



Tool Development for the Optimal Supply of Medical Oxygen Delivered at Home

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Abstract. Due to the increase in demand for medical oxygen, there is a need to improve the oxygen home delivery service. In this project, we will work with the data of a company specialized in the management of medicinal gases, which has a group of patients who receive medicinal oxygen through cylinders. The goal is to develop a tool that automates the assignment of the optimal distribution center to each patient's zip code, taking into account the distance, supply capacity, and delivery time.

Keywords: Assignment problem · Medical oxygen · Optimization

1 Introduction

Medical oxygen is a gas used for patients who need oxygen therapy. This therapeutic measure has been shown to increase survival in patients with chronic obstructive pulmonary disease (COPD) and respiratory failure, COVID-19, cystic fibrosis, severe asthma attack, pneumonia, etc. [7].

The proper choice of oxygen source depends on many factors, including the amount of oxygen required by the patient; the infrastructure, cost, convenience, patient adaptability, capacity, distribution, and supply chain available for the local production and delivery of medicinal gases; the reliability of the electricity supply; and access to maintenance services and spare parts, etc. [7, 9].

Common sources of oxygen are liquid oxygen and oxygen generating plants in bulk storage tanks and oxygen concentrators. The most common source of oxygen storage used in healthcare settings is a cylinder (tank) containing compressed oxygen [9], which can be of different dimensions depending on the patient's need.

Therefore, guarantee the delivery of this gas on time is of utmost importance for the well-being of the patients, which at the same time translates into greater eligibility, trust, and permanence of the clients in the company.

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1.1 Optimization of Medical Oxygen Cylinder Deliveries

Due to the precise, delicate, and accurate service guarantee in the supply and delivery of medical oxygen cylinders, planning, and logistics distribution represents a key factor in this market. The interaction of transportation in the supply and distribution programs constitutes a dynamic process that requires high coordination.

Without taking into account that the timely delivery of oxygen guarantees customer loyalty and a high level of confidence in the service of their home delivery. Being able to derive an increase in capacity by having a better organization, allowing greater sales, and reducing costs [5].

In the same way, proper management of the supply chain is important since the availability of oxygen must be ensured when the user requires it and even anticipated. In this way, the growing market for this type of services will increase, , since the existence expectancy of a heightened incidence of chronic respiratory diseases, together with the augmentation on the demand for home health care [6].

That's why a proper assignation of the distribution center to the patients is essential to increase the quality of the delivery service.

1.2 Generalized Assignment Problem, GAP

The classic assignment problem (AP) consists in that given two sets of tasks and agents, assign one agent-to-one task, minimizing costs or maximizing profits [1]. Instead the generalized assignment problem (GAP) each task is assigned to one agent, as in the classic AP, but it allows for the possibility that an agent may be assigned to more than one task, while recognizing that a task may use only part of an agent's capacity rather than all of it [2,3].

GAP has many real-life applications, like a subproblem in routing problem, resource scheduling, scheduling of project networks, storage space allocation, scheduling of payments, assignment jobs to computers, assignment ships to overhaul, fleets of aircraft to trips, or the assignment of school buses to routes [11–13].

However, in most practical applications, each agent requires a quantity of some limited resource to process a given job. Therefore, the assignments have to be made taking into account the resource availability of each agent. The GAP is in practice even more difficult, since most of its applications have a stochastic nature [13].

Stochasticity can be due to two different sources. On the one hand, it appears when the actual capacity of the agents or the amount of resource needed to process the jobs is not known in advance [13].

The second source of stochasticity is uncertainty about the presence or absence of individual jobs. In such cases, there is a set of potential jobs, but only a subset of them will have to be processed. This subset is not known when the assignment has to be decided. e.g. the case of emergency services [13].

The general formulation of the problem is:

$$\min \sum_i \sum_j c_{ij} x_{ij} \quad (1)$$

$$\text{s.t.} \sum_j a_{ij} x_{ij} \leq b_i \quad \forall i \in I \quad (2)$$

$$\sum_i x_{ij} = 1 \quad \forall j \in J \quad (3)$$

$$x_{ij} = \{0, 1\} \quad \forall i \in I, \forall j \in J \quad (4)$$

where c_{ij} is the cost assigning job j to agent i , a_{ij} is the capacity absorption when job j is assigned to agent i , b_i the available capacity of agent i . The assignment variable x_{ij} equals 1 if agent i is to perform job j , 0 otherwise.

The objective function (1) is to minimize the total assignment cost of jobs (j) to agents (i). Constraints (2) designate the capacity availability restriction of each agent. Constraints (3) ensure that each job is assigned to exactly one agent. Finally, constraints (4) are the condition on the decision variables.

2 Problem Description

In this article a company specialized in industrial and medical gases is studied. Nowadays, this company counts 32 DC in all Mexican Republic (Fig. 1), which only deliver medical oxygen to homecare patients

Of all the DCs, only nine will be analyzed, currently each of these DCs has a group of patients that they visit periodically. For this research, the locations of the patients' zip codes will be taken as the delivery points.

Historically, in this company, the assignments of DCs to each patient have been made manually and with no mathematical background, so not in all cases the closest DC is assigned to the customers, causing a delay in delivery, greater distance traveled and therefore, it makes the service less profitable to the company.

In this case, not vehicular routes will be proposed. Hence, the scope of the project is both the optimal assignation of DC to the patient and, in the future the generation of a schedule of visiting days (according to the distance between a group of patients assigned to a DC, vehicle capacity, and the area in which the client/patient belong).

A generalized assignment problem is proposed, where the objective function is to minimize the distance between the DC and the patient, where a distribution center i can serve n patients j (which in this case are located by ZIP code), and a patient can only be served by one distribution center. And also each DC has a maximum number of patients that can attend (zip codes).

These types of problems had been described in different papers, however, each of them had different application purposes. Table 1 gives some examples.



Fig. 1. Locations of distribution centers

Table 1. Literature review

Author	Characteristic
Qian X, 2017 [14]	The Taxi group ride (TGR) is one popular case of taxi ride sharing, where passenger trips with nearby origins and destinations and similar departure times are grouped into a single ride. This study investigates the optimal assignment of a set of passengers for the sake of maximizing total saved travel miles
Albareda-Sambola, 2006 [13]	This paper deals with a stochastic Generalized Assignment Problem with recourse. An assignment of each job to an agent is decided a-priori, and once the subset of jobs that have to be executed is now, reassignments can be performed if there are overloaded agents
Öncan, T., 2007 [12]	In this survey, it was concentrated on real-life applications in scheduling, timetabling, telecommunication, facility location, transportation, production planning, etc. Where Generalized Assignment Problem (GAP) is used to find the optimum assignment of each item to exactly one knapsack, without exceeding the capacity of any knapsack

3 Methodology

3.1 Data Collection

To obtain the exact location of all the national zip codes, it was necessary to use the data from Correos de México, which through treatment in MapInfo [15] software it was able to calculate the centroids of each zip code, these locations (longitude, latitude) were used as a reference of the patients found in that area. The quantity and location of the distribution centers and their location were data provided by the company, also the preference of which DC will be analyzed were made by their requirement.

3.2 Geodesic Distance

For the matrix of distance data, the Geodesic distance formula was used to measure the distance d_{ij} between the distribution center and every zip code. This formula has been used to generate assignments, which don't necessarily have to have high accuracy concerning the distance between two points, as well as the lack of data on the distances of roads between the different points [10].

$$d_{ij} = \cos^{-1}(b * d * \cos c + \cos b * \cos d * \cos(c - a)) * R \quad (5)$$

where: (a, b) and (c, d) represent the pair of longitude and latitude in radians of location i and j , respectively, and R the average radius of Earth.

3.3 Distribution Center Assignment

Being the objective to minimize the distance between the distribution center and the centroids of the ZIP code. Our formulation uses binary variables x_{ij} for each arc (i, j) to denote whether or not the distribution center i is assigned to the zip code j .

$$\min \sum_i \sum_j d_{ij} x_{ij} \quad (6)$$

The constraints were defined by: One Zip code just can be attended by one distribution center.

$$\sum_i x_{ij} = 1 \quad \forall j \in J \quad (7)$$

Also, every DC has a zip code limit assignment (L).

$$\sum_j l_{ij} x_{ij} \leq b_i \quad \forall i \in I \quad (8)$$

Finally, the constraints enforce the integrality condition on the decision variables.

$$x_{ij} \in \{0, 1\} \quad \forall i \in I, \forall j \in J \quad (9)$$

3.4 Python as a Programming Tool

To solve the GAP, Python was used as the programming tool to achieve the assignation of the optimal DC to each patient zip code, and it was used the Cplex method to achieve the purpose of this optimization problem.

4 Results

So far, the optimal distribution centers for every patient’s zip code were obtained using only distance and a maximum quantity of zip codes that a DC could attend. Comparing the assignments that the company previously used, it was found that 10% of the assignments made by the company were not optimal.

At the same time, it was observed that the decisions being made for assignments was based on the coincidence of whether or not they were within the same State, which caused that the zip codes that were in the outskirts between two DC delivery areas were not attended necessarily by the closest one, these can be seen in Fig. 2. Where it is quite evident that there are better DC options that could serve patients who are in the State of Puebla but who should not necessarily be attended by the DC belonging to that same State.

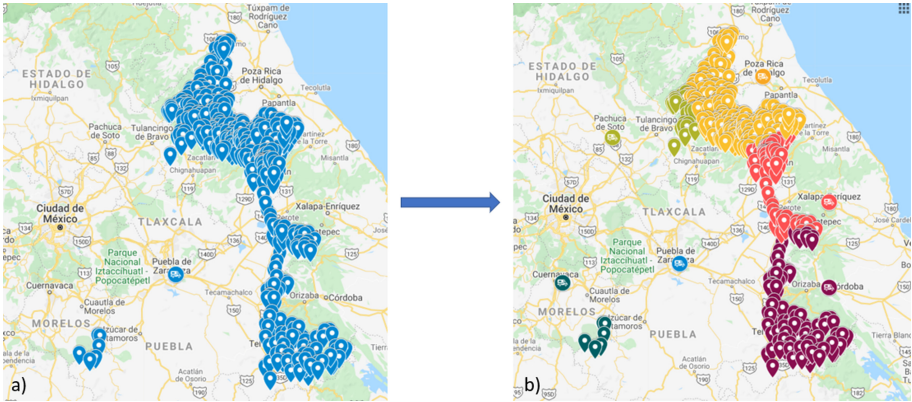


Fig. 2. a) Assignment made for the state of Puebla on its border zip codes that don’t belong to their optimal DC , b) The new assignation proposed by the GAP implementation

By distance, it is convenient to change the current patients who belong to the DC of Puebla to the 3 DCs that belong to the state of Veracruz, as well as a DC from the state of Pachuca and another from the state of Morelos, where there would be a saving of 383 km traveled.

This problem can also be observed in the state of Jalisco, where likewise most of the patients on the outskirts of the city of Guadalajara and other nearby towns of the state of Jalisco were assigned to the DC of Guadalajara.

However, due to optimality, these patients should be attended by the DCs of Puerto Vallarta and Ocotlan that belong to the same state, as well as the DCs that are in different states such as Durango, Aguascalientes, Guanajuato and Colima, having a saving of 548 km traveled for the delivery of oxygen cylinders Fig. 3.

In these cases, it was observed that it is due to the fact the patients who belong to the two States mentioned before, are on the outskirts or limits of the urban area, they are adjacent to other states, which causes that they haven't had a correct DC assignment from the beginning.

This phenomenon occurs in most of the non-urban areas of the other states that were analyzed. For the patients that belong to nine states that were analyzed, previously they were served by 15 DCs, however, after applying the GAP it was concluded that it is necessary to include 5 extras DCs to better distribute the workload.

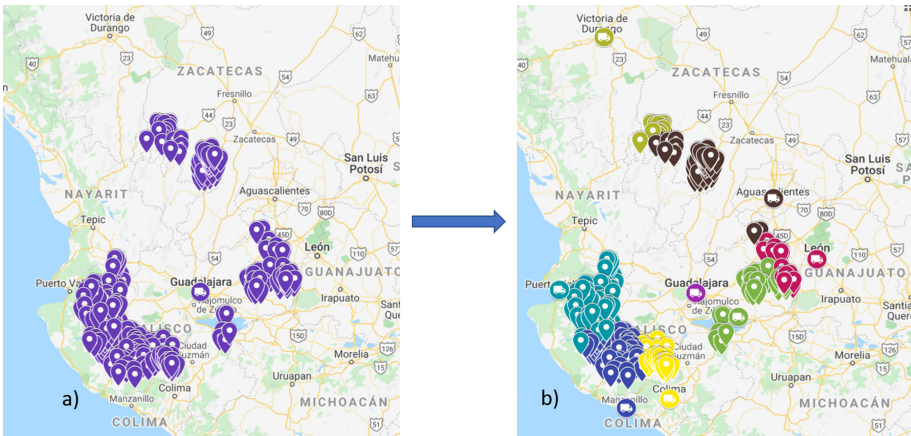


Fig. 3. a) Assignment made for the state of Jalisco on its border zip codes that don't belong to their optimal DC, b) The new assignment proposed by the GAP implementation

An analysis was carried out by States, to determine the savings in distance traveled from the changes that were made when replacing the DC used previously by those that the GAP grants as optimal (Table 2).

Carrying out these changes did not cause any DC to be over-saturated with zip codes to serve, since DCs, have a significant slack for the maximum limits of delivery areas. Although these results still do not translate into monetary savings, it is assumed that the 1,830 km of savings in distance traveled will positively impact the earnings for the home delivery service.

Table 2. Savings on travel distance to deliver medical oxygen at the patients zip codes ubication.

Patient state	DC company assignment	DC GAP assignment	Distance savings (Km)
Colima	Colima, Colima	Puerto Vallarta, Jalisco	128
Guanajuato	Celaya, Guanajuato	Leon, Guanajuato	50
Guanajuato	Celaya, Guanajuato	Queretaro, Queretaro	45
Guanajuato	Celaya, Guanajuato	Ocotlan, Jalisco	50
Hidalgo	Pachuca, Hidalgo	Queretato, Queretaro	50
Hidalgo	Pachuca, Hidalgo	Poza Rica, Veracruz	45
Jalisco	Guadalajara, Jalisco	Durango, Durango	52
Jalisco	Guadalajara, Jalisco	Aguascalientes, Aguascalientes	58
Jalisco	Guadalajara, Jalisco	Leon, Guanajuato	48
Jalisco	Guadalajara, Jalisco	Manzanillo, Colima	165
Jalisco	Guadalajara, Jalisco	Puerto Vallarta, Jalisco	65
Jalisco	Guadalajara, Jalisco	Colima, Colima	90
Jalisco	Guadalajara, Jalisco	Ocotla, Jalisco	70
Puebla	Puebla, Puebla	Cuernavaca, Morelos	38
Puebla	Puebla, Puebla	Pachuca, Hidalgo	70
Puebla	Puebla, Puebla	Cordoba, Veracruz	70
Puebla	Puebla, Puebla	Xalapa, Veracruz	65
Puebla	Puebla, Puebla	Poza Rica, Veracruz	140
Oaxaca	Oaxaca, Oaxaca	—	0
Sinaloa	Mazatlán	Culiacan, Sinaloa	160
Tabasco	Villa Hermosa, Tabasco	Coatzacoalcos, Veracruz	60
Veracruz	Poza Rica, Veracruz	Xalapa, Veracruz	70
Veracruz	Coatzacoalcos, Veracruz	Cordoba, Veracruz	200
Veracruz	Coatzacoalcos, Veracruz	Tuxtepec, Veracruz	41

5 Conclusions

It was found that there are indeed errors in the manual allocation carried out in the company, so generating a method of choosing the closest distribution center will generate greater benefits in time and savings, by reducing the distances traveled from the distribution center to customers.

However, the project is in the first phase of its development since it's also desired to propose a scheduling scheme for the delivery of medicinal oxygen.

Although, routes will not be proposed, it is necessary to do a territory analysis to assign the days of visit for clients depending on the area in which they reside, and therefore generate savings in the delivery of the product, as well, ensuring the days of visit and at the same time generate reliability in the patients towards the company.

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