



Cross-term Suppression in Cyclic Spectrum Estimation Based on EMD

Jurong Hu (✉), Long Lu, and Xujie Li

Hohai University, Nanjing Jiang Su 211100, China
2990693712@qq.com

Abstract. It is inevitable to generate cross term when calculating the cyclic spectrum estimation of complex electromagnetic environment interference signals. Aiming at the problem of cross term in multiple signal cycle spectrum in complex electromagnetic environment, this paper presents a method for cross-term suppression in cyclic spectrum estimation based on empirical mode decomposition (EMD). The effective information of complex electromagnetic environment signals is extracted by compression and reconstruction algorithm, and the effective information is decomposed by empirical mode. Results of simulation and experiment show that the proposed method can effectively suppress the cross term.

Keywords: Radar · Cyclic spectrum · Empirical mode decomposition · Cross term

1 Introduction

With the development of communication technology, the distribution of communication base stations is more and more intensive, and the frequency range of communication signals is continuously expanded, which makes the radar [1] more and more interfered by the same frequency communication signals. In the case of same frequency, the jamming of communication signal to radar belongs to compression jamming.

In the electromagnetic space where the radar works, there are interference signals emitted by adjacent radars or hostile radars. Interference signals of various modulation forms, bandwidths and frequencies are superimposed together to form a dense and complex electromagnetic environment. In the electromagnetic environment of radar, the prior information of various radiation sources is unknown, the transmitting waveform is various, and forms [2, 3] of interference signal are various. The research of high precision spectrum estimation method of electromagnetic environment signal is the core of improving radar anti-jamming ability. The mean value, autocorrelation and other statistics of the signal are generally periodic. If the periodicity changes with time, the signal is called cyclic stationary signal, such as communication, radar, remote sensing and other signals [4, 5, 6]. The second order cyclic spectrum has low estimation complexity, fast calculation speed and strong anti-noise capability, which can transform complex non-linear signals and make them periodic. The second order cyclic spectrum can estimate

and detect the frequency band of interference signal in the electromagnetic environment of MIMO radar better.

In this paper, a spectrum analysis method based on second order cyclic spectrum estimation is studied. It is inevitable to generate cross terms when calculating the cyclic spectrum estimation of complex electromagnetic environment interference signals. Aiming at the problem of cross term in cyclic spectrum estimation, this paper studies the effect of cross-term suppression based on empirical mode decomposition^l(EMD), and analyzes the performance of the algorithm through simulation experiment.

2 Cyclic Spectrum Estimation

Radar received signal can be defined as signals and noise. Noise is a random signal with zero mean value, and they are independent of each other. So the signal can be expressed as

$$r(t) = \sum_{i=1}^m s_i(t) + n(t) \quad (i = 1, 2, \dots, m) \tag{1}$$

where $\sum_{i=1}^p s_i(t)$ represent signals and $n(t)$ represents noise.

For formula (1), the second-order hysteresis product of the signal is calculated and expressed as follows

$$q(t) = r(t)r * (t + \tau) \tag{2}$$

where τ is time offset. * is representation conjugate. The statistical average for $q(t)$ is calculated and expressed as follows

$$R_{11}(t, \tau) = E\{r(t)r * (t + \tau)\} \tag{3}$$

$R_{11}(t, \tau)$ is a function of t and τ , called autocorrelation function.

$$R_{11}(t, \tau) = \sum_{p=-\infty}^{\infty} R_{11}(\tau)e^{j2\pi w t} \tag{4}$$

Fourier expansion of Eq. (3) is

$$R_{11}(\tau) = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} r(t)r * (t+\tau)e^{-j2\pi w t} dt \tag{5}$$

So the cyclic autocorrelation function is expressed as

$$S_{11}(f) = \lim_{T \rightarrow \infty} \lim_{\Delta t \rightarrow \infty} \int_{-\Delta t/2}^{\Delta t/2} \frac{1}{T} R(t, f + w/2)R * (t, f - w/2)dt \tag{6}$$

$R(f)$ is the Fourier transform of $r(t)$; w is the cycle frequency, and $w = p/T_0$. T_0 is the period.

3 Cross Term Suppression based on EMD

3.1 Compression and Reconstruction

In order to express a signal more concisely, the signal is usually transformed into a new base or frame. When the number of non-zero coefficients is far less than the number of terms of the original signal, these small number of non-zero coefficients can be called the sparse expression [7, 8] of the original signal.

Collect the signal $r(t)$ at point N . Define \mathbf{r} as the projection coefficient vector of x on the orthogonal basis y . We can use orthogonal basis vectors to represent complex electromagnetic environment signals in space.

$$\mathbf{r} = \sum_{ns=1}^N \Psi_{ns} \mathbf{a}_{ns} = \Psi \mathbf{a} \quad (7)$$

Ψ is orthogonal basis matrix, and the dimension of Ψ is $N \times N$. The number of non-zero elements in projection coefficient vector \mathbf{a} is defined as:

$$K = \|\mathbf{a}\|_0 \quad (8)$$

$$\mathbf{y} = \Phi \mathbf{r} = \Phi \Psi \mathbf{a} = \Theta \mathbf{a} \quad (9)$$

Φ is a $M \times N$ dimensional observation matrix. $\Theta = \Phi \Psi$ is called the recovery matrix.

It is difficult to directly solve the projection coefficient vector \mathbf{a} , so it is transformed into \mathbf{a}' , the approximate value of matrix \mathbf{a} , that is

$$\hat{\mathbf{a}} = \arg \min \|\mathbf{a}'\|_1 \quad s.t. \quad \Theta \mathbf{a}' = \mathbf{y} \quad (10)$$

Therefore, the reconstructed signal is

$$\mathbf{r}' = \Psi \mathbf{a}' \quad (11)$$

3.2 Empirical Mode Decomposition

The purpose of EMD algorithm is to decompose the signal with poor performance into a set of Intrinsic Mode Functions (IMF) with good performance. The specific steps of EMD algorithm are as follows:

Step1: The local maximum and minimum points of signal $r(t)$ are found out, and the upper envelope $H(t)$ and the lower envelope $V(t)$ are obtained by cubic spline interpolation.

Step2: Calculate the envelope mean as

$$y_1(t) = \frac{H(t) + V(t)}{2} \quad (12)$$

Step3: The components are obtained by subtracting the envelope mean from the radar received signal.

$$e_1(t) = r(t) - y_1(t) \quad (13)$$

Step4: Whether $e_1(t)$ satisfies the condition of IMF. If not, return to step1 and replace $e_1(t)$ with $r(t)$ for the second screening.

$$e_2(t) = e_1(t) - y_2(t) \quad (14)$$

Repeat k times, until the condition is satisfied, and get the kth component.

$$e_k(t) = e_{k-1}(t) - y_k(t) \quad (15)$$

The first order IMF is

$$b_1(t) = e_k(t) \quad (16)$$

Step5: The remainder of the first order IMF is obtained

$$r_1(t) = r(t) - b_1(t) \quad (17)$$

Step6: Repeat step1 to Step5 with $r_1(t)$ as the new signal. When the residual amount of the nth order IMF is constant or cannot be further decomposed, the EMD algorithm is completed.

$$r_n(t) = r_{n-1}(t) - b_n(t) \quad (18)$$

As is shown in Fig. 1, the improved algorithm is the combination of CS and EMD. We receive electromagnetic environmental signal and get the sparse representation of signal. The signal is compressed and measured, then we reconstruct the signal and conduct EMD, a series of IMF function components are obtained. The cyclic spectrum of each IMF component was calculated separately. The cyclic spectrum of each component is accumulated linearly, so we can get the cyclic spectrum of the original electromagnetic signal.

4 Simulation

In this section, representatives of signal FM and LFM will be selected to simulate the complex electromagnetic environment that radar works in. The carrier frequency of FM signal is 1600 MHz. The amplitude of the carrier signal is 1.0. The initial phase of the carrier signal is 0. The frequency of the baseband signal is 1 MHz. The frequency modulation sensitivity is 14. The amplitude of LFM signal is 0.5. The initial frequency is 0. Signal bandwidth B is 150 MHz. FM slope is 2×10^{12} . It is assumed that the noise in the complex electromagnetic environment is gaussian white noise, and the SNR of radar in complex electromagnetic environment is 20 dB. At this time, the signal model of radar operating complex electromagnetic environment can be expressed as follow.

$$r(t) = s_{FM}(t) + s_{LFM}(t) + n(t) \quad (19)$$

Figure 2 shows cyclic spectrum estimation results of FM and LFM. f is frequency. alpha is cycle frequency. s is cyclic spectrum. Figure 3 shows cyclic spectrum estimation results of FM and LFM based on EMD.

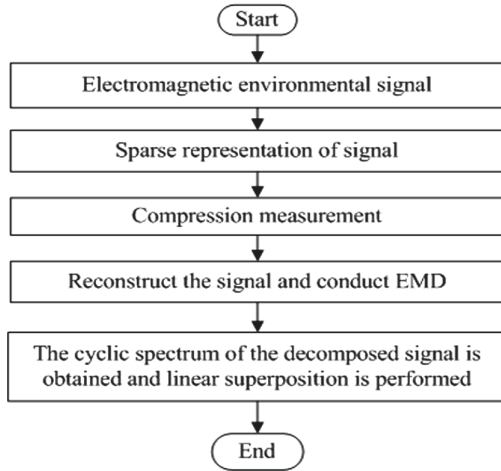


Fig. 1. Schematic diagram improved algorithm

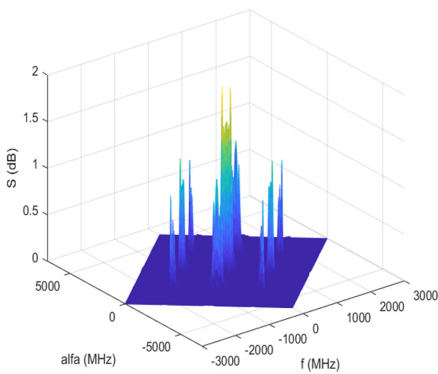


Fig. 2. Cyclic spectrum estimation

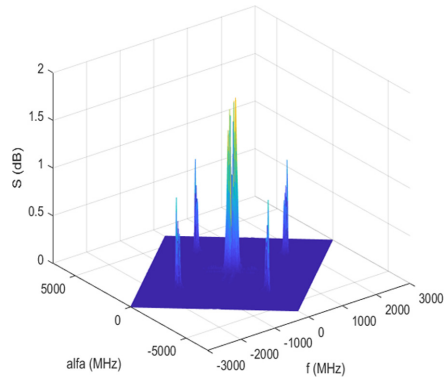


Fig. 3. Cyclic spectrum estimation of FM of FM and LFM and LFM based on EMD

In order to analyze the cyclic spectrum of FM and LFM more clearly, the schematic diagram of cyclic spectrum estimation contour of FM signal and LFM signal in Fig. 2 and the schematic diagram of cyclic spectrum estimation contour based on EMD in Fig. 3 were obtained as follows.

Figure 4 shows that the frequency band of jamming signal FM is $-1680\text{ MHz} - 1515\text{ MHz}$, $-75\text{ MHz} - 75\text{ MHz}$, $1525\text{ MHz} - 1680\text{ MHz}$. The frequency band of jamming signal LFM is $-135\text{ MHz} - 130\text{ MHz}$. The frequency band where the cross terms are generated by M and LFM is $-905\text{ MHz} - 685\text{ MHz}$, $680\text{ MHz} - 915\text{ MHz}$.

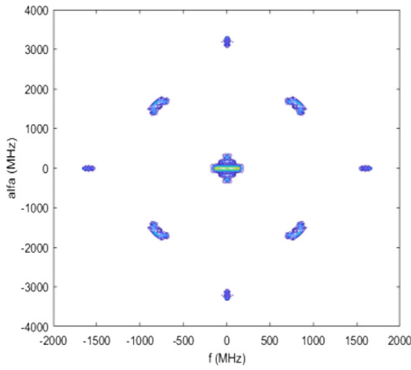


Fig. 4. Cyclic spectrum estimation

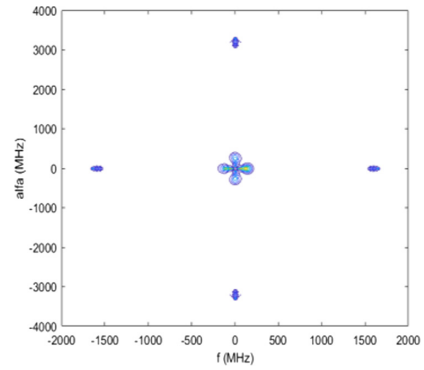


Fig. 5. Cyclic spectrum estimation contour map contour map based on EMD

Figure 5 shows that the frequency band of jamming signal FM is $-1670 \text{ MHz} \sim -1535 \text{ MHz}$, $-75 \text{ MHz} \sim 75 \text{ MHz}$, $1535 \text{ MHz} \sim 1685 \text{ MHz}$. The frequency band of jamming signal FM is $-135 \text{ MHz} \sim 130 \text{ MHz}$.

Compare Fig. 5 with Fig. 4, the cross term in Fig. 5 was completely suppressed. As the complexity of signal increases, the complexity of cyclic spectrum estimation of signal will increase, and the cross term will also become complex. The improved cyclic spectrum estimation algorithm not only improves the readability of cyclic spectrum estimation of complex electromagnetic environment interference signals, but also effectively eliminates the cross terms.

5 Conclusion

This paper proposes cross term suppression method for cyclic spectrum estimation based on EMD. The cyclic spectrum of common jamming signals such as FM signal and LFM signal is studied emphatically. As the complexity of signals increase, the complexity of cyclic spectrum estimation of signals will increase, and the cross terms will also become more complex. Simulation results show that the proposed method can effectively suppress the cross term of multiple signals in a complex environment and improve the resolution of the self-term.

References

1. Li, J., Stoica, P.: MIMO Radar Signal Processing [M] (2009)
2. White, L.B., et al.: Signal design for MIMO diversity systems. In Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004, Vol. 1, pp. 973-977. IEEE (2004)
3. Fishler, E., et al.: Spatial diversity in radars—models and detection performance. *IEEE Trans. Signal Process.* **54**(3), 823–838 (2006)
4. Thameri, M., Abed-Meraim, K., Foroozan, F., Boyer, R., Asif, A.: On the statistical resolution limit (SRL) for time-reversal based MIMO radar. *Signal Process.* **144**, 373–383 (2018). <https://doi.org/10.1016/j.sigpro.2017.10.029>

5. Lehmann, N.H., et al.: High resolution capabilities of MIMO radar. In: Fortieth Asilomar Conference on Signals, Systems and Computers, pp. 25-30. IEEE (2006)
6. Zhang, J., Zhang, R.R., Yang, R.W., et al.: Signal subspace reconstruction method of MIMO radar. *Electron. Lett.* **46**(7), 531–533 (2010)
7. Karthikeyan, C.S., Suganthi, M.: Optimized spectrum sensing algorithm for cognitive radio. *Wireless Pers. Commun.* **94**(4), 2533–2547 (2017)
8. Gan, H., et al.: Circulant and toeplitz chaotic matrices in compressed sensing. *J. Comput. Inf. Syst.* **11**(4), 1231–1238 (2015)
9. Candes, E.J., Romberg, J., Tao, T.: Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. Inf. Theory* **52**(2), 489–509 (2006). <https://doi.org/10.1109/TIT.2005.862083>