Fast and Simple Method for Weapon Target Assignment in Air Defense Command and Control System

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Abstract. The assignment problem is a fundamental optimization problem which can be applied in many real-life automation tasks including weapon target assignment (WTA) as a basic functional module of automated command and control centers. The WTA problem is a class of optimization problems in the field of optimization and operation research. It consists of finding an optimal assignment of a set of weapons of various types to a set of targets in order to maximize the total expected damage to the opponent. In this paper, we propose an optimal air target distribution method in a mixed air defense cluster scenario. Our mathematical formulation is built with both weapons units and targets which are diverse in types in the practical complex anti-aircraft combat conditions. Then, we proposed a method based on Kuhn-Munkres assignment algorithm which has low computational cost and complexity, allowing quick estimation of the optimal distribution plan based on the criterion "efficiency: maximum hostile target destruction, minimum cost of weapon consumed and surface target damage". The experimental results have clarified that the proposed method has a great efficiency in real-time and optimal requirements.

Keywords: Assignment problem · Weapon targets assignment · Command and control · Air defense system

1 Introduction

Experience in wars and conflicts since the World War II shows that most of the victories were achieved by one side owning the superiority in the air. Therefore, air defence systems are the most sophisticated and specialized defense systems in any modern army. The air defense systems is a combination of technical measures including weapon systems, associated sensor systems, command and control arrangements. The air defense automated command and control systems ensure the complete implementation of the following tasks in real-time:

- collecting, processing and fusing sensor information on air and ground situations;

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- forming the most accurate model of the real situation in the combat area from sensors data, GIS and reports;
- automatically analyzing aerial situations and provide operational decision-proposals on the basis of available options.

One of the key parts of the operational decision is the allocation of defense weapons to threatening enemy targets, this process will be called weapon target assignment (WTA) further in this paper. In an air defense situation, operators have to evaluate the most cost-effective WTA in real-time. Due to human factors, operators tend to perform worse as the number of targets increase [1]. This is the reason there is a need for semi-automated and automated decision support systems, that can evaluate the air situation and find the optimal assignment decision to control weapon terminals.

The weapon target assignment problem is a class of computational optimization problems present in the fields of optimization and operations research. It consists of finding an optimal assignment of a set of weapons of various types to a set of targets in order to maximize the total expected damage done to the opponent. The WTA problem can be formulated as a nonlinear integer programming problem and is known to be NP-complete. There are constraints on weapons available of various types and on the minimum number of weapons by type to be assigned to various targets. The constraints are linear, and the objective function is nonlinear. The objective function is formulated in terms of probability of damage of various targets weighted by their military value [2].

In this paper, we propose an optimal air target distribution method for the weapons unit of a mixed air defense cluster. The key difference of assignment problem in our formulation is both weapons units and targets are diverse in types in real life complex anti-aircraft combat conditions. The proposed method is built on the basis of Kuhn-Munkres assignment algorithm, features with low computational cost and complexity, allowing quick estimation of the optimal distribution plan based on the criterion "efficiency: maximum hostile target destruction, minimum cost of weapon consumed and surface target damage". Thanks to the simplicity of Kuhn-Munkres assignment algorithm [3], the computational cost and complexity are reduced, allowing quick estimation of the optimal distribution. In the next part of this paper, we will evaluate the algorithm on typical WTA scenarios and compare the computational cost of the algorithm in different problem sizes.

2 Conventional Approaches for Solving WTA Problems

There are several popular methods to solve the WTA problem, such as:

– Dynamic programming methods include linear programming, stochastic programming, and mixed-integer programming [4]. Linear programming is an optimization technique of a linear objective function, subject to linear equality and linear inequality constraints. It is a mathematical method that is used to determine the best possible outcome or solution from a given set of parameters or a list of requirements, which are represented in the form of linear relationships. Stochastic programming offers a solution by eliminating uncertainty and characterizing it using probability distributions. A mixed-integer programming problem is a mathematical optimization in which some of the variables are restricted to be integers. There are many methods of solving programming problems, but in general, these problems are very complex in computational cost, especially with large size.

– Meta-heuristic optimization algorithms for static WTA [5], including the algorithms Ant Colony Optimization (ACO), Genetic Algorithm (GA), Enhanced Maximal Marginal Return (EMMR) and Particle Swarm Optimization (PSO). In [6], the authors proposed to use a hybrid algorithm including the improved algorithm Artificial Fish Swarm Algorithm (AFSA) and Harmony Search (HS) to solve the problem of WA for anti-air weapons. The comparison of heuristic WTA algorithms may seem trivial, but there are a number of aggravating circumstances. Firstly, there is a lack of unclassified real-world data sets on which to evaluate the algorithms. For this reason, it seems to be standard procedure to evaluate the algorithms on randomly generated synthetic data sets. Secondly, most heuristic algorithms have a large degree of inbuilt randomness. It is therefore important to base the evaluation of such algorithms on a statistically sufficient number of runs, rather than a single one. Thirdly, algorithms for static WTA only have a very limited time available for searching for good or optimal solutions, due to the real-time requirements, thus an algorithm can generate very good solutions after a long search time can return quite bad solutions when the available search time is shorter.
– In addition to these complicated methods, a study in [7] has proposed the target distribution algorithm for fast-response weapon systems in multi-target and multi-weapon scenarios. The target distribution is built based on the target's azimuth range, distance and threat level value. However, for the purpose of simplicity, the author has assumed that all fire powered targeted vehicles are of the same type. This is not feasible when being applied to a mixed air defense formation with many types of weapons and heterogeneous features.

Our method differs from the above listed approaches in the manner that we propose an optimal and fast assignment solution for the WTA problem of a mixed air defense cluster with strict-deadline real time requirements, which means the algorithm has determined computational time for each problem size. The method is to be applied in mixed-weapons systems with each weapon having different cost and effectiveness, and each target having different threat levels and kill probabilities.

3 Mathematical Formulation of WTA Problem

The target distribution problem is illustrated in Fig. 1, [2, 8], where: $\mathbf{T} = \{T_1, T_2, \ldots, T_N\}$ - set of hostile air targets; $\mathbf{O} = \{O_1, O_2, \ldots, O_K\}$ - collection of objects to be protected; $\mathbf{W} = \{W_1, W_2, \ldots, W_M\}$ - collection of anti-aircraft weapon systems. Each dashed arrow represents a possible assignment between a target and a weapon unit. The number of possible assignments is $N \times M$.

In mixed air defense cluster, each weapon unit is characterized by the boundaries of the affected area in height $H_{i\ \max}$, $H_{i\ \min}$ and range D_{near} and D_{far}. Determining the range of impact of the weapon unit, the possibility of firing at the target by speed v_i and by the course parameter S_i, the probability of hitting the target P_i, firing time t_i. Each

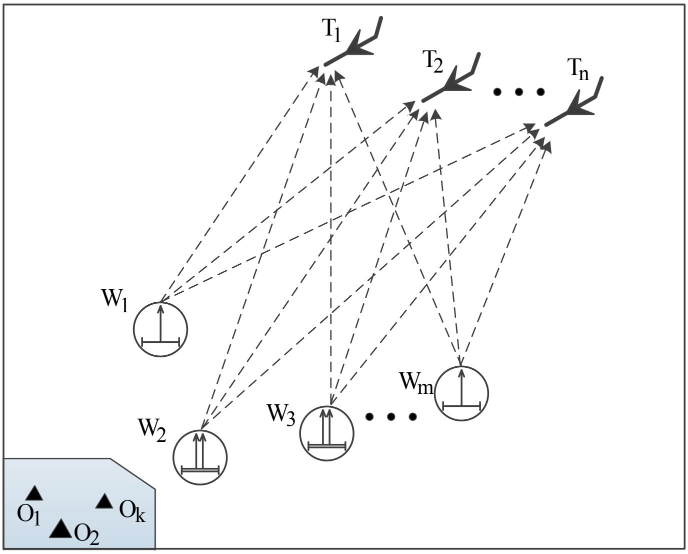

Fig. 1. Illustration of WTA scenario with N targets and M weapon units.

air target is characterized by flight altitude H_j, speed v_j, course parameter S_j and flight time t_j of reaching the $j-th$ target of the impact boundary of the $i-th$ weapon unit.

The firing process of the target by a weapon unit is possible under the following restrictions [9]:

$$\begin{cases} H_{i\ \min} \leq H_j \leq H_{i\ \max} \\ v_j \leq v_i \\ S_j \leq S_i \\ t_j \geq t_{i\ \min} + t_{i\ (j-1)} \end{cases} \tag{1}$$

where, $t_{i\ \min}$ - minimum firing cycle of weapon unit W_i; $t_{i\ (j-1)}$ - time of occupation of the weapon unit W_i by firing the previously assigned target T_{j-1}.

Killing probability is a joint probability of independent events, in this case including possibility of firing and possibility of hitting. If one of listed restrictions is not satisfied, the firing process is impossible and the kill probability of an assignment is equal to zero. Otherwise, the joint probability of a target kill event is formulated as follows [10]:

$$P_{unit\ ij} = 1 - (1 - P_{1\,i})^n \tag{2}$$

where: $P_{1\,i}$ - kill probability of a single missile in weapon unit W_i.

In regard to given condition, impact level of fire command can be measured from kill probability $P_{unit\,ij}$ and target threat level V_j as follows:

$$F_{impact} = \sum_{j=1}^{N} \sum_{i=1}^{M} (P_{unit\,ij} \times V_j)\, m_{ij} \tag{3}$$

In this case effectiveness of a fire command can be formulated as subtraction of impact to cost of the assignment. According to typical tactic requirements, F_{impact} is usually considered as a constant value, as the weapons unit can fire multiple times to an object to achieve the required P_{unit}. The required number of missiles to be fired per target can be calculated as follows:

$$n = \frac{(1 - \log P_{unit\,ij})}{(1 - \log P_{1\,i})} \tag{4}$$

In this condition, the cost of a WTA assignment can be estimated according to the following expression:

$$F_{cost} = \sum_{j=1}^{N}\sum_{i=1}^{M}\overline{C}_{ij}m_{ij} \tag{5}$$

The global function to optimization is called loss function, which can be calculated from cost value and impact value as follows:

$$\begin{aligned} F_{lost} = F_{cost} - F_{impact} &= \sum_{j=1}^{N}\sum_{i=1}^{M}\overline{C}_{ij}m_{ij} - \sum_{j=1}^{N}\sum_{i=1}^{M}(P_{unit\,ij} \times V_j)m_{ij} \\ &= \sum_{j=1}^{N}\sum_{i=1}^{M}\left(C_{1\,i} \times n - P_{unit\,ij} \times V_j\right)m_{ij} \\ &= \sum_{j=1}^{N}\sum_{i=1}^{M}\left[C_{1\,i} \times \frac{(1 - \log P_{unit\,ij})}{(1 - \log P_{1\,i})} - P_{unit\,ij} \times V_j\right]m_{ij} \end{aligned} \tag{6}$$

where:

$P_{unit\,ij}$ - tactical minimum required kill probability (>0.75);

V_j - individual target value estimated for each target T_j.

$C_{1\,i}$ - cost of one missile in weapon unit W_i.

m_{ij} - specific assignment parameters for the T_j target and W_i weapon, $m_{ij} = 1$ if weapon W_i is assigned to target T_j, $m_{ij} = 0$ otherwise [11].

The optimization problem is now formulated as finding assignment values m_{ij} to minimize value of F_{lost}. The main purpose of the assignment problem is to minimize the cost of WTA decision, represented mathematically as F_{lost}. Decision variables of F_{lost} function are the values of the matrix of control parameters m_{ij} regarding to the following constraints:

$$\sum_{i=1}^{M} m_{ij} = 1 \quad (j = \overline{1;N}) \tag{7}$$

$$\text{and} \quad \sum_{j=1}^{N} m_{ij} = 1 \quad (i = \overline{1;M}) \tag{8}$$

The constraint in (7) indicates that only one weapon can be assigned to a target at the current moment of time. Similarly, the constraint in (8) allows only one target to be assigned with a weapon in the current moment of time.

4 Proposed Air Target Distribution Method

In this section, we propose a method to solve the air target distribution problem based on the criteria: maximizes effective protection of objects on the ground; minimizes the cost of using weapon units to destroy air targets; minimizes computation time for consistent application in real-time command and control systems. Due to the air defense combat situation constantly changing, resolving all control tasks in repeated after a specified time, called the control task resolution cycle. The proposed target distribution method is to be performed in each control cycle according to the following steps (illustrated in Fig. 2).

1) Based on sensor information about the targets, such as flight direction, speed, altitude, distance, type… and status of protected objects, it is possible to calculate the threat values $V_{jk} \in [0, 1]$ of each target in relation to each protected object (T_j, O_k). Using these threat values, formation of the threat values matrix for target-protected pairs [8]

$$\mathbf{V} = \begin{bmatrix} V_{11} & V_{12} & \dots & V_{1K} \\ V_{21} & V_{22} & \dots & V_{2K} \\ \dots & \dots & \dots & \dots \\ V_{N1} & V_{N2} & \dots & V_{NK} \end{bmatrix}$$

   An individual target value V_j is estimated for each target T_j. To compute the target value is to simply go through all threat values for a specific target T_j and pick the largest one

$$V_j = \max(V_{jk} w_k), \quad k = 1, \dots, K \tag{9}$$

   where $\omega_k \in [0, 1]$ is a weight representing the important of the protected object O_k, K is the total number of the protected objects.
2) Generation of the air target impact matrix **A** on based on the target value V_j and target kill probability $P_{unit\ ij}$

$$A_{ij} = P_{unit\ ij} \times V_j. \tag{10}$$

3) Generation of the lost matrix $\overline{\mathbf{C}}$ based on the weapon cost and impact value A_{ij}

$$\overline{C}_{ij} = C_{1\,i} \times n - A_{ij}. \tag{11}$$

4) Applying Kuhn-Munkres optimization algorithm to find m_{ij} in (6) in 6 substeps as follows (Fig. 3):

In the input of Kuhn-Munkres algorithm, we are given a non-negative $m \times n$ matrix $\overline{\mathbf{C}}$, where the element $\overline{C}_{ij}$ in the $i - th$ row and $j - th$ column represents the cost of assigning the $j - th$ target to the $i - th$ weapon unit. We have to find an assignment of the target to the weapon unit, such that each target is assigned to one weapon unit and each worker is assigned one target, such that the total cost of assignment is minimum.

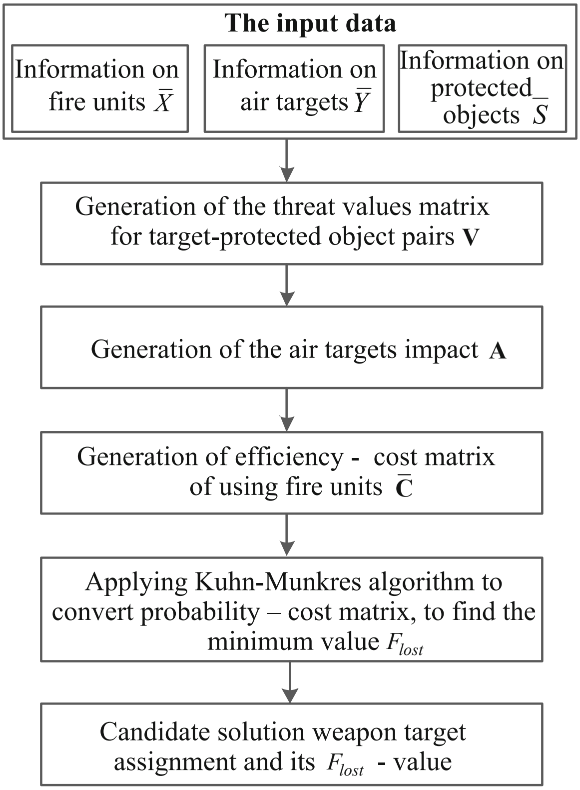

Fig. 2. Flowchart of proposed method to solve the WTA problem.

Step 1: Then we perform row operations on the matrix. To do this, the lowest of all $\overline{\mathbf{C}}$ is taken and is subtracted from each element in that row. This will lead to at least one zero in that row (We get multiple zeros when there are two equal elements which also happen to be the lowest in that row). This procedure is repeated for all rows. We now have a matrix with at least one zero per each row. Go to Step 2.
Step 2: Find a zero (Z) in the resulting matrix. If there is no starred zero in its row or column, star Z. Repeat for each element in the matrix. Go to Step 3.
Step 3: Cover each column containing a starred zero. If K columns are covered, the starred zeros describe a complete set of unique assignments. In this case, Go to END, otherwise, Go to Step 4.
Step 4: Find a non-covered zero and prime it. If there is no starred zero in the row containing this primed zero, Go to Step 5. Otherwise, cover this row and uncover the column containing the starred zero. Continue in this manner until there are no uncovered zeros left. Save the smallest uncovered value and Go to Step 6.
Step 5: Construct a series of alternating primed and starred zeros as follows. Let Z0 represent the uncovered primed zero found in Step 4. Let Z1 denote the starred zero in the column of Z0 (if any). Let Z2 denote the primed zero in the row of Z1 (there will always be one). Continue until the series terminates at a primed zero that has no starred

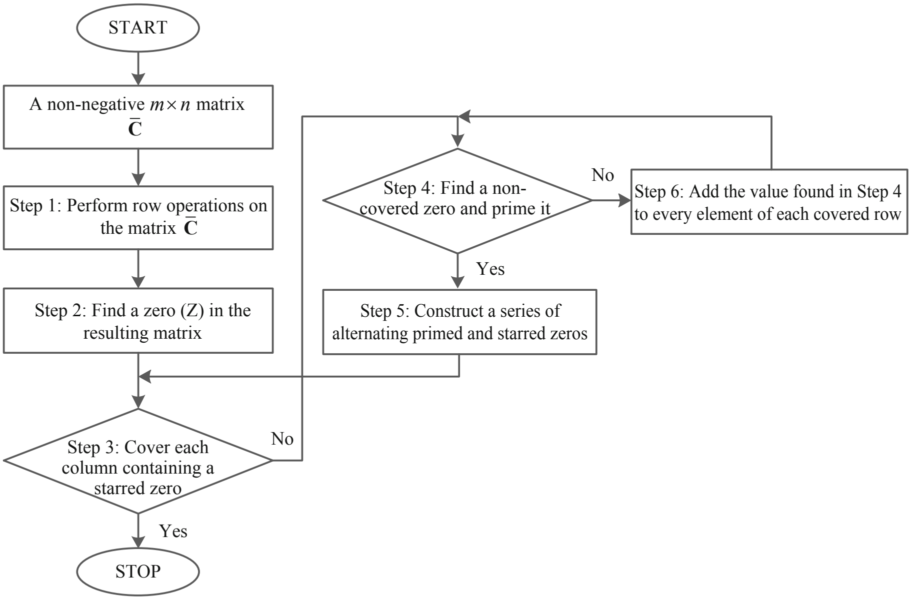

Fig. 3. Flow chart Kuhn-Munkres optimization algorithm.

zero in its column. Remove star each starred zero of the series, star each primed zero of the series, erase all primes and uncover every line in the matrix. Return to Step 3.
Step 6: Add the value found in Step 4 to every element of each covered row, and subtract it from every element of each uncovered column. Return to Step 4 without altering any stars, primes, or covered lines.

5 Experimental Results

For testing and evaluation, the target distribution algorithm was implemented in a computer CPU Intel(R) Core i3-4150, 3.50 GHz, RAM 4 GB, Windows 10 OS and Python 3.8 development environment. In the tests we evaluate the accuracy and effectiveness of the proposed method for different air defense scenarios. The input parameters of the problem such as the target's threat value, the kill probability and the cost of use of a missile for each weapon unit are randomly generated with independent distribution in the segment [0; 1]. As follows:

We create a matrix of threat value of an aerial target regarding to ground-protected objects ($0.5 \leq V_{jk} \leq 0.9$) and a weight vector representing the important of the protected object ($0 \leq \omega_k \leq 1$). To calculate the required number of missiles of weapon unit W_i to destroy target T_j with the probability ($P_{unit\,ij} \geq 0.75$), we create a matrix of 1 missile kill probability ($0.5 \leq P_{1\,i} \leq 1.0$), and the required number n of fire units is calculated according to the formula (4). From the required number of fire units we calculate the average cost of using weapon W_i to destroy target T_j.

Experiment 1: In the first test, we simulate an air defense scenario and apply the proposed method to solve the scenario target distribution problem to analyze and evaluate the accuracy of the proposed method. In this scenario, we create 6 aerial target, 6 surface weapon to protect ground objects. The size of the target distribution problem is 6 × 6. From the randomly generated input parameters with independent distribution in the range of values as shown above, we calculate the lost matrix of using weapon W_i to target T_j as follows:

$$\overline{\mathbf{C}} = \begin{bmatrix} 0.18115 & 0.180771 & 0.476646 & 0.324114 & 0.0691015 & 0.123247 \\ 0.321566 & 0.399824 & 0.232902 & 0.673997 & 0.451779 & 0.505241 \\ 0.480866 & 1.023720 & 0.714871 & 1.268250 & 0.861215 & 0.921880 \\ 0.233978 & 0.597722 & 0.314498 & 0.467106 & 0.198332 & 0.260932 \\ 0.234936 & 0.677352 & 0.320142 & 0.435908 & 0.292508 & 0.429728 \\ 0.298668 & 0.484624 & 0.757474 & 0.761298 & 0.504243 & 0.254476 \end{bmatrix}$$

Applying Kuhn-Munkres algorithm to convert lost matrix, to find the minimum value, the matrix after transformation is as follows:

$$\overline{\mathbf{C}} = \begin{bmatrix} 0.112049 & 0 & 0.407545 & 0.054040 & 0 & 0.054145 \\ 0.088665 & 0.055253 & 0 & 0.240124 & 0.218878 & 0.272340 \\ 0 & 0.431184 & 0.234005 & 0.586410 & 0.380349 & 0.441014 \\ 0.035645 & 0.287720 & 0.116166 & 0.067802 & 0 & 0.062599 \\ 0 & 0.330746 & 0.085206 & 0 & 0.057572 & 0.194792 \\ 0.044191 & 0.118478 & 0.502998 & 0.305850 & 0.249766 & 0 \end{bmatrix}$$

From the above matrix, we find the WTA solution as follows:

```
--------------------------------------------------
Candidate WTA solution and its F_lost-value
(1, 2) -> 0.18077090531346085
(2, 3) -> 0.232901549508034
(3, 1) -> 0.4808662061728568
(4, 5) -> 0.1983324667735118
(5, 4) -> 0.43590779964434334
(6, 6) -> 0.25447647156617825
--------------------------------------------------
Total  lowest lost: 1.7832553989783853
```

The solution can be interpreted as: First weapon is assigned to 2^{nd} target with cost value 0.180771, second weapon is assigned to 3^{rd} target with cost value 0.232902, third weapon is assigned to 1^{st} target and so on.

Experiment 2: Evaluate the calculation time complexity of the proposed method. We created various air defense scenarios with increasing complexity to calculate execution time.

As shown in Fig. 4, the time complexity of proposed method obeyed the $O(n^3)$ complexity. For WTA scenarios, with the required time for generating assignment problems about 2–3 s, the algorithm can generate solution for up to 90 targets and 90 weapon units shown in Fig. 5.

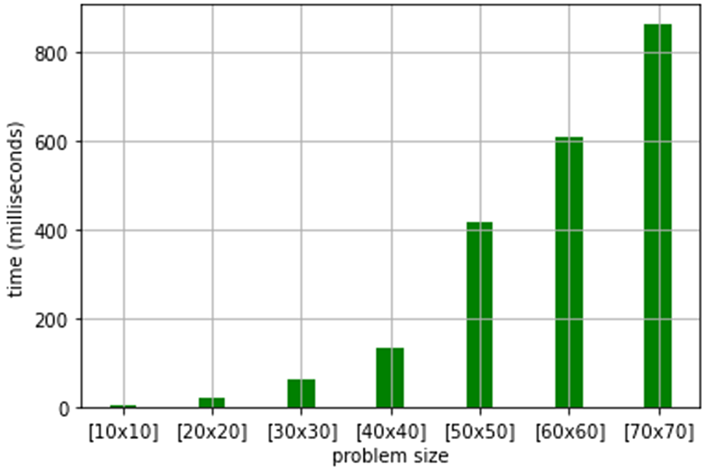

Fig. 4. Computational time for generating a candidate solution and calculating its F_{lost} value.

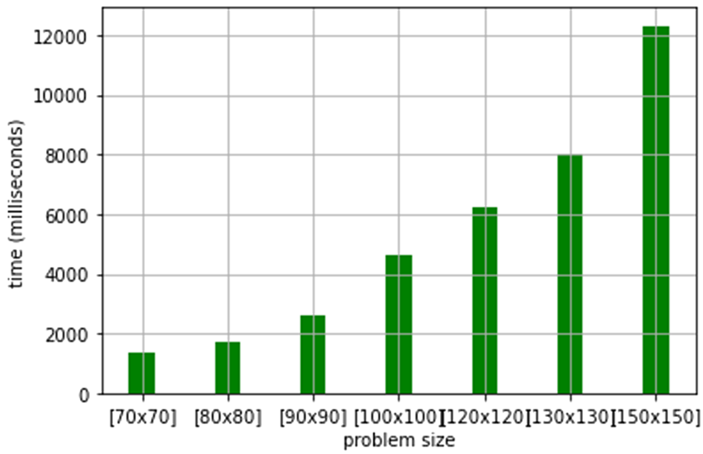

Fig. 5. Computational time for generating a candidate solution and calculating its F_{lost} value with large problem size.

Experiment 3: Experiment with different quantity combination of weapons and targets.

In [11], the real-time condition is assumed the solutions which are obtained in about one second. Hence, to satisfy this condition, the size of a WTA problem which is solved in the static form using exhaustive search algorithm should be smaller than 7×7 (weapons $\times$ targets) (Table 1).

Table 1. Computational time (in milliseconds) for 25 different scenarios.

Weapons	Targets				
	6	12	18	24	32
6	4	5	7	10	11
12	5	10	14	16	21
18	7	15	18	26	37
24	8	17	28	37	53
32	10	21	44	61	98

The comparison of calculation time with the same assignment problem size on the algorithm categories [4–6] (Search algorithms, MMR algorithms, evolutionary algorithms and suggested algorithm) is demonstrated in Fig. 6. Although the implementation of the MMR algorithms is simpler, they produce better performance than search algorithm and evolutionary algorithm. Among the listed method, the proposed method (using Kuhn-Munkres algorithm) is significantly faster.

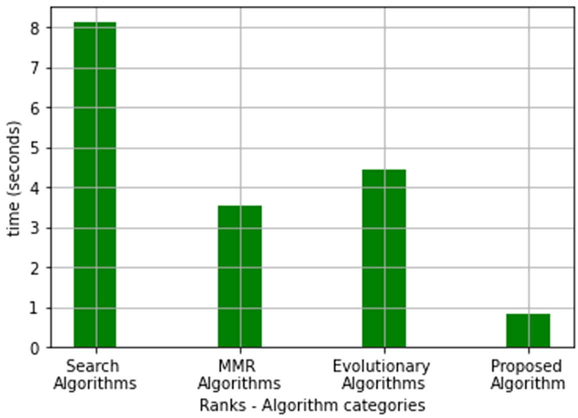

Fig. 6. Ranks – algorithm categories.

Applying our real-time requirement (less than 4 s calculation time in a single core of usual x86 PC) and optimal requirement (guaranteed convergence to global optimum solution), we can sum up the general comparison of WTA algorithms in the aspects of being real-time and optimal in Table 2.

Table 2. A general comparison of weapon assignment algorithms.

Weapon assignment algorithms	Real-time	Optimal
Maximum marginal return algorithm	Yes	No
Genetic algorithm	Yes	No
Ant colony optimization algorithm	Yes	No
Exhaustive search algorithm	No	Yes
Proposed algorithm	Yes	Yes

6 Conclusion

In this paper, we have analyzed the WTA problem in simulated real-combat situation. This is a typical problem for optimizing and planning methods. Finding solutions to this problem in real-time is a high practical research for the military, since these kinds of allocation problems are present in many real-world air defense situations. A mathematical formulation of the real-world problem was proposed to convert WTA problem to a matrix assignment problem. We also proposed an application of a simple and effective assignment algorithm which can guarantee minimization of calculation time for real-time requirement of combat scenarios. In the experiments, the method can generate WTA solution proposal to operator for most air situation (with assignment problem size up to 90×90) in less than seconds. In our future work, the mathematical model and proposed method can be further improved to be more realistic regarding the importance of protected objects and status of weapon units using probability models.

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