals impinging on a vector sensor array for high frequency surface wave radar (HFSWR). The vector array effectively utilizes the polarization domain information of incident signals, and the quaternion model is adopted for signals polarization characteristic maintenance and computational burden reduction. The quaternion data model based on vector uniform linear array (ULA) is established. Based on the model, a quaternion MUSIC algorithm based on HFSWR signal processing is proposed for HFSWR DOA estimation. The algorithm combines the HFSWR signal processing and vector ULA to enhance the DOA estimation performance. Analytical simulations are operated to certify the capability of the algorithm.

Keywords: Direction of arrival · Vector uniform linear array · Polarization · Quaternion model

1 Introduction

Array signal processing is a basic theory in the fields of radar, sonar, navigation, etc. [1, 2]. With the increasing reliability of vector sensors, polarization is added to the DOA estimation as a basic information attribute. Therefore, many researchers have proposed multi-component data processing algorithms. For polarization vector sensor arrays, MUSIC-like algorithms are introduced in [3] and [4] and ESPRIT techniques in [5–7]. [8] and [9] have studied the Cramer–Rao bound for the vector-sensor arrays. However, these methods presume that the complex-valued data model represents incident signals frequency domain samples. A data covariance matrix is next described as second-order statistical magnitude between all sensor components. In the recent few decades, algorithms based on quaternion were introduced [10] for array signal processing. A multidimensional complex signals hypercomplex version [11] was also introduced by Bülow and Sommer [12]. In [13], Sebastian Miron proposes a quaternion data model algorithm based on vector-sensor arrays providing an estimation of DOA and polarization parameters, which decreases the data covariance model representation memory size leading to an efficient algorithm.

In HFSWR signal processing, the polarization domain information of the incident signal is effectively utilized by vector array. Quaternion data model is adopted to increase

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the orthogonality constraint of received data and maintain the polarization characteristics of the signal. Since the shift from the scalar sensor to the polarization vector sensor increases the amount of data to be processed, the quaternion model can reduce the amount of computation. For HFSWR DOA estimation, a quaternion data model based on vector array is firstly established, and then combined with HFSWR signal processing, a quaternion DOA estimation algorithm suitable for HFSW is proposed. The purpose is to improve the DOA estimation performance by using the polarization information and to improve the operation rate and convergence rate by combining the quaternion model. Finally, the effectiveness of the algorithm is verified by computer simulation.

This paper consists of the following sections. In Sect. 2, a short quaternions description is introduced and a polarized signal model is given. In Sect. 3, a description of the quaternion DOA estimation based on vector array is given. In Sect. 4, the algorithm performances are evaluated by computer simulations. Finally, the conclusions of this paper are presented.

2 Polarized Quaternion Signal Model

2.1 Quaternion

Quaternions discovered by Hamilton in 1843 are four-dimensional (4-D) hypercomplex numbers, which are an extension of complex numbers into 4-D space. A quaternion is defined via one real and three imaginaries, whose Cartesian form can be described as

$$q = w + x\mathrm{i} + y\mathrm{j} + z\mathrm{k}, \tag{1}$$

where $\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = \mathrm{ijk} = -1$, $\mathrm{ij} = \mathrm{k}, \mathrm{ji} = -\mathrm{k}, \mathrm{ki} = \mathrm{j}, \mathrm{ik} = -\mathrm{j}, \mathrm{jk} = \mathrm{i}, \mathrm{kj} = -\mathrm{i}$.

Several complex numbers properties may be extended into quaternions.

(1) the conjugate of q, q^*, is expressed as $q^* = w - x\mathrm{i} - y\mathrm{j} - z\mathrm{k}$;.
(2) a pure quaternion is a null real part quaternion: $q = x\mathrm{i} + y\mathrm{j} + z\mathrm{k}$;.
(3) the quaternion modulus q is $\|q\| = \sqrt{qq^*} = \sqrt{q^*q} = \sqrt{w^2 + x^2 + y^2 + z^2}$;
(4) its inverse is expressed as

$$q^{-1} = q^*/\|q\|; \tag{2}$$

(5) a null quaternion is given by $w = x = y = z = 0$;
(6) the quaternions set, noted $\mathbb{H}$, forms a noncommutative normed algebra, meaning

$$q_1 q_2 \neq q_2 q_1; \tag{3}$$

(7) conjugation over $\mathbb{H}$ is an anti-involution

$$(q_1 q_2)^* = q_2^* \cdot q_1^*. \tag{4}$$

In [16], a Cayley–Dickson form quaternion is expressed as: $q = q_1 + q_2\mathrm{j}$, where $q_1 = w + x\mathrm{i}$ and $q_2 = y + z\mathrm{i}$. That is

$$q = [1 \quad \mathrm{j}][q_1 \quad q_2]^{\mathrm{T}}. \tag{5}$$

2.2 Polarization Model

In this section, the concept of electromagnetic wave polarization is firstly expounded, then the joint expression of spatial domain and polarization domain of signal is modeled and the geometric structure of electromagnetic vector sensor array is modeled, and finally, the signal receiving a model of the electromagnetic vector sensor array is established. This article mainly studies fully polarized electromagnetic waves.

Assume that there is a point signal source with elevation angle θ and azimuth angle ϕ at a certain point in space, which is mutually excited by the magnetic field and the electric field. The ideal transmission medium is transmitted to the array along the direction of the Poynting vector, as shown in Fig. 1.

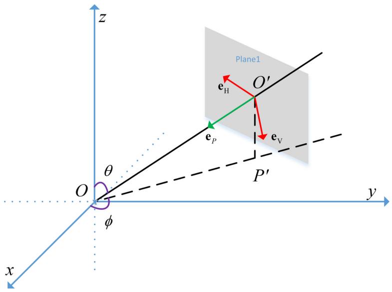

Fig. 1. Characterization of fully polarized electromagnetic waves

The unit vectors pointing in the positive direction of x, y, and z axis are denoted as $\mathbf{e}_{\mathrm{x}}$, $\mathbf{e}_{\mathrm{y}}$, $\mathbf{e}_{\mathrm{z}}$ respectively. The coordinate origin serves as the reference phase point of the electromagnetic vector sensor array. As can be seen from Fig. 1, the unit vector of the incident direction of the signal is the propagation vector of the signal:

$$\mathbf{e}_P = [-\sin\theta\cos\phi \quad -\sin\theta\sin\phi \quad -\cos\theta]^{\mathrm{T}}. \tag{6}$$

The horizontal vector $\mathbf{e}_{\mathrm{H}}$ and the vertical vector $\mathbf{e}_{\mathrm{V}}$ constitute a set of standard orthogonal bases perpendicular to the plane of the propagation direction. The three form a right-handed coordinate system, which is related to the angle of arrival of the signal. According to Fig. 1:

$$\mathbf{e}_{\mathrm{H}} = -\mathbf{e}_P\left(\phi + \frac{\pi}{2}, \frac{\pi}{2}\right) = [-\sin\phi \quad \cos\phi \quad 0]^{\mathrm{T}}, \tag{7}$$

$$\mathbf{e}_{\mathrm{V}} = -\mathbf{e}_{P}\left(\phi, \theta + \frac{\pi}{2}\right) = [\cos\theta\cos\phi \quad \cos\theta\sin\phi \quad -\sin\theta]^{\mathrm{T}}. \tag{8}$$

The signal polarization information may be described via the instantaneous ratio of the electric field amplitude and phase in the H and V directions, and recorded $\tan\gamma = E_{\mathrm{Vm}}/E_{\mathrm{Hm}}, \gamma \in [0, \pi/2]$ is polarization assist angle, $\eta = \varphi_{\mathrm{V}} - \varphi_{\mathrm{H}}, \eta \in [0, 2\pi]$ is polarization phase difference. For a fully polarized wave, its endpoint polarization trajectory is an ellipse with a fixed long and short axial ratio and inclination. The coordinates of a complete six-dimensional electric and magnetic field in a plane rectangular coordinate system are:

$$\begin{bmatrix} \mathbf{E}_{\mathrm{x}}(t) \\ \mathbf{E}_{\mathrm{y}}(t) \\ \mathbf{E}_{\mathrm{z}}(t) \\ \mathbf{H}_{\mathrm{x}}(t) \\ \mathbf{H}_{\mathrm{y}}(t) \\ \mathbf{H}_{\mathrm{z}}(t) \end{bmatrix} = \begin{bmatrix} -\sin\phi & \cos\theta\cos\phi \\ \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\theta \\ \cos\theta\cos\phi & \sin\phi \\ \cos\theta\sin\phi & -\cos\phi \\ -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\gamma \\ \sin\gamma e^{\mathrm{i}\eta} \end{bmatrix} E_{\mathrm{c}}(t) \tag{9}$$

where

$$\mathbf{A}_{P}(\phi, \theta, \gamma, \eta) = \begin{bmatrix} -\sin\phi & \cos\theta\cos\phi \\ \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\theta \\ \cos\theta\cos\phi & \sin\phi \\ \cos\theta\sin\phi & -\cos\phi \\ -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\gamma \\ \sin\gamma e^{\mathrm{i}\eta} \end{bmatrix} \tag{10}$$

$\mathbf{A}_{P}(\phi, \theta, \gamma, \eta)$ is polarization steering vector, and **E** is the electric field component and **H** is the magnetic field component.

There is a uniform linear electromagnetic vector sensor array composed of N electromagnetic vector sensors arranged in sequence along the positive direction of the y axis. The electric dipoles of the electric field are orthogonal to each other, and the distance between the electromagnetic vector sensor array elements is d. The y-axis coordinate y_n of the nth electromagnetic vector sensor is $(n-1)d$; $n = 1, 2, \ldots, N$ in Fig. 2. And make $M (M < N)$ far-field expected signals (fully polarized waves) incident on the array and elevation angle $\theta = 90°$. Let the azimuth of the desired signal, the spatial phase delay between the nth element and the reference element ($d = \lambda/2$) is

$$\alpha_n = -\pi(n-1)\sin\phi. \tag{11}$$

The steering vector

$$\mathbf{A}_s = \left[e^{\mathrm{i}\alpha_1} \quad e^{\mathrm{i}\alpha_2} \quad \cdots \quad e^{\mathrm{i}\alpha_N}\right]^{\mathrm{T}}. \tag{12}$$

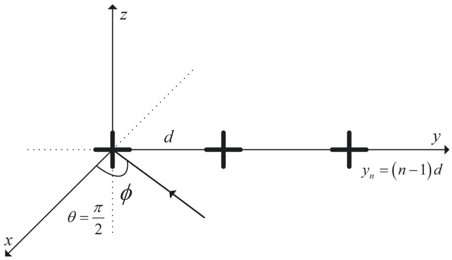

Fig. 2. Uniform linear array

The data received by the entire array is

$$\mathbf{X}(t) = \begin{bmatrix} e^{i\alpha_1}\mathbf{A}_P s(t) \\ e^{i\alpha_2}\mathbf{A}_P s(t) \\ \vdots \\ e^{i\alpha_N}\mathbf{A}_P s(t) \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1(t) \\ \mathbf{n}_2(t) \\ \vdots \\ \mathbf{n}_N(t) \end{bmatrix} = \mathbf{A}_s(\phi,\theta) \otimes \mathbf{A}_P(\phi,\theta,\gamma,\eta)s(t) + \mathbf{N}(t) \tag{13}$$

where $\otimes$ represents tensor product and $\mathbf{n}_n(t) = \begin{bmatrix} n_{ny}(t) & n_{nz}(t) \end{bmatrix}^{\mathrm{T}}, n = 1, 2, \ldots, N$ is noise vector in directions of yz. It is assumed that the noise is independent between each array element, and the noise is independent between the vector components within each array element, and the signal and noise are relatively independent. Noise is an independent Gaussian white noise with zero mean and σ^2 variance.

3 HFSWR Polarized Quaternions DOA Estimation Algorithm

3.1 HFSWR Signal Processing

This section describes target azimuth acquisition based on the Range Doppler spectrum. The LFM continuous-wave transmitting signal with an FM period is set as:

$$S_{\mathrm{T}}(t) = \cos\left[2\pi\left(f_c t + 0.5kt^2\right)\right] = \cos[\phi_T(t)], \tag{14}$$

where f_c is center frequency and k is frequency modulation slope. After passing through the radio frequency amplifier at the receiver, the received echo signal is mixed with the transmitted signal, that is:

$$\begin{aligned} S_{\mathrm{d}}(t) &= A\cos\left(2\pi\left(f_c t + 0.5kt^2 - f_c(t - t_{\mathrm{d}}) - 0.5k(t - t_{\mathrm{d}})^2\right)\right) \\ &= A\cos\left(2\pi\left(f_c t_{\mathrm{d}} + kt_{\mathrm{d}}t - 0.5kt_{\mathrm{d}}^2\right)\right) \end{aligned} \tag{15}$$

where t_{d} is time delay. Based on the phase of the difference frequency signal of the nth frequency modulation period, the Fourier transform result is:

$$V_n(f) = \frac{AT}{2}\left\{ \begin{array}{l} \frac{\sin[2\pi(f-f_{dn})T/2]}{2\pi(f-f_{dn})T/2} e^{-j\phi_0 + j2\pi f_c \frac{2v}{c} nT_r} \\ + \frac{\sin[2\pi(f+f_{dn})T/2]}{2\pi(f+f_{dn})T/2} e^{+j\phi_0 - j2\pi f_c \frac{2v}{c} nT_r} \end{array} \right\} \quad (16)$$

where v is speed and T_r is Frequency sweep cycle. Take out the positive frequency part of V, a total of M sampling points. The process of obtaining the range spectrum from the differential frequency signal from Eq. (16) is called the solution range transformation in the HFSWR signal processing.

Continuous transmitting N FM signals, and the difference frequency signal of each frequency modulation period do the distance transformation, can get $N \times M$ samples

$$\mathbf{V} = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,M} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,M} \end{bmatrix} \quad (17)$$

The mth (m = 1, 2, …, M) column is N samples taken from the mth distance gate, namely the time series on the distance gate. The Fourier transform of each column is the solution velocity transform. The Result of the Fourier transform is:

$$Q_{nm} = KNT_r \frac{\sin[2\pi(f - 2f_c v/c)NT_r/2]}{2\pi(f - 2f_c v/c)NT_r/2} \quad (18)$$

After the signal processing solution velocity transformation, the Range-Doppler matrix can be obtained:

$$\boldsymbol{Q} = \begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,M} \\ Q_{2,1} & Q_{2,2} & \cdots & Q_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{N,1} & Q_{N,2} & \cdots & Q_{N,M} \end{bmatrix} \quad (19)$$

The Range-Doppler (RD) spectrum of the target can be drawn from the matrix. It can be seen from the theoretical derivation that the Range-Doppler spectrum is the 2-dimensional envelope form of the sinc function.

After Range processing and Doppler processing, the echo data can be represented as a 3-dimensional data block containing Range-Doppler-Array information. Based on the RD spectrum, the points with the same range and Doppler are extracted to form time series. Replace the previous polarization signal with this time series:

$$\mathbf{X}(t) = \mathbf{A}_s(\phi, \theta) \otimes \mathbf{A}_P(\phi, \theta, \gamma, \eta) s_{\mathrm{RD}}(t) + \mathbf{N}(t) \quad (20)$$

3.2 DOA Estimation Algorithm

Take the $\mathbf{e}_y$ and $\mathbf{e}_z$ in formula (16) to obtain the polarization domain steering vector of the mth signal received by this electromagnetic vector sensor array:

$$\mathbf{A}_P(\phi_m, \theta_m, \gamma_m, \eta_m) = \begin{bmatrix} \cos\phi_m & \cos\theta_m \sin\phi_m \\ 0 & -\sin\theta_m \end{bmatrix} \begin{bmatrix} \cos\gamma_m \\ \sin\gamma_m e^{\mathrm{i}\eta_m} \end{bmatrix} \tag{21}$$

The elevation $\theta = 90°$, after simplification:

$$\mathbf{A}_P(\phi_m, \gamma_m, \eta_m) = \begin{bmatrix} \cos\phi_m \cos\gamma_m \\ -\sin\gamma_m e^{\mathrm{i}\eta_m} \end{bmatrix} \tag{22}$$

Applying formula (5) here, the quaternion Cayley-Dickson representation synthesizes the polarization domain steering vector in the complex number domain into a polarization domain steering vector in the quaternion domain, namely:

$$p(\phi_m, \gamma_m, \eta_m) = \begin{bmatrix} 1 & \mathrm{j} \end{bmatrix} \begin{bmatrix} \cos\phi_m \cos\gamma_m \\ -\sin\gamma_m e^{\mathrm{i}\eta_m} \end{bmatrix} \in \mathbb{H} \tag{23}$$

after simplification:

$$p = \cos\phi_m \cos\gamma_m - \sin\gamma_m \cos\eta_m \mathrm{j} - \sin\gamma_m \sin\eta_m \mathrm{k} \tag{24}$$

According to formula (20), it can be known that the mth signal received by the entire electromagnetic vector sensor array is:

$$\mathbf{X}_m(t) = \mathbf{A}_s(\phi_m) p(\phi_m, \gamma_m, \eta_m) s_{\mathrm{RD}m}(t) + \mathbf{N}(t) \tag{25}$$

where

$$\mathbf{A}_s(\phi_m) = \begin{bmatrix} e^{\mathrm{i}\alpha_{m,1}} & e^{\mathrm{i}\alpha_{m,2}} & \cdots & e^{\mathrm{i}\alpha_{m,N}} \end{bmatrix}^{\mathrm{T}} \tag{26}$$

The quaternion noise is synthesized from the complex noise vector by Cayley-Dickson representation as:

$$n = \begin{bmatrix} 1 & \mathrm{j} \end{bmatrix} \begin{bmatrix} n_{n\mathrm{y}}(t) \\ n_{n\mathrm{z}}(t) \end{bmatrix} = n_{n\mathrm{y}}(t) + n_{n\mathrm{z}}(t)\mathrm{j} \tag{27}$$

All signals received by the entire array can be expressed as:

$$\mathbf{X}(t) = \sum_{m=1}^{M} \mathbf{A}_s(\phi_m) p(\phi_m, \gamma_m, \eta_m) s_{\mathrm{RD}m}(t) + \mathbf{N}(t) \tag{28}$$

where

$$\mathbf{N}(t) = \begin{bmatrix} n_1 & n_2 & \cdots & n_N \end{bmatrix}^{\mathrm{T}} \tag{29}$$

Make $\mathbf{a}_m = \mathbf{A}_s(\phi_m)p(\phi_m, \gamma_m, \eta_m)$ write formula (26) as a matrix:

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{A}_{s1} & \mathbf{A}_{s2} & \cdots & \mathbf{A}_{sM} \end{bmatrix} \cdot \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_M \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_M(t) \end{bmatrix} + \mathbf{N}(t) \tag{30}$$

A is called the spatial-polarization joint steering vector matrix. In this way, a signal receiving data model of the electromagnetic vector sensor array in the quaternion domain is established.

The equivalent second order representation for a vector-sensor array using Quaternion spectral matrix (QSM)

$$\mathbf{\Omega} = \mathrm{E}\{\mathbf{X}\mathbf{X}^{\triangleleft}\} \tag{31}$$

where $\mathrm{E}\{\bullet\}$ is the mathematical expectation operator and $^{\triangleleft}$ represents quaternion conjugate transpose.

Assuming the decorrelation between sources themselves and between the noise and the sources, the quaternion spectral matrix becomes

$$\mathbf{\Omega} = \mathrm{E}\{\mathbf{A}\mathbf{S}\mathbf{S}^{\triangleleft}\mathbf{A}^{\triangleleft}\} + \mathrm{E}\{\mathbf{N}\mathbf{N}^{\triangleleft}\} = \mathbf{\Omega}_S + \mathbf{\Omega}_N \tag{32}$$

where

$$\mathbf{\Omega}_S = \mathbf{A}\mathrm{E}\{\mathbf{S}\mathbf{S}^{\triangleleft}\}\mathbf{A}^{\triangleleft} = \sum_{m=1}^{M} \sigma_m^2 \mathbf{A}\mathbf{A}^{\triangleleft} = \sum_{m=1}^{M} \sigma_m^2 \|p_m\|^2 \mathbf{A}_{sm}\mathbf{A}_{sm}^{\triangleleft} \tag{33}$$

and $\mathbf{\Omega}_N = \mathrm{E}\{\mathbf{N}\mathbf{N}^{\triangleleft}\}$ is a matrix containing noise second order statistics. In (33), $\mathbf{\Omega}_S$ is the signal part and $\sigma_m^2\|p_m\|^2$ is mth source antenna power. Let the eigenvalue decomposition (EVD) be given by

$$\mathbf{\Omega} = \mathbf{U}\mathbf{D}\mathbf{U}^{\triangleleft} \tag{34}$$

with $\mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_N] \in \mathbb{H}^{N \times N}$ containing the N eigenvectors and **D** is the diagonal matrix of its eigenvalues.

Define two matrices $\mathbf{U}_S \in \mathbb{H}^{N \times M}$ and $\mathbf{U}_G \in \mathbb{H}^{N \times (N-M)}$, such as

$$\mathbf{U}_S = [\mathbf{u}_1, \cdots, \mathbf{u}_M] \tag{35}$$

$$\mathbf{U}_G = [\mathbf{u}_{M+1}, \cdots, \mathbf{u}_N] \tag{36}$$

$\mathbf{U}_\mathrm{S}$ contains the signal subspace eigenvectors and $\mathbf{U}_\mathrm{G}$ the noise subspace eigenvectors. Because

$$\mathbf{A}^{\triangleleft}\mathbf{U}_G = 0 \tag{37}$$

If (37) is multiplied on the right by $(\mathbf{A}^{\triangleleft}\mathbf{U}_G)^{\triangleleft}$, (37) can be expressed using columns of $\mathbf{A}$ as

$$\mathbf{a}_m^{\triangleleft}(\phi_m, \theta_m, \gamma_m, \eta_m)\mathbf{U}_G\mathbf{U}_G^{\triangleleft}\mathbf{a}_m(\phi_m, \theta_m, \gamma_m, \eta_m) = 0 \tag{38}$$

for all sets of $\{\phi_m, \theta_m, \gamma_m, \eta_m\}$ corresponding to parameters of M signal sources. $\mathbf{\Pi}_N = \mathbf{U}_G\mathbf{U}_G^{\triangleleft} \in \mathbb{H}^{N\times N}$ represents the noise subspace projector.

The DOA estimator based on quaternion is then achieved by projecting the quaternion steering vector $\mathbf{a}_m(\phi_m, \theta_m, \gamma_m, \eta_m) \in \mathbb{H}^N$

$$\mathbf{a}_m = \begin{bmatrix} e^{\mathrm{i}\alpha_{m,1}} & e^{\mathrm{i}\alpha_{m,2}} & \cdots & e^{\mathrm{i}\alpha_{m,N}} \end{bmatrix}^{\mathrm{T}} p(\phi_m, \theta_m, \gamma_m, \eta_m) \tag{39}$$

on the noise subspace as:

$$SP_\mathrm{Q}(\phi_m, \theta_m, \gamma_m, \eta_m) = \frac{1}{\mathbf{a}_m^{\triangleleft}\hat{\Pi}_N\mathbf{a}_m} \tag{40}$$

The functional in (40) has maxima for sets of $\{\phi_m, \theta_m, \gamma_m, \eta_m\}$ corresponding to sources present in the signal. In this article, the DOA estimation of the signal is mainly concerned and it is assumed that the polarization of the signal is known.

4 Simulation Analysis

In this section, numerical examples illustrate the polarization DOA estimation performance superiority based on quaternion in HFSWR. Assume that the number of the sources is known and all sources are equal power. RD spectrum is obtained after range and velocity processing in HFSWR, as shown in Fig. 3. Figure 3 shows the single-channel result, and the RD spectrum of other channels is similar. As the RD spectrum acquisition period is relatively long, the number of snapshots should not be too much. To combine the HFSWR signal processing, the snapshot number is about ten.

The next simulations consider the RMSE performance versus the input SNR, the number of snapshots. The fixed parameter setting is following: SNR is from –10 dB to 10 dB because SNR varies under different conditions, T are 10 snapshots. Figure 4 shows the RMSE of the DOA estimates versus the SNR. As the SNR increases, all the RMSEs decrease. Moreover, the Quaternion MUSIC achieves smaller RMSE than Long vector MUSIC and MUSIC across a wide range of the SNR. Figure 5 illustrates the RMSE of the DOA estimates versus the number of the snapshot, all DOA estimates become more accurate and stabilized as snapshots increase, and Quaternion MUSIC behaves better than Long vector MUSIC and MUSIC.

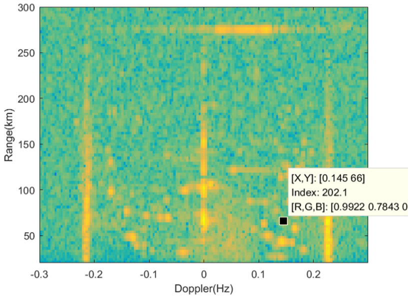

Fig. 3. RD spectrum

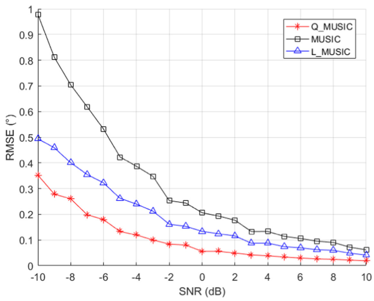

Fig. 4. RMSE versus the SNR

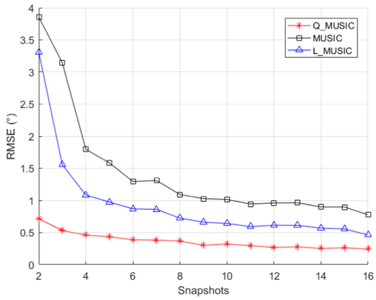

Fig. 5. RMSE versus the number of snapshots

5 Conclusion

In this paper, combining HFSWR signal processing with a quaternion data model, we propose a quaternion-based polarization DOA estimation algorithm for HFSWR. The proposed method has been compared to the classical MUSIC algorithm and the long vector MUSIC algorithm. Q-MUSIC is more accurate than them, reducing the computational burden the memory required. The results indicate the quaternion potential to model polarized signals and the possibility of describing more than two complex-valued components signals using higher-dimensional hypercomplex algebras in HFSWR signal processing.

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