Sliding Mode Adaptive Control for Sensorless Permanent Magnet Synchronous Motor

Lei Wang[1], Tongwei Liang[2], and Shengjun Wen[3](✉)

[1] HRG (Shandong) Intelligent Equipment Research Institute, No. 1268 Gongye 4 Road, Jinan 250000, Shandong, China
wangleisd@hitrobotgroup.com

[2] Department of Electric and Information Engineering, Zhongyuan University of Technology, No. 41 Zhongyuan Road, Zhengzhou 450007, China

[3] Zhongyuan-Peterburg Aviation College, Zhongyuan University of Technology, No. 41 Zhongyuan Road, Zhengzhou 450007, China
wsj@zut.edu.cn

Abstract. In this paper, a sliding mode control technology based on sensorless adaptive estimation is proposed for a permanent magnet synchronous motor. Firstly, the motor position and speed is estimated by using an adaptive estimator based on model reference instead of the traditional mechanical sensor, where the stability of the estimating system is ensured by using Popov hyperstability. Then, the sliding mode control is designed to track the rotor position accurately. Finally, the simulation results are given to show the presented scheme, where, the estimation error is less than 1.5% and the tracing error of the rotor speed 0.7% under the sliding mode control scheme.

Keywords: Permanent magnet synchronous motor · Sliding mode control · Adaptive estimation · Sensorless technology

1 Introduction

Permanent magnet synchronous motor (PMSM) is widely used in various complicated working situations due to its high speed, wide speed range, good reliability and excellent control characteristics, especially in high-power AC drive systems. Various sensorless control methods of PMSM with high performance are proposed, inductance method [1], carrier frequency component method [2], high frequency signal injection method [3, 4], which can be applied to low speed control. Most of these methods are considered by analyzing the output response of the motor to suppress the external input signal. The complicated signal processing and analytical algorithm make it difficult to apply these methods to the motor control at a higher speed. Some methods based on the motor back-EMF are suitable for medium speed and high speed control, which include direct calculation method based on the mathematical equations of the motor [5], extended Kalman filter method [6], adaptive observer method [7, 8], sliding mode observer method

© ICST Institute for Computer Sciences, Social Informatics and Telecommunications Engineering 2021
Published by Springer Nature Switzerland AG 2021. All Rights Reserved
W. Fu et al. (Eds.): ICMTEL 2021, LNICST 387, pp. 214–222, 2021.
https://doi.org/10.1007/978-3-030-82562-1_19

[9] and artificial intelligence algorithm [10]. However, for above methods, the estimation accuracy is poor, or there exist complicated calculation problems which need to take up many resources in actual applications.

In this paper, a sliding mode control technology based on sensorless adaptive estimation is proposed for the permanent magnet synchronous motor. The motor position and speed is estimated by using an adaptive estimator based on model reference. And the sliding mode control is designed to ensure the tracking performance accurately.

2 Model of Permanent Magnet Synchronous Motor

The voltage equation of a three-phase permanent magnet synchronous motor can be described as in (1) in the natural coordinate system.

$$\begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & R_1 \end{bmatrix} \begin{bmatrix} i_{\mathrm{a}} \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_a(\theta, i) \\ \psi_b(\theta, i) \\ \psi_c(\theta, i) \end{bmatrix} \tag{1}$$

Here, U_a, U_b, U_c is the three-phase winding voltage, R_1 is the stator winding resistance; i_a, i_b, i_c is the three-phase winding line current;$\psi_a(\theta, i)$, $\psi_b(\theta, i)$, $\psi_c(\theta, i)$ is the three-phase winding full flux linkage, θ is the space between the rotating coordinate d axis and the three-phase stationary coordinate A axis Electrical angle.

The flux linkage equation for a three-phase winding is:

$$\begin{bmatrix} \psi_A(\theta, i) \\ \psi_B(\theta, i) \\ \psi_C(\theta, i) \end{bmatrix} = \begin{pmatrix} L_1 & -M_1 & -M_1 \\ -M_1 & L_1 & -M_1 \\ -M_1 & -M_1 & L_1 \end{pmatrix} \bullet \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} + A \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} \tag{2}$$

where

$$A = \begin{pmatrix} -L_2 \cos 2\theta & M_2 \cos 2(\theta + 30°) & M_2 \cos 2(\theta + 150°) \\ M_2 \cos 2(\theta + 30°) & -L_2 \cos 2(\theta - 120°) & M_2 \cos 2(\theta - 90°) \\ M_2 \cos 2(\theta + 150°) & M_2 \cos 2(\theta - 90°) & -L_2 \cos 2(\theta + 120°) \end{pmatrix}$$

and L_1 and L_2 are the average and second harmonic amplitude of the phase stator winding self-inductance. M_1 and M_2 are the absolute values of the mutual inductance average of the two-phase stator winding and the amplitude of the second harmonic. The electromagnetic torque of a permanent magnet synchronous motor can be expressed as:

$$T_e = \begin{cases} p \cdot \begin{bmatrix} i_a & i_b & i_c \end{bmatrix} T \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{p}{\omega} \begin{bmatrix} i_a & i_b & i_c \end{bmatrix} \cdot \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} \\ \frac{J}{p} \frac{d\omega}{dt} - T_f \end{cases} \tag{3}$$

where p is the pole pair number of the permanent magnet synchronous motor, ω_e is the motor angular frequency, and E_a, E_b, and E_c are the back electromotive forces generated in the stator winding when the motor rotor rotates. J is the moment of inertia and T_f is the load torque at the shaft end of the motor rotor.

Permanent Magnet Synchronous Motor will be control in two phase d-, q- reference coordinate system through coordinate transformation. In this coordination, the d-axis is the direct-axis, which is related to the rotor flux linkage. The q-axis is quadrature-axis, which is 90° leading d-axis. The PMSM model is shown in Eq. (4).

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = R\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_\alpha \\ \psi_\beta \end{bmatrix} \tag{4}$$

We have

$$\begin{bmatrix} \psi_\alpha \\ \psi_\beta \end{bmatrix} = \begin{bmatrix} L + \triangle L cos2\theta_r & -\triangle L sin2\theta_r \\ -\triangle L sin2\theta_r & L - \triangle L cos2\theta \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \psi_f\begin{bmatrix} cos\theta_r \\ sin\theta_r \end{bmatrix}$$

where u_α, u_β i_α i_β ψ_α ψ_β are the stator phase voltage, phase current, and phase total flux linkage, respectively. $L = (L_d + L_q)/2$ is the average inductance; $VL = (L_d - L_q)/2$ is the semi-difference inductance, and θ_r is the rotor position.

3 Adaptive Estimation of the Rotor Speed

The adaptive estimation structure of the rotor speed based on reference model is shown in Fig. 1 for the sensorless permanent magnet synchronous motor. The rotor position is obtained by the adaptive estimation method, and the reference model is selected without the position reference. A reasonable parameter adaptive law is designed to adjust the parameters of the adjustable model and accurately track and estimate the rotation speed.

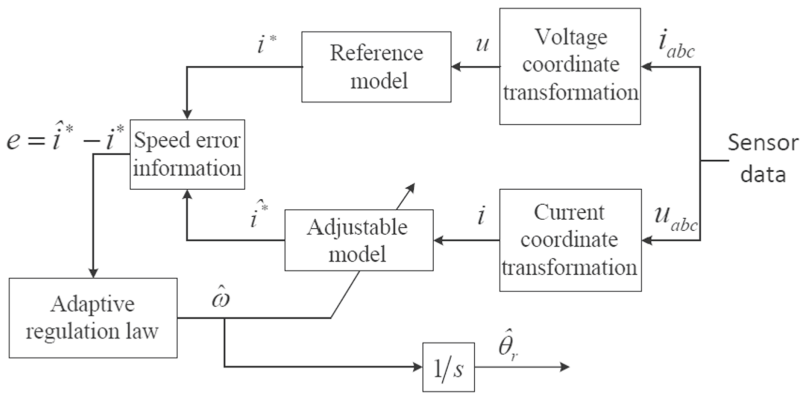

Fig. 1. Block diagram of sensorless control based on speed estimation

Combining the motor mathematical model established by Eqs. (1)–(4), the current equation of the motor in the synchronous rotating coordinate system can be obtained as follows.

$$\begin{cases} \frac{d}{dt}i_d = -\frac{R}{L_d}i_d + \omega\frac{L_q}{L_d}i_q + \frac{1}{L_d}u_d \\ \frac{d}{dt}i_q = -\frac{R}{L_q}i_q - \omega\frac{L_d}{L_q}i_d - \frac{\psi}{L_q}\omega + \frac{1}{L_q}u_q \end{cases} \tag{5}$$

Then, we have the adjustable model equation of the system as follows,

$$\frac{d}{dt}\begin{bmatrix}\hat{i}_d^*\\ \hat{i}_q^*\end{bmatrix}=\begin{bmatrix}-\frac{R}{L_d} & \hat{\omega}\frac{L_q}{L_d}\\ -\hat{\omega}\frac{L_d}{L_q} & -\frac{R}{L_q}\end{bmatrix}\begin{bmatrix}\hat{i}_d^*\\ \hat{i}_q^*\end{bmatrix}+\begin{bmatrix}\frac{1}{L_d}u_d^*\\ \frac{1}{L_q}u_q^*\end{bmatrix} \tag{6}$$

where $\hat{i}_d^* = \hat{\mathrm{i}}_d + \frac{\psi}{L_d}, \hat{i}_q^* = \hat{\mathrm{i}}_q, u_d^* = u_d + \frac{R_1}{L_d}\psi, u_q^* = u_q$.

Define generalized errors of the currents $e = i^* - \hat{i}^*$, we have

$$\frac{d}{dt}\begin{bmatrix}e_d\\ e_q\end{bmatrix}=\begin{bmatrix}-\frac{R}{L_d} & \omega\frac{L_q}{L_d}\\ -\omega\frac{L_d}{L_q} & -\frac{R}{L_q}\end{bmatrix}\begin{bmatrix}e_d\\ e_q\end{bmatrix}-(\hat{\omega}-\omega)\mathrm{J}\begin{bmatrix}\hat{i}_d^*\\ \hat{i}_q^*\end{bmatrix} \tag{7}$$

It can be rewritten as

$$\begin{cases}\frac{d}{dt}e = \boldsymbol{Ae} - \boldsymbol{W}\\ \boldsymbol{v} = \boldsymbol{C}\cdot\boldsymbol{e}\end{cases} \tag{8}$$

where $e_d = \hat{i}_d^* - i_d^*$, $e_q = \hat{i}_q^* - i_q^*$, $\boldsymbol{W} = (\hat{\omega}-\omega)\boldsymbol{J}\hat{\boldsymbol{i}}^*$, $\boldsymbol{B} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $\boldsymbol{C} = \boldsymbol{I} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $\boldsymbol{J} = \begin{bmatrix}0 & -\frac{L_q}{L_d}\\ \frac{L_d}{L_q} & 0\end{bmatrix}$, $\boldsymbol{A} = \begin{bmatrix}-\frac{R}{L_d} & \omega\frac{L_q}{L_d}\\ -\omega\frac{L_d}{L_q} & -\frac{R}{L_q}\end{bmatrix}$.

According to Popov hyperstability [24], to make the system stable, the system (8) shall meet the requirement

$$\eta(0,t') = \int_0^{t'} v^T \boldsymbol{W}\, dt \geq -\gamma_0^2 \;\left(t' \geq 0,\; \gamma_0 \geq 0\right)$$

where $\boldsymbol{W} = (\hat{\omega}-\omega)\boldsymbol{J}\hat{\boldsymbol{i}}^{\boldsymbol{*}}$ satisfies the condition of the positive real.

In order to the above condition, we assume that the parameter adaptive regulation law has the following proportional integral form:

$$\hat{\omega} = \int_0^t F_1(v,t,\tau)d\tau + F_2(v,t) + \hat{\omega}(0) \tag{9}$$

where $\hat{\omega}(0)$ is the angular velocity of the motor at the initial moment, and we have.

$$F_1(v,t,\tau) = k_i e^{\mathrm{T}}\boldsymbol{J}\hat{\boldsymbol{i}}^{\boldsymbol{*}}, \qquad F_2(v,t) = k_p e^T \boldsymbol{J}\hat{\boldsymbol{i}}^{\boldsymbol{*}} \tag{10}$$

It can be proved, the above design satisfies the condition of Popov integral inequality. Therefore, the adaptive estimation parameter is convergence. Substitute Eq. (10) into Eq. (9) and the adaptive regulation law of the motor speed can be got

$$\begin{aligned}\hat{\omega} &= \textstyle\int_0^{t'} k_i e^{\mathrm{T}}\boldsymbol{J}\hat{\boldsymbol{i}}^{\boldsymbol{*}} d\tau + k_p e^{\mathrm{T}}\boldsymbol{J}\hat{\boldsymbol{i}}^{\boldsymbol{*}} + \hat{\omega}(0)\\ &= \textstyle\int_0^{t'} k_i D d\tau + k_p D + \hat{\omega}(0)\end{aligned} \tag{11}$$

where

$$D = e^{\mathrm{T}} \boldsymbol{J}\hat{\boldsymbol{i}}^{*} = \frac{L_q}{L_d} i_d \hat{i}_q - \frac{L_d}{L_q} i_q \hat{i}_d + \frac{\psi}{L_d}\frac{L_d}{L_q}\left(\hat{i}_q - i_q\right) + \hat{i}_d \hat{i}_q \left(\frac{L_d}{L_q} - \frac{L_q}{L_d}\right).$$

The estimation position of the rotor can be obtained

$$\hat{\theta}_r = \int_0^t \hat{\omega} dt \tag{12}$$

4 Sliding Mode Control of Permanent Magnet Synchronous Motor

Then, the sliding mode control scheme is considered to track the rotor position accurately along an exponentially approaching rule. The sliding mode control diagram based on speed estimation is shown in Fig. 2.

In this paper, an exponentially approaching rule $\dot{s} = -\varepsilon\,\mathrm{sgn}(s) - qs,\ \varepsilon, q > 0$ is used to design the controller, where we consider $i_q = 0$. In the synchronous rotating coordinate system, the current equation of the motor is

$$\begin{cases} \dfrac{di_q}{dt} = \dfrac{1}{L}\left(u_q - Ri_q - p\psi_f\right) \\ \dfrac{d\omega}{dt} = \dfrac{1}{J}\left(\dfrac{3p\psi_f}{2} i_q - T_L\right) \end{cases} \tag{13}$$

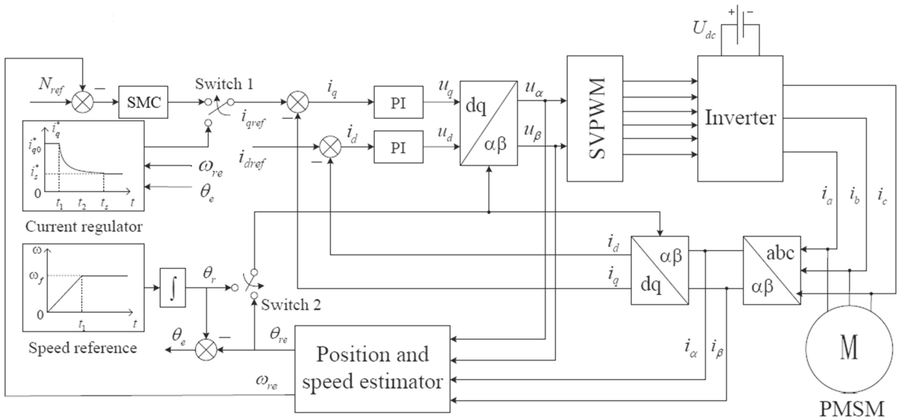

Fig. 2. Block diagram of sliding mode control based on speed estimation

Define the state varies of the motor as

$$\begin{cases} x_1 = \omega_{ref} - \omega \\ x_2 = \dot{x}_1 = -\dot{\omega} \end{cases} \tag{14}$$

where ω_{ref} and ω are the reference speed and real speed of the motor. So, the state equation can be described as

$$\begin{cases} \dot{x}_1 = -\dot{\omega}_m = \frac{1}{J}\left(T_L - \frac{3p\psi_f}{2} i_q\right) \\ \dot{x}_2 = -\ddot{\omega}_m = -\frac{3p\psi_f}{2J}\dot{i}_q \end{cases} \quad (15)$$

Assume that $u = \dot{i}_q$, $H = \frac{3p\psi_f}{2J}$, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -H \end{bmatrix} u \quad (16)$$

We design a sliding variable as $s = cx_1 + x_2$, where c is the design parameter and larger than 0. Then, its derivation is

$$\dot{s} = c\dot{x}_1 + \dot{x}_2 = cx_2 + \dot{x}_2 = cx_2 - Hu \quad (17)$$

Ignoring the disturbance estimation error, the equivalent control can be obtained by

$$u = \frac{1}{H}\left[cx_2 + \varepsilon\,\mathrm{sgn}(s) + qs\right] \quad (18)$$

That is, the projection current of the rotating coordinate system is

$$i_q^* = \frac{1}{H}\int_0^t \left[cx_2 + \varepsilon\,\mathrm{sgn}(s) + qs\right] d\tau \quad (19)$$

From the above projection current, we can see that an integral term is included in the design. So that, it can eliminate effectively the chattering phenomenon of the sliding mode control system and reduce the steady-state error of the system. Also, the system is gradually stable under the condition $s\dot{s} = -\varepsilon s\,\mathrm{sgn}(s) - qs^2 < 0$.

5 Simulation Results

In this section, MATLAB Simulink is used to verify the proposed adaptive estimation and sliding mode control scheme. The motor parameters used in the simulation experiment are shown in Table 1.

Table 1. Parameters of PMSM for experiment

Parameters	Value	Parameters	Value
Rated frequency f_n	50 Hz	The rotor ux linkage ψ_f	0.213 wb
Rated power P_w	1 Kw	Stator resistance R	2.878 Ω
Rated speed N_r	1000 r/min	The moment of inertia J	1.94×10^{-3} kg · m^2
Axis inductance L_d, L_q	0.85 mH	Pole pairs p	4

The results of the adaptive estimation are shown in Fig. 3 and Fig. 4, where Fig. 3 shows speed estimation of sensorless motor during load fluctuation and Fig. 4 shows position error of sensorless motor. In order to verify the adaptive performance under the load disturbance, the electromagnetic torque of the motor is transformed from 0 Nm to 2 Nm at 0.2 s and from 2 Nm to 1 Nm at 0.4 s. From the results, we can see that the speed and position can be estimated accurately under the load fluctuation. Also, it shows that the estimation error of the motor position is less than 1.5%.

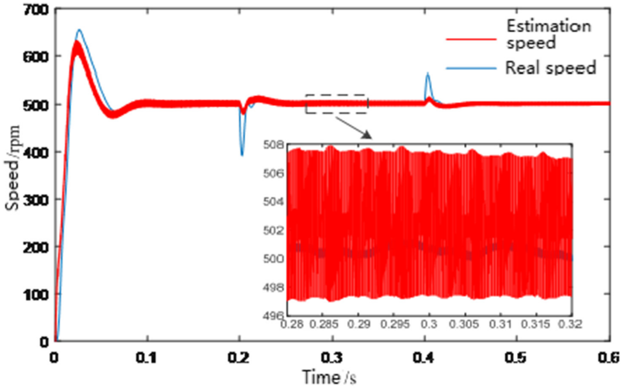

Fig. 3. Real speed and estimation speed**.**

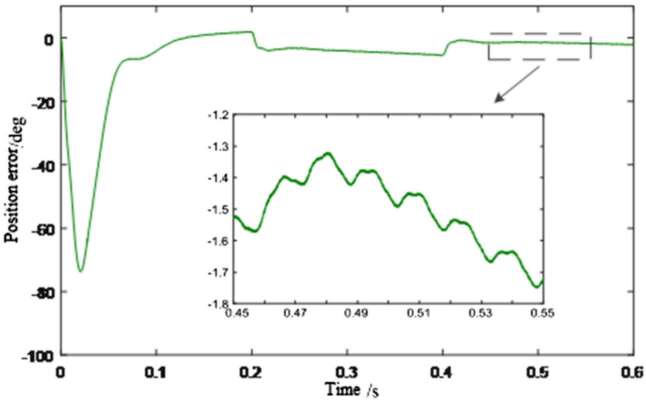

Fig. 4. Position error under adaptive estimation

Figure 5 shows the speed response of sliding mode control during load fluctuation and Fig. 6 shows the speed response of sliding mode control under variable reference input. From Fig. 5, it shows that the robust stability of the system can be ensured when the load torque of the motor is suddenly changed (increase by 0.5 Nm). In Fig. 6, the motor is operated at the rising and falling speed. These results show that the steady-state error of the system is less than 0.9% by using the presented sliding mode control method.

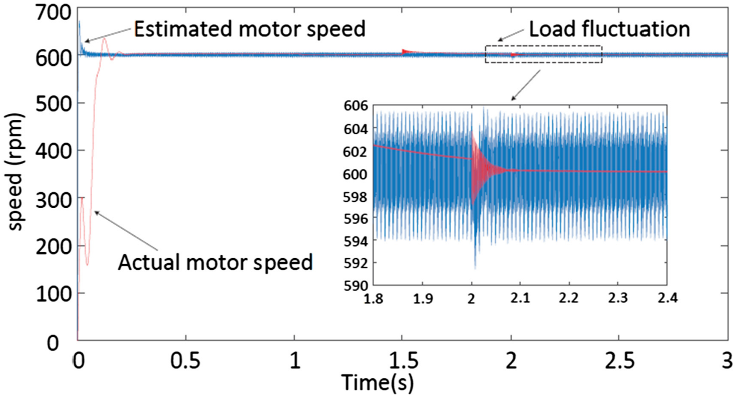

Fig. 5. Speed response with load fluctuation under adaptive estimation

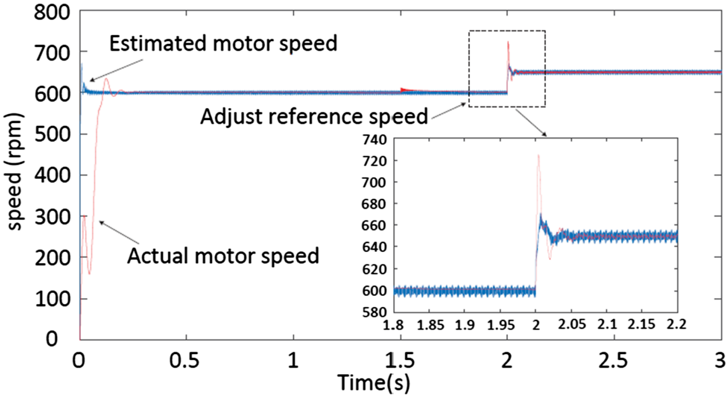

Fig. 6. Speed response with variable reference input under adaptive estimation

6 Conclusion

In this paper, a sliding mode control technology based on sensorless adaptive estimation is proposed for a permanent magnet synchronous motor. Firstly, the motor position and speed is estimated by using an adaptive estimator based on model reference instead of the traditional mechanical sensor, where the stability of the estimating system is ensured by using Popov hyperstability. Then, the sliding mode control is designed to track the rotor position accurately. Finally, the simulation results are given to show the presented scheme, where, the estimation error is less than 1.5% and the tracing error of the rotor speed 0.7% under the sliding mode control scheme.

Acknowledgment. This work was supported in part by the National Natural Science Foundation of China (U1813201, 62073297, 61973157); National Key Research and Development Project of China (2020YFB1712403); Fundamental Research Foundation of Zhongyuan University of Technology (K2020TD005).

References

1. Schmidt, E., Eilenberger, A.: Calculation of position-dependent inductances of a permanent magnet synchronous machine with an external rotor by using voltage-driven finite element analyses. IEEE Trans. Magn. **45**(3), 1788–1791 (2009)
2. Jeong, Y., Lorenz, R.D., Jahns, T.M., Sul, S.: Initial rotor position estimation of an interior permanent-magnet synchronous machine using carrier-frequency injection methods. IEEE Trans. Ind. Appl. **41**(1), 38–45 (2005)
3. Li, Z., Xia, C., Chen, W.: Position sensorless control of brushless DC motor based on line back EMF. J. Electric. Eng. **25**(7), 24–41 (2010)
4. Yuan, L., Hu, B., Wei, K., Chen, W.: The Control Principle and MATLAB Simulation of Modern Permanent Magnet Synchronous Motor. Beijing University of Aeronautics and Astronautics Press, Beijing (2015)
5. Raca, D., Harke, M.C., Lorenz, R.D.: Robust magnet polarity estimation for initialization of PM synchronous machines with near-zero saliency. IEEE Trans. Ind. Appl. **44**(4), 1199–1209 (2008)
6. Wang, G., Yang, R., Yu, Y., et al.: Initial position estimation method for built-in permanent magnet synchronous motor rotor. J. Electric Mach. Control **14**(6), 51–60 (2010)
7. Louis, J.P., Zhu, X.: Synchronous Motor Control. Mechanical Industry Press, Beijing (2016)
8. Wang, Z., Ye, Y.: Self-starting process of permanent magnet synchronous motor without position sensor based on back EMF algorithm. J. Electric Mach. Control **10**(15), 36–41 (2011)
9. Zhaowei, Q., Tingna, S., Yindong, W.: New sliding-mode observer for position senseless control of permanent-magnet synchronous motor. IEEE Trans. Ind. Electron. **60**, 710–741 (2013)
10. Liu, J., Xiao, F., Shen, Y., Mai, Z., Li, C.: A review of the research on position sensorless control technology for permanent magnet synchronous motors. J. Electric. Eng. **16**(32), 76–85 (2010)