Beyond Anchors: Optimal Equality Constraints in Cooperative Localization

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Abstract. Although anchors are the most common in cooperative localizations, they are not the optimal in the class of equality constraints which provide the global reference information for deriving absolute locations. Using Cramér-Rao lower bound (CRLB) to evaluate the localization accuracy, this paper derives the optimal equality constraints that achieve the lowest CRLB trace under given constraint number, and analyzes the feasibility of constructing the optimal constraints before knowing the node ground truth locations. Simulations compare the performance between the anchor-type constraints and the optimal ones, and suggest a cooperative localization algorithm by using the optimal equality constraints.

Keywords: Cramér-Rao Lower Bound (CRLB) · Equality constraints · Maximum Likelihood Estimate (MLE) · Singular Value Decomposition (SVD) · Wireless Sensor Networks (WSN)

1 Introduction

Knowing the locations of a large number of nodes is essential to performing location based service (LBS) in wireless sensor networks (WSN), 5G, and Internet of things (IoT) [1,2]. Compared with the traditional localization approaches such as global positioning system (GPS), cooperative localization introduces the measurements between the unknown nodes to construct a lowcost localization strategy, which can be implemented during internode communications [3,4]. But internode measurements provide only the node location information relative to each other. To get the absolution locations, global constrains, *e.g.*, anchor locations, should be introduced to fix the relative locations onto a coordinate system [5].

Restricted to the anchor-type constraints, where to deploy the anchors is critical for ensuring the localizability and improving localization accuracy, which is

L. Peñalver and L. Parra (Eds.): Industrial IoT 2020, LNICST 365, pp. 47–59, 2021.
https://doi.org/10.1007/978-3-030-71061-3_4

known as anchor selection problem. Based on the exhaustive search of all possible anchor locations, some empirical results are derived to offer some guidance on anchor deployment [6]. Despite the difficulty in finding the optimal anchors, whether introducing anchors is the optimal for constructing global constraints is questionable.

This paper extends the anchor-type constraints to general equality constraints, and derives the optimal ones that can provide the most accurate location estimate. The accuracy of the location estimate is quantified by performing Cramér-Rao lower bound (CRLB) analysis [7] under given internode measurements and global constraints, which offers the lower bound of the variance of any unbiased estimate and is independent of specific localization algorithms. From the CRLB analysis, it can be found that anchor-type constraints are far from the optimal in terms of localization accuracy.

The practicability of constructing optimal equality constraints is also considered in this paper. Compared with the anchor-type constraints which are constructing by locating a small portion of the nodes, constructing the optimal equality constraints needs the ground truth node locations apriori. Actually, it is proved in this paper that no global constraints can be taken as the optimal for all possible node locations. Therefore, it is suggested that the rough prior locations of the nodes can be used to derive suboptimal equality constraints. In our prior work [8], a specifical case under minimally constraint number has been investigated for calibrating rough GPS locations, which is extended to general equality constraints in this paper.

The following of this paper is organized as follows. Section 3 reviews the literature related to the problem. Section 3 introduces the internode measurement model, and provides the CRLB under general equality constraints. Section 4 derives the optimal equality constraints and discuss how to use the optimal equality constraints in practice. Simulations in Sect. 5 compare the anchor-type constraints with the optimal ones, and exhibit the performance of the maximum likelihood estimates of the node locations under the optimal equality constraints.

2 Related Work

In cooperative localization, the measurements between the nodes can be used to aid the location estimate [9], so that the nodes at unknowns locations can be located in a multi-hop manner under the existence of a small portion of anchors/references (*i.e.*, nodes at known absolute locations) [9,10]. This greatly alleviates the burden on the task of node localization as opposed to manual calibrations, or reduces the cost on the devices as opposed to being equipped with global positioning system (GPS) modules.

The above process is conventionally called anchor-based localization, where abundant algorithms are built by fusing the relative location information contained in internode measurements and absolute location information embodied in anchor positions [11–17]. These algorithms provide the absolute location estimates for unknown nodes, but it is somewhat complicated to answer the question that which information dominates the localization accuracy. Conventionally, one may fix the anchors and investigate the influence from the internode measurements, where the Cramér-Rao lower bound (CRLB) analysis serves as a convenient tool [9]. Or on the contrary, one can also fix the internode measurements to investigate how the number and positions of anchors affect the localization accuracy, which is the well-known anchor selection problem [18–21]. Although these approaches help to answer that question in some extent, the relatively high cost in producing/deploying the anchors and the difficulty in finding the optimal anchor deployment pose challenges in the design of efficient and applicable anchor-based localization system.

On the other hand, anchor-free Localization uses only internode measurements, where no anchors are involved [22]. Since the internode measurements involve merely relative location information, only the network's *relative configuration* [23], or called *relative map* [24], can be estimated, while the network's transformation uncertainty, including global translation, rotation, reflection and in some cases scaling, can not be specified yet. To investigate the relative configuration, statistical shape analysis methods are introduced to explore the "shape" of the network [5], where the optimal minimally constraint system is derived to specify the location, orientation and sometimes scaling of the network [8]. However, in existing work, the optimality is restricted to the minimally constraint systems, which usually 3 for 2-dimensional rigid network. For other equality numbers, the optimality has not been explored.

3 Problem Formation

3.1 Internode Measurements

Let us consider n nodes, whose locations are $\mathbf{s}_i = [s_{i,x}, s_{i,y}]^T$, $i = 1, 2, \ldots, n$, distributed on a two-dimensional plane. Each node, say the ith node, emits wireless signal with a preset strength P_i (dBm), and any other node, say the jth node, receives the signal and records the received signal strength (RSS) P_j (dBm). When both the emission strength P_i and the received strength P_j are available, the signal attenuation $r_{i,j} = P_i - P_j$ can be measured, which follows the signal attenuation model

$$r_{i,j} = 10\alpha \log_{10} \|\mathbf{s}_i - \mathbf{s}_j\| + \epsilon_{i,j}, \ (i,j) \in \mathcal{E} \tag{1}$$

where $\alpha > 2$ is a path-loss exponent, $\| \cdot \|$ denotes Euclidean norm, and $\epsilon_{i,j}$, $(i,j) \in \mathcal{E}$, are independent and identically distributed Gaussian noises with mean 0 and variance σ^2. $\mathcal{E}$ is the set of the connected edges, where $(i,j) \in \mathcal{E}$ means the signal emitted from the ith signal can be received by the jth node. Here, the measurements are assumed to be symmetrical, so that any $(i,j) \in \mathcal{E}$ is required to fulfill $i < j$ for simplicity.

3.2 Global Constraints

The internode measurement model (1) specifies only the relative location information between the nodes. To get the absolute locations, global constraints, or named global reference information are needed. In this letter, the global constraints are restricted to equality constants

$$\mathbf{g}(\mathbf{s}) = \mathbf{0} \tag{2}$$

where the location vector $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_n^T]^T$, and the constraint number is k. For example, when anchor-type constraints are chosen, the equality constraints can be represented as

$$\mathbf{g}(\mathbf{s}) = [\mathbf{s}_j - \mathbf{a}_j]_{j \in \mathcal{A}} = \mathbf{0} \tag{3}$$

where $\mathbf{a}_j$, $j \in \mathcal{A}$, are known anchor locations, $\mathcal{A}$ is the index set of the anchors, and the constraints number is twice of the number of the anchors.

The introduction of global constraints contributes both the network localizability and localization accuracy. For a two-dimensional localization problem, when the internode (distance) measurements are sufficient to make the network globally rigid, *e.g.*, fully connected, only 3 equality constraints are required to specify the two-dimensional location and orientation (up to a global reflection) of the network. When the internode measurements are not sufficient, more equality constraints are required. The minimum number of the equality constraints (2) to make the network rigid, named minimally constrained system (MCS) [23], depends on the spatial dimension and network connectivity, where the performance has been explored in [8]. But due to the internode measurements noise, introducing more global constraints might be required to improve the localization accuracy, where the optimal ones are explored in this letter. Notably, we do not use the equality constraints to identify the local/global reflections, where the latter can identified by some inequality constraints or proper initials in iterative localization algorithms.

3.3 CRLB

Constrained CRLB analysis is performed on the measurement model (1) under the constraints (2). In statistics, CRLB serves as a lower bound for the variance of any unbiased estimate, and can be asymptotically achieved by the maximum likelihood estimate (MLE). To avoid the discussion of specific localization algorithms, the performance metric in this paper is set as the CRLB trace, which serves as a benchmark for the performance of most localization algorithms [3,15,25].

Under the measurement model (1), the log-likelihood function of $\mathbf{s}$ is

$$l(\mathbf{s}) = \frac{1}{2\sigma^2} \sum_{(i,j) \in \mathcal{E}} \left(10\alpha \log_{10} \|\mathbf{s}_i - \mathbf{s}_j\| - r_{i,j}\right)^2 + c \tag{4}$$

where the constant c is independent of $\mathbf{s}$. This log-likelihood function leads to the (Fisher information matrix) FIM of $\mathbf{s}$ as

$$\mathbf{J} = \mathrm{E}\left[\frac{\partial l(\mathbf{s})}{\partial \mathbf{s}}\frac{\partial l(\mathbf{s})}{\partial \mathbf{s}^T}\right] = \sigma^{-2}\mathbf{F}^T\mathbf{F} \tag{5}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operation and $\mathbf{F}$ is stacked in row by

$$\left[\mathbf{0}_{1\times 2(i-1)}, \mathbf{a}_{i,j}^T, \mathbf{0}_{1\times 2(j-i-1)}, \mathbf{a}_{j,i}^T, \mathbf{0}_{1\times 2(n-j)}\right] \tag{6}$$

where

$$\mathbf{a}_{i,j} = -\mathbf{a}_{j,i} = \frac{10\alpha}{\log_e 10}\frac{\mathbf{s}_i - \mathbf{s}_j}{\|\mathbf{s}_i - \mathbf{s}_j\|^2}. \tag{7}$$

The FIM $\mathbf{J}$ is rank deficient. It requires the global constraints (2) as a regularization condition to obtain the CRLB. By implementing the constrained CRLB theory [26,27], the CRLB of $\mathbf{s}$ can be represented as

$$\mathbf{C} = \mathbf{U}\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right)^{-1}\mathbf{U}^T \tag{8}$$

where $\mathbf{U}$ is a $2n$-by-$(2n-k)$ matrix whose columns form an orthonormal basis of the null space of $\mathbf{G}^T = \frac{\partial \mathbf{g}(\mathbf{s})}{\partial \mathbf{s}^T}$, and $\mathbf{g}(\mathbf{s})$ is properly designed so that $\mathbf{U}^T\mathbf{J}\mathbf{U}$ is invertible.

Throughout the following of this paper, finding the optimal equality constraints is formulated as constructing $\mathbf{g}(\mathbf{s}) = \mathbf{0}$ to minimizing the CRLB trace $\mathrm{tr}(\mathbf{C})$.

4 Optimal Equality Constraints

The optimal equality constraints refer to the equality constraints $\mathbf{g}(\mathbf{s}) = \mathbf{0}$ that minimizes the trace of the constrained CRLB (8). These equality constraints can be constructed by corresponding global measurements, which produce the most accurate location estimates together with the internode measurements.

4.1 Construction

We first derive the lower bound of the trace of (8), and provide a method to construct the equality constraints to achieve this bound, seen in Proposition 1.

Proposition 1. *The trace of the CRLB (8) is lower bounded by* $\sum_{i=1}^{2n-k}\lambda_i^{-1}$, *where* λ_i *is the* ith *largest eigenvalue of the FIM (5).*

Proof 1. We first represent the CRLB trace $\mathrm{tr}(\mathbf{C})$ *by the eigenvalues of* $\mathbf{U}^T\mathbf{J}\mathbf{U}$ *as*

$$\begin{aligned}\mathrm{tr}(\mathbf{C}) &= \mathrm{tr}\left(\mathbf{U}\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right)^{-1}\mathbf{U}^T\right)\\ &= \mathrm{tr}\left(\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right)^{-1}\right)\\ &= \sum_{i=1}^{2n-k}\lambda_i^{-1}\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right)\end{aligned} \tag{9}$$

where $\lambda_i\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right)$ *denotes the* ith *largest eigenvalue of* $\mathbf{U}^T\mathbf{J}\mathbf{U}$.

Note that $\mathbf{U}^T\mathbf{J}\mathbf{U} = \sigma^{-2}\mathbf{U}^T\mathbf{F}^T\mathbf{F}\mathbf{U}$*, it owns the same non-zero eigenvalues as* $\sigma^{-2}\mathbf{F}\mathbf{U}\mathbf{U}^T\mathbf{F}^T$*. Since*

$$\sigma^{-2}\mathbf{F}\mathbf{U}\mathbf{U}^T\mathbf{F}^T \leq \sigma^{-2}\mathbf{F}\mathbf{F}^T = \mathbf{J} \tag{10}$$

we have

$$\lambda_i\left(\mathbf{U}^T\mathbf{J}\mathbf{U}\right) \leq \lambda_i \tag{11}$$

where λ_i *denotes the* ith *largest eigenvalue of the FIM* $\mathbf{J}$*.*

Substituting (11) into (9), we get a lower bound of the CRLB trace as

$$\mathrm{tr}(\mathbf{C}) \geq \sum_{i=1}^{2n-k} \lambda_i^{-1}. \tag{12}$$

□

The lower bound $\sum_{i=1}^{2n-k} \lambda_i^{-1}$ is achieved when the columns of $\mathbf{U}$ are the eigenvectors corresponding to the $2n-k$ largest eigenvalues of $\mathbf{J}$. In other words, the lower bound $\sum_{i=1}^{2n-k} \lambda_i^{-1}$ is achieved when the columns of $\mathbf{G} = \frac{\partial \mathbf{g}^T(\mathbf{s})}{\partial \mathbf{s}}$ span the subspace related to the k smallest eigenvalues of $\mathbf{J}$. For example, the following constraints

$$\mathbf{G}^T\mathbf{s} - \mathbf{b} = \mathbf{0} \tag{13}$$

are optimal constraints, where the columns are $\mathbf{G}$ the eigenvectors refers to the k smallest eigenvalues of $\mathbf{J}$.

But constructing (13) needs the node locations, which is possible only in simulations or experiments where the ground truth locations of the nodes are known. In practice, no ground truth locations are available, so we should find other approach to construct the equality constraints which are the optimal for all possible node locations. Do these constraints exist?

4.2 Feasibility

Unfortunately, no equality constraints keep the optimal for all possible node locations, as proved in Proposition 2.

Proposition 2. *There is no* $\mathbf{g}(\mathbf{s}) = \mathbf{0}$ *whose CRLB trace achieves the lower bound* $\sum_{i=1}^{2n-k} \lambda_i^{-1}$ *for all* $\mathbf{s} \in \mathbb{R}^{2n}$*.*

Proof 2. It is easy to verify from the proof of Proposition 1 that the trace of the CRLB (8) achieves its lower bound $\sum_{i=1}^{2n-k} \lambda_i^{-1}$ *if and only if the columns of* $\mathbf{U}$ *span the eigenspace corresponding to the* $2n-k$ *largest eigenvalue of* $\mathbf{J}$*, and thus the columns of* $\mathbf{G} = \frac{\partial \mathbf{g}^T(\mathbf{s})}{\partial \mathbf{s}}$ *span the eigenspace corresponding to the* k *smallest eigenvalues of* $\mathbf{J}$*.*

Note that the null space of $\mathbf{J}$ *involves the vectors* $\mathbf{1}_x = [1, 0, \ldots, 1, 0]^T \in \mathbb{R}^{2n}$*,* $\mathbf{1}_y = [0, 1, \ldots, 0, 1]^T \in \mathbb{R}^{2n}$*, and* $\mathbf{v}_\mathbf{s} = [s_{1,y}, -s_{1,x}, \ldots s_{n,y},, -s_{n,x}]^T \in \mathbb{R}^{2n}$*, there should exist a 3-by-k matrix* $\mathbf{T}(\mathbf{s})$ *which makes the equalities*

$$[\mathbf{1}_x, \mathbf{1}_y, \mathbf{v}_\mathbf{s}]^T = \mathbf{T}(\mathbf{s})\frac{\partial \mathbf{g}(\mathbf{s})}{\partial \mathbf{s}^T} \tag{14}$$

hold.

Since $\mathbf{g}(\mathbf{s}) = \mathbf{0}$*, we have*

$$[\mathbf{1}_x, \mathbf{1}_y, \mathbf{v}_\mathbf{s}]^T = \mathbf{T}(\mathbf{s})\frac{\partial \mathbf{g}(\mathbf{s})}{\partial \mathbf{s}^T} = \frac{\partial \mathbf{T}(\mathbf{s})\mathbf{g}(\mathbf{s})}{\partial \mathbf{s}^T}. \tag{15}$$

Therefore, there must exist a function $\tilde{g}(\mathbf{s})$*, which is the last element of* $\mathbf{T}(\mathbf{s})\mathbf{g}(\mathbf{s})$*, satisfying* $\frac{\partial \tilde{g}(\mathbf{s})}{\partial \mathbf{s}} = \mathbf{v}_\mathbf{s}$ *to make the last equality in (15) hold. But such* $\tilde{g}(\mathbf{s})$ *does not exist since* $\frac{\partial \tilde{g}(\mathbf{s})}{\partial s_{1,x}} = s_{1,y}$ *and* $\frac{\partial \tilde{g}(\mathbf{s})}{\partial s_{1,y}} = -s_{1,x}$ *lead to two contradictory expressions of* $\tilde{g}(\mathbf{s})$ *as*

$$\begin{aligned} \tilde{g}(\mathbf{s}) =& s_{1,x}s_{1,y} + c_1 \\ =& - s_{1,x}s_{1,y} + c_2, \end{aligned} \tag{16}$$

where c_1 *and* c_2 *are independent of* $s_{1,x}$ *and* $s_{1,y}$*, respectively. Therefore, there exists no* $\mathbf{g}(\mathbf{s}) = \mathbf{0}$ *whose CRLB trace achieves the lower bound* $\sum_{i=1}^{2n-k} \lambda_i^{-1}$ *uniformly across all* $\mathbf{s} \in \mathbb{R}^{2n}$*.* □

Although constructing the optimal equality constraints is unfeasible in practice, one can still construct suboptimal constraints to fulfil practical requirement. For example, if the nodes' rough locations are available, (13) can be constructed and its performance can be evaluated through biased CRLB analysis similar to Proposition 6 in [5], where the constraint number k can be selected according to the quality and the quantity of the internode measurements. Anchor-type constraints are also admissible, where anchor selection problem can also be viewed as a sparse construction of $\mathbf{G}$ (13).

When the internode measurements are sufficient to make the network rigid, at least $k = 3$ global constraints, named minimally constrained system [5], are required to locate the nodes up to local reflections. In this case, one can maximize the log-likelihood (4) directly, then superimpose the results onto the node ground truth locations through global translations and rotations/reflections. This superimposing operation produces an MLE under the optimal equality constraints. It achieves the lower bound of the CRLB (8) asymptotically as the measurement noise decreases to zero, seen in [28].

5 Simulations

Anchor selection in cooperative localization refers to providing absolute locations of a small portion of the nodes to ensure the localizability and increase the localization accuracy. In previous work, although some empirical strategies are suggested, finding the optimal anchor set actually requires the exhaustive search of all possible anchor sets [6]. Viewed the optimal selection of the anchors as some equality constraints, we use the optimal equality constraints as a benchmark for anchor selection problem, and investigate its relationship with the optimal anchor set.

Table 1. Sensor locations

Sensor no. i	$s_{i,x}$	$s_{i,y}$	Sensor no. i	$s_{i,x}$	$s_{i,y}$
1	6.5574	2.7603	16	8.2346	9.5929
2	0.3571	6.7970	17	6.9483	5.4722
3	8.4913	6.5510	18	3.1710	1.3862
4	9.3399	1.6261	19	9.5022	1.4929
5	6.7874	1.1900	20	0.3445	2.5751
6	7.5774	4.9836	21	4.3874	8.4072
7	7.4313	9.5974	22	3.8156	2.5428
8	3.9223	3.4039	23	7.6552	8.1428
9	6.5548	5.8527	24	7.9520	2.4352
10	1.7119	2.2381	25	1.8687	9.2926
11	7.0605	7.5127	26	4.8976	3.4998
12	0.3183	2.5510	27	4.4559	1.9660
13	2.7692	5.0596	28	6.4631	2.5108
14	0.4617	6.9908	29	7.0936	6.1604
15	0.9713	8.9090	30	7.5469	4.7329

We randomly generate 30 nodes on a 10-by-10 plane, whose locations are given in Table 1. Under the assumption that network is full-connected and the measurement variance is 1, we can derive the FIM (5) and get the CRLB (8) under the anchor-type constraints for a given anchor set. In simulations, the exhaustive search of all possible anchor set is performed under given anchor number respectively.

Figure 1 shows the boxplot of the logarithm of the CRLB traces where the anchor number varies from 3 to 8, with the comparison of the corresponding optimal equality constraints. Similar trends can be found between the optimal anchor set and the optimal equality constraints, which hints the use of the optimal equality constraints as a benchmark for anchor selection. In fact, the CRLB trace under the optimal equality constraints evaluates the quality of the internode measurements, which can be used to determine the refinement of the localization accuracy should be achieve by improving internode measurement accuracy or introducing more global constraints.

We also investigate the constrained MLE of the node locations, which maximizes the likelihood (4) under the optimal equality constraints (13). In our simulations, the nodes deployed at the locations Table 1, and the variance of the measurement error ranges from -30 dB to 30 dB. To construct equality constraints, we introduce three types of reference, *i.e.*, ground truth reference, low error reference obtained by disturbing the ground truth locations using zero mean Gaussian noise with variance 0.01, and high error reference obtained by disturbing the ground truth locations using zero mean Gaussian noise with variance 1. 1000 simulations are performed to get the mean squared error of the estimates.

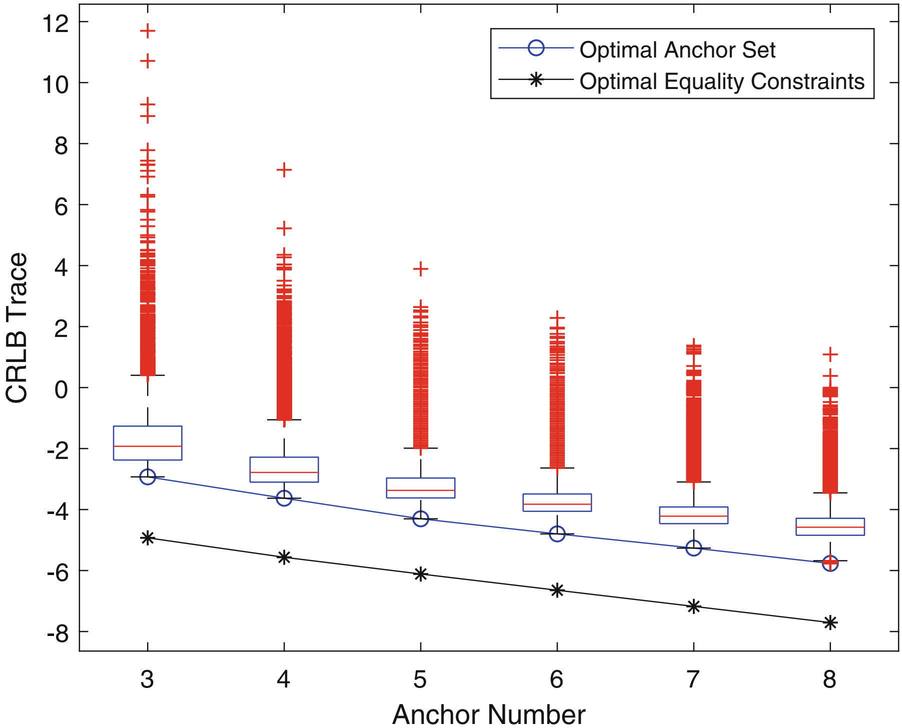

Fig. 1. Boxplot of CRLB traces: Under given anchor number, the CRLB trace of the optimal anchor selection (blue circles: ∘) from all possible anchor sets is compared with the CRLB trace of corresponding optimal equality constraints (black asterisk: $*$). (Color figure online)

Just as the theoretical result that the mean squared error of the MLE approaches the CRLB asymptotically when measurement noise approaches zero, the mean squared error of constrained MLE derived through 1000 simulations approaches the CRLB under the optimal equality constraints constructed by using the node ground truth locations, seen in Fig. 2. In practice, suboptimal equality constraints constructed by using the reference locations, which can be viewed as the rough estimates of the node locations, can be used to derive the constrained MLE. Compared with the mean squared error of the reference locations, which are -2.4772 for the low error reference and 18.3801 for the high error reference, the location estimates under both the case are refined under moderate internode measurement noise. However, unlike using the ground truth reference, using the error reference introduce some bias which cannot be eliminated by improving internode measurement accuracy.

How to choose the suitable number of the equality is also investigated in simulations, seen in Fig. 3. Similar as the simulations in Fig. 2, we fix the variance

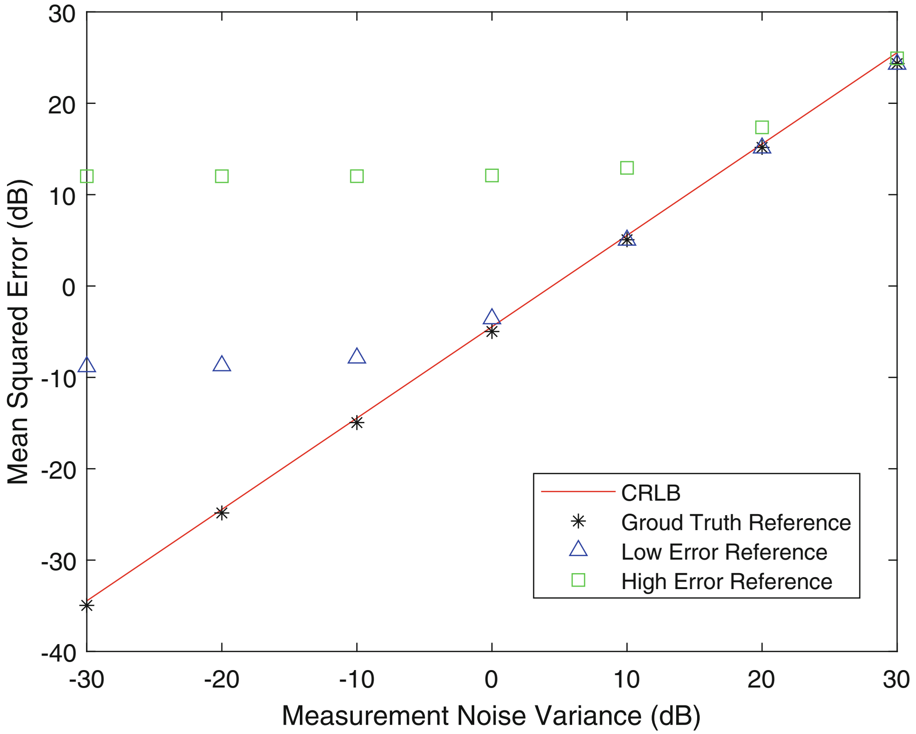

Fig. 2. Mean squared error of constrained MLE: Under given the variance of the measurement noise, the mean squared error of constrained MLE under ground truth reference (black asterisk: $*$)), low error reference (blue triangles: $\triangle$), and high error reference (green square: $\square$) are compared with the CRLB trace (red line: –). (Color figure online)

of the measurement noise at 1, and ranges the constraint number from 3 to 30. Other settings are the same as the ones in Fig. 2.

Using the ground truth reference, it is obvious that introducing more equality constraints can reduce the location estimate error asymptotically to zero. But taken the reference error into consideration, more equality constraints may cause large bias that can not be eliminated by improving the internode measurement accuracy. Therefore, there exists a trade-off between the contribution of equality constraints and the fault caused by reference error, which should be balanced by choosing suitable number of the equality constraints. In Fig. 3, it is preferred to choice only 3 equality constraints in high error reference case and 13 equality constraints low error reference case.

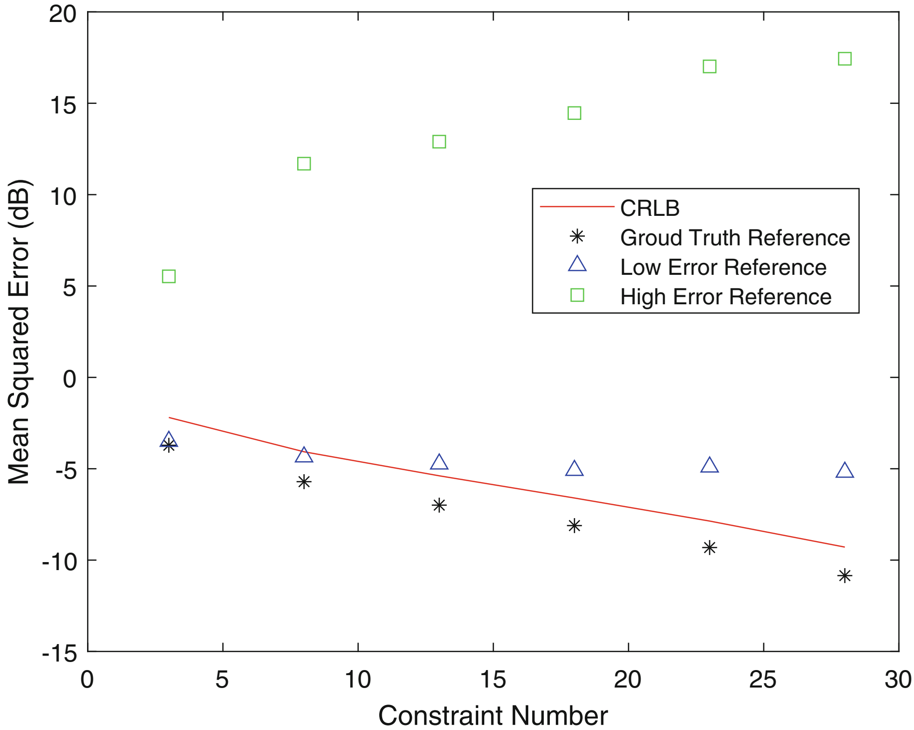

Fig. 3. Mean squared error of constrained MLE: Under given constraint number, the mean squared error of constrained MLE under ground truth reference (black asterisk: $*$)), low error reference (blue triangles: △), and high error reference (green square: □) are compared with the CRLB trace (red line: –). (Color figure online)

6 Conclusions

The optimal equality constraints for the internode RSS measurements are derived by constrained CRLB analysis, which outperform the anchor-type constraints in terms of localization accuracy. Compared with the exhaustive search of the optimal anchor sets, the optimal equality constraints can be directly derived from the eigen subspace of the FIM of the internode measurements, which serve as a bench mark for the anchor selections. In practice, although the optimal equality constraints cannot be derived because of the unknown ground truth locations, the references with locations error can be introduced to construct suboptimal constraints to provide relatively accurately location estimated by exploring the internode measurements. The results are not restricted to the internode RSS measurements. It can be directly derived from other range based measurements such as distances or time-of-arrivals (ToAs), and extended to angle-of-arrivals (AoAs) with some modifications.

7 Discussions and Future Work

Although introducing node rough locations can produce suboptimal constraints which fix the relative locations provided by the internode measurements onto a predefined coordinate system, the performance needs further study. Some directions are given below.

1. How to set the optimal constraint number? Using the ground truth locations, it is obvious that introducing more constraints can improve the localization accuracy. But in practice, since the constraints are constructed by the locations with error, the relative locations provided by the internode measurements may be contaminated by the constraints. Therefore, we hope the suboptimal constraints contribute more on the node locations that cannot provide by the internode measurements, which may be controlled by the constraint number.
2. Whether recursively updating the locations used in constructing suboptimal constraints can improve the localization accuracy?
3. Under the consideration of constraint error, whether the performance of the linear constraints constructed by the eigenvalue decomposition of the FIM is a good choice compared with other type constraints.

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