2D DOA Estimation Based on Modified Compressed Sensing Algorithm

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Abstract. In order to realize high-precision DOA tracking in space, researches on two-dimensional DOA estimation have been conducted in recent years. The existing algorithms often need large snapshots for estimation accuracy, going against the fast solution. Considering the low sensitivity of DOA estimation algorithm based on compressed sensing theory to the number of snapshots and the correct estimation with less sampling data, a modified two-dimensional multitask compressed sensing algorithm based on SVD decomposition is proposed in this paper. This algorithm makes up for the drawbacks of existing compressed sensing algorithms in dealing with multi snapshot problem and reduces the unnecessary calculation. Simulation results show that the proposed algorithm can solve the off-grid problem in compressed sensing, and has better estimation performance than other algorithms under the condition of low SNR and few snapshots.

Keywords: 2D DOA estimation · Compressed sensing · SVD

1 Introduction

First proposed by Malioutov et al. in 2005, the concept of CS-DOA is called l1-SVD [1], with the core idea of transforming the DOA estimation problem into a sparse reconstruction problem for solving an underdetermined system of equations. It has become a classic algorithm in the field of CS-DOA. The array signal models of the following algorithms are roughly the same, and the differences mainly focus on the selection of optimal reconstruction algorithms. To reduce the calculation amount in OMP algorithm used in sparse signal reconstruction, Wang Shuhao et al. proposed DOA Estimation of LFM Signals Based on Compressed Sensing [2], which makes use of the bat algorithm's population search mode and excellent echo localization ability in flight to achieve fast optimization. To solve the problem of poor accuracy of DOA estimation based on compressed sensing when the array antenna has amplitude and phase errors, Zuo Luo et al. proposed a super-resolution DOA estimation method based on TLS-CS [3], combined with singular value decomposition (SVD) and greedy iterative pursuit algorithm for CS sparse reconstruction to obtain the azimuth information of the target. In order to solve the problem of low efficiency when orthogonal matching pursuit algorithm is used for sparse recovery of high dimensional signals, Zhao

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Hongwei et al. proposed a DOA estimation algorithm combined with particle swarm optimization [4]. The algorithm uses PSO algorithm to solve the optimization problem, and improves the particle renewal mechanism and inertia weight.

At present, among the problems in DOA estimation algorithm based on compressed sensing theory, the main one is off-grid. No matter how fine the grid is divided, the target signal may be located between the grids, resulting in mismatch between the sparse base of the artificially constructed redundant dictionary and the real base of the target, so the off-grid problem is also known as the base mismatch problem. In the field of DOA, many researches on off-grid problem have been carried out, such as the alternating iteration method [5], and the sparse Bayesian method [6] used to solve the off-grid problem in DOA. Some variant algorithms on this basis also appear, such as the introduction of second-order Taylor polynomial, and the idea of noise subspace [7]. However, the limitation of these algorithms is that most of the signal models are for one-dimensional DOA estimation, which cannot be effectively extended to two-dimensional cases. And the two-dimensional off-grid problem is more universal. In order to improve the accuracy of DOA estimation, this paper will focus on the realization of two-dimensional DOA estimation based on compressed sensing and the off-grid problem.

2 Problem Formulation

In the case of one-dimensional space, there are Q incident signals with azimuth angles $\{\theta_1, \cdots \theta_Q\}$. Meanwhile, we divide the space $[0^\circ, 90^\circ]$ into N grids as $[\alpha_1, \alpha_2, \cdots, \alpha_N]$ with $Q << N$. When there is an incident signal at a certain grid, the value at the corresponding position of sparse representation signal is not zero; otherwise, it is zero.

The relationship between the division of spatial discrete grids and the angles of incident signals is shown in Fig. 1, where "•"is the real spatial incident signals, and "○"the potential ones.

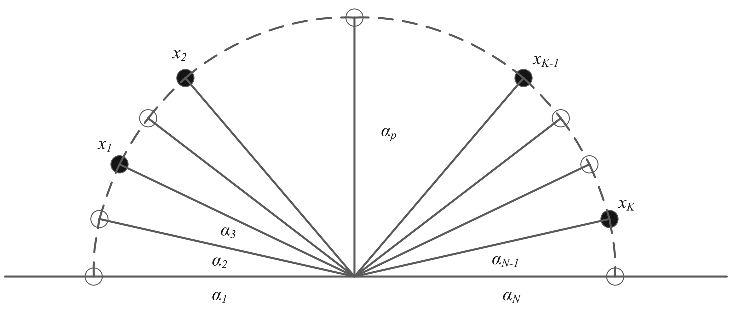


Fig. 1. Spatial discrete grids and the angles of incident signals

In practice, the direction of arrival of the signal is continuous in the spatial domain, but we discretized space angles in the process of DOA estimation. The incoming direction of the actual source may fall between the adjacent grid points in spite of dense grid sampling, as shown in Fig. 2. The off-grid problem hence appears, leading to errors in DOA estimation. Although the dense division of spatial grids can alleviate this problem, it will increase the dimension of redundant dictionary (array manifold) and the amount of calculation and slow down the solution, which restricts the application of DOA estimation algorithm based on compressed sensing theory in practice.

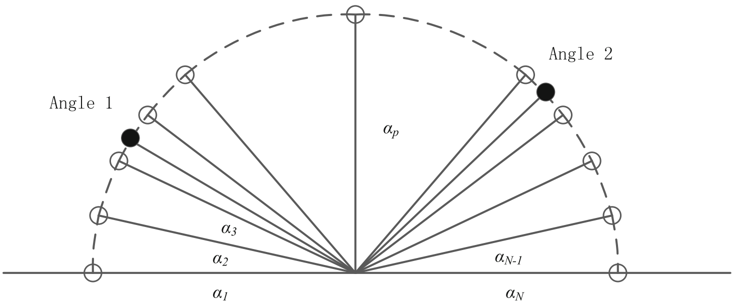


Fig. 2. Off-grid problem

Due to the coupling of pitch angles and azimuth angles in conventional array manifold, the steering matrix $\boldsymbol{A}$ cannot be decomposed. Hence, new array model is introduced, as shown in Fig. 3.

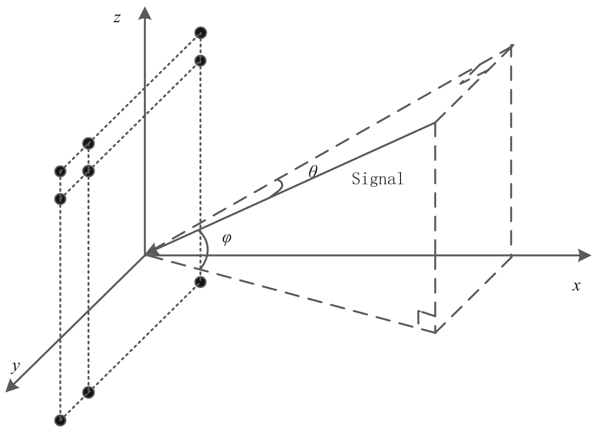


Fig. 3. Array model

The uniform square array is set as the object, with dimension as $M \times N$ in zoy plane. The number of spatial domain grids is shown as $P \times Q$, and the signal time delay $\tau_{(m,n),(p,q)}$ of the (p,q)-th grid received by the (m,n)-th array element as

$$\tau_{(m,n),(p,q)} = \frac{(n-1)d\sin\theta_{p,q} + (m-1)d\sin\varphi_{p,q}}{c} \tag{1}$$

Where p is the pitch dimension of the grid, and q is the azimuth dimension. For a clearer derivation, the azimuth angle $\theta_{p,q}$ is θ_q, and the pitch angle $\varphi_{p,q}$ is φ_p. The formula (1) can be expressed as

$$\tau_{(m,n),(p,q)} = \tau_m\left(\varphi_p\right) + \tau_n\left(\theta_q\right) \tag{2}$$

Where

$$\begin{aligned} \tau_m\left(\varphi_p\right) &= \frac{(m-1)d\sin\varphi_p}{c} \\ \tau_n\left(\theta_q\right) &= \frac{(n-1)d\sin\theta_q}{c} \end{aligned} \tag{3}$$

And the steering matrix $\boldsymbol{A}$ can be expressed as

$$\begin{aligned} A = &\begin{bmatrix} e^{-j2\pi f_0\tau_1(\varphi_1)} & e^{-j2\pi f_0\tau_1(\varphi_2)} & \cdots & e^{-j2\pi f_0\tau_1(\varphi_P)} \\ e^{-j2\pi f_0\tau_2(\varphi_1)} & e^{-j2\pi f_0\tau_2(\varphi_2)} & \cdots & e^{-j2\pi f_0\tau_2(\varphi_P)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi f_0\tau_M(\varphi_1)} & e^{-j2\pi f_0\tau_M(\varphi_2)} & \cdots & e^{-j2\pi f_0\tau_M(\varphi_P)} \end{bmatrix} \\ &\otimes \begin{bmatrix} e^{-j2\pi f_0\tau_1(\theta_1)} & e^{-j2\pi f_0\tau_1(\theta_2)} & \cdots & e^{-j2\pi f_0\tau_1(\theta_Q)} \\ e^{-j2\pi f_0\tau_2(\theta_1)} & e^{-j2\pi f_0\tau_2(\theta_2)} & \cdots & e^{-j2\pi f_0\tau_2(\theta_Q)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi f_0\tau_N(\theta_1)} & e^{-j2\pi f_0\tau_N(\theta_2)} & \cdots & e^{-j2\pi f_0\tau_N(\theta_Q)} \end{bmatrix} \end{aligned} \tag{4}$$

i.e.

$$\boldsymbol{A} = \boldsymbol{\Psi} \otimes \boldsymbol{\Theta}^T \tag{5}$$

Restore vector $\boldsymbol{s}$ to matrix as $\boldsymbol{S}$, a $P \times Q$ dimensional matrix; the matrix form of vector $\boldsymbol{n}$ is $\boldsymbol{N}$, an $M \times N$ dimensional matrix. Therefore,

$$\boldsymbol{y} = \left(\boldsymbol{\Psi} \otimes \boldsymbol{\Theta}^T\right)\boldsymbol{s} + \boldsymbol{n} = vec(\boldsymbol{\Psi S \Theta} + \boldsymbol{N}) \tag{6}$$

Where *vec* shows that the matrix is arranged into column vector in row priority order. Then,

$$Y = \boldsymbol{\Psi} S \boldsymbol{\Theta} + N \tag{7}$$

Where $\boldsymbol{Y}$ is $M \times N$ dimensional, and the matrix form of vector $\boldsymbol{y}$.

3 2D Multitasking CS Algorithm Based on SVD

3.1 2D Off-Grid Algorithm Based on Taylor Expansion

The first order approximation of Taylor expansion can be used to solve the basis mismatch problem in two-dimensional DOA estimation by applying the above signal model with separable steering matrix.

Discrete the spatial domain; suppose the angle of a target signal is $\left(\hat{\theta}, \hat{\varphi}\right)$, and the angle of its nearest grid is $\left(\theta_p, \varphi_q\right)$. δ_{θ_p} and δ_{φ_q} represent the estimation bias of the pitch dimension and azimuth dimension. Here, $\delta_\theta = [\delta_{\theta_1}, \delta_{\theta_2}, \cdots, \delta_{\theta_P}]^T \delta_\varphi = \left[\delta_{\varphi_1}, \delta_{\varphi_2}, \cdots, \delta_{\varphi_Q}\right]^T$.

Discrete steering vector as follows:

$$a\left(\hat{\varphi}, \hat{\theta}\right) = \psi(\hat{\varphi}) \otimes \phi\left(\hat{\theta}\right) \tag{8}$$

Where $\otimes$ is Kronecker product.

$$a\left(\hat{\varphi}, \hat{\theta}\right) = \left[e^{-j2\pi f_0 \tau_{(1,1)}\left(\hat{\varphi},\hat{\theta}\right)} \quad e^{-j2\pi f_0 \tau_{(1,2)}\left(\hat{\varphi},\hat{\theta}\right)} \quad \cdots \quad e^{-j2\pi f_0 \tau_{(m,n)}\left(\hat{\varphi},\hat{\theta}\right)} \quad \cdots \quad e^{-j2\pi f_0 \tau_{(M,N)}\left(\hat{\varphi},\hat{\theta}\right)} \right]^T \tag{9}$$

$$\tau_{(m,n)}\left(\hat{\varphi}, \hat{\theta}\right) = \frac{(n-1)d\sin\hat{\theta} + (m-1)d\sin\hat{\varphi}}{c} \tag{10}$$

$\psi(\hat{\varphi})$ is the steering vector of pitch dimension, expressed as follows.

$$\psi(\hat{\varphi}) = \left[e^{-j2\pi f_0 \tau_1(\hat{\varphi})} \quad e^{-j2\pi f_0 \tau_2(\hat{\varphi})} \quad \cdots \quad e^{-j2\pi f_0 \tau_m(\hat{\varphi})} \quad \cdots \quad e^{-j2\pi f_0 \tau_M(\hat{\varphi})} \right]^T \tag{11}$$

$\phi\left(\hat{\theta}\right)$ is the steering vector of azimuth dimension, expressed as follows.

$$\phi\left(\hat{\theta}\right) = \left[e^{-j2\pi f_0 \tau_1\left(\hat{\theta}\right)} \quad e^{-j2\pi f_0 \tau_2\left(\hat{\theta}\right)} \quad \cdots \quad e^{-j2\pi f_0 \tau_n\left(\hat{\theta}\right)} \quad \cdots \quad e^{-j2\pi f_0 \tau_N\left(\hat{\theta}\right)} \right]^T \tag{12}$$

Express the steering vectors of pitch and azimuth dimensions of the target signal separately with first order approximation of Taylor expansion as follows.

$$a\left(\hat{\varphi}, \hat{\theta}\right) = \left(\psi(\varphi_p) + b(\varphi_p)(\hat{\varphi} - \varphi_p)\right) \otimes \left(\phi(\theta_q) + c(\theta_q)\left(\hat{\theta} - \theta_q\right)\right) \tag{13}$$

Where $b(\varphi_p)$ and $c(\theta_q)$ are first derivative vectors of $\psi(\varphi_p)$ and $\phi(\theta_q)$.

Put the above into matrix form, and the signal model of 2D DOA estimation can be expressed as follows.

$$\boldsymbol{Y} = \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi}\right)\boldsymbol{S}(\boldsymbol{\Theta} + \boldsymbol{\Delta}\delta_{\theta}\boldsymbol{C}) + \boldsymbol{N} \tag{14}$$

Where $\boldsymbol{\Delta}\delta_{\varphi} = diag\left(\delta_{\varphi}\right)$ and $\boldsymbol{\Delta}\delta_{\theta} = diag(\delta_{\theta})$, $\boldsymbol{B}$ is the matrix obtained by deriving each element in the steering vector of pitch dimension $\boldsymbol{\Psi}$, so does $\boldsymbol{C}$, and $\boldsymbol{N}$ is the noise.

In (14), there are three unknown variables $\boldsymbol{\Delta}\delta_{\varphi}$, $\boldsymbol{\Delta}\delta_{\theta}$ and $\boldsymbol{S}$. Solve the matrix $\boldsymbol{S}$, then solve $\boldsymbol{\Delta}\delta_{\varphi}$ and $\boldsymbol{\Delta}\delta_{\theta}$ with the method of alternate iteration as follows.

In the solution of signal matrix $\boldsymbol{S}$, initialize $\boldsymbol{\Delta}\delta_{\varphi}$ and $\boldsymbol{\Delta}\delta_{\theta}$ as zero matrixes, and (14) can be expressed as follows.

$$\boldsymbol{Y} = \boldsymbol{\Psi}\boldsymbol{S}\boldsymbol{\Theta} + \boldsymbol{N} \tag{15}$$

The original problem degenerates into the basic problem of separable DOA estimation, which can be regarded as a rough solution of $\boldsymbol{S}$. Due to the off-grid problem, the solution obtained must be deviated from the true value.

Then solve $\boldsymbol{\Delta}\delta_{\varphi}$ and $\boldsymbol{\Delta}\delta_{\theta}$.

In the solution of $\boldsymbol{\Delta}\delta_{\varphi}$, initialize $\boldsymbol{\Delta}\delta_{\theta}$ as unit matrix, regard $\boldsymbol{S}(\boldsymbol{\Theta} + \boldsymbol{\Delta}\delta_{\theta}\boldsymbol{C})$ in (14) as a fixed value, let $\boldsymbol{H} = \boldsymbol{S}(\boldsymbol{\Theta} + \boldsymbol{\Delta}\delta_{\theta}\boldsymbol{C})$, then

$$\boldsymbol{Y} = \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi}\right)\boldsymbol{H} + \boldsymbol{N} \tag{16}$$

The minimum deviation is required between $\left(\theta_p, \varphi_q\right)$, the angle of signal matrix, and the true angle of the target signal, which means δ_{φ} and δ_{θ} are restrained to the minimum. Constrain their sparsity with 2-norm, and the optimization problem can be obtained.

$$\min\left\|\delta_{\varphi}\right\|_2^2 + \left\|\boldsymbol{Y} - \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi}\right)\boldsymbol{H}\right\|_F^2 \tag{17}$$

Least-squares solution is applied, and each column in matrix $\boldsymbol{Y}$ in (17) meets the following equation:

$$\boldsymbol{Y}[n] = \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi}\right)\boldsymbol{H}[n] \tag{18}$$

Where $\bullet[n]$ is the n-th column, $n = 1, 2, \cdots, N$. Given the following equation

$$\boldsymbol{\Delta}\delta_{\varphi}\boldsymbol{H}[n] = \boldsymbol{\Delta}\boldsymbol{H}[n]\varphi \tag{19}$$

Where $\boldsymbol{\Delta}\boldsymbol{H}[n] = diag(\boldsymbol{H}[n])$. The optimization problem can be transformed into the problem of obtaining the least square solution.

$$\begin{bmatrix} \boldsymbol{Y}[1] - \boldsymbol{\Psi H}[1] \\ \boldsymbol{Y}[2] - \boldsymbol{\Psi H}[2] \\ \vdots \\ \boldsymbol{Y}[N] - \boldsymbol{\Psi H}[N] \end{bmatrix} = \begin{bmatrix} \boldsymbol{B\Delta H}[1] \\ \boldsymbol{B\Delta H}[2] \\ \vdots \\ \boldsymbol{B\Delta H}[N] \end{bmatrix} \delta_{\varphi} \tag{20}$$

The solution to δ_{φ} can be obtained with the least square method.

$$\delta_{\varphi} = (\boldsymbol{B}_{\Delta H})^{+} \boldsymbol{Y}_{\Psi H} \tag{21}$$

Where $(\bullet)^{+}$ is the generalized inverse of the matrix.

$$\begin{aligned} \boldsymbol{B}_{\Delta H} &= [\boldsymbol{B\Delta H}[1], \boldsymbol{B\Delta H}[2], \cdots, \boldsymbol{B\Delta H}[N]]^{T} \\ \boldsymbol{Y}_{\Psi H} &= [\boldsymbol{Y}[1] - \boldsymbol{\Psi H}[1], \boldsymbol{Y}[2] - \boldsymbol{\Psi H}[2], \cdots, \boldsymbol{Y}[N] - \boldsymbol{\Psi H}[N]]^{T} \end{aligned} \tag{22}$$

Substitute δ_{φ} into (14), and let $\boldsymbol{G} = \left(\boldsymbol{\Psi} + \boldsymbol{B\Delta}\delta_{\varphi}\right)\boldsymbol{S}$, and solve δ_{θ} in the same way. Calculate δ_{φ} and δ_{θ} alternatively, and set the condition to finish iterative process. $\boldsymbol{S}$, $\boldsymbol{\Delta}\delta_{\varphi}$ and $\boldsymbol{\Delta}\delta_{\theta}$ are ultimately solved, then the true angles of the target signal can be expressed as $\hat{\theta}_p = \theta_p + \delta_{\theta_p}$ and $\hat{\varphi}_q = \varphi_q + \delta_{\varphi_q}$.

3.2 OMP Reconstruction Method Based on SVD

The two-dimensional off-grid algorithm based on the first-order Taylor expansion has some limitations. When reconstructing the sparse matrix $\boldsymbol{S}$, the noise of the received data matrix $\boldsymbol{Y}$ is zero by default, which leads to a poor accuracy of the solution to the sparse matrix and a less likely improvement in the following solution. In addition, in the case of a low SNR, more sampling snapshots can achieve certain estimation accuracy, and the algorithm is only suitable for single snapshot, which needs to be extended to multiple snapshots and solve the problem of high computational complexity.

In order to reduce the computational complexity and improve the anti-noise performance, the subspace methods are often applied in array signal processing. In this paper, singular value decomposition (SVD) is used to extract signal subspace to process array received signal matrix.

Without considering the coherence among signal sources, this paper conducts the singular value decomposition to the received data matrix $\boldsymbol{Y}$ with the purpose of removing noise from the received signal.

$$\boldsymbol{Y} = \boldsymbol{ULV}^{H} = [\boldsymbol{U}_S \boldsymbol{U}_N]\boldsymbol{LV}^{H} \tag{23}$$

Where V is an orthogonal matrix, and U is a matrix formed by arranging singular values from large to small. U_S is the signal subspace formed by singular vectors corresponding to the preceding K singular values. U_N is the noise subspace. Suppose the number of signal source K is known, take the preceding K columns of matrix U, and the $N \times K$ dimensional matrix $\boldsymbol{Y}_S = \boldsymbol{ULD}_K = \boldsymbol{YVD}_K$ formed by signal components is obtained, where $\boldsymbol{D}_K = [\boldsymbol{I}_K \quad \boldsymbol{O}]^H$. $\boldsymbol{I}_K$ is the $K \times K$ dimensional unit matrix, O is the $K \times (T - K)$ dimensional zero matrix, and the low-dimensional form of $\boldsymbol{Y}$ can be expressed as follows.

$$\boldsymbol{Y}_S = \boldsymbol{\Psi S}_S \boldsymbol{\Theta} + \boldsymbol{N}_S \tag{24}$$

Where $\boldsymbol{S}_S = \boldsymbol{SVD}_K$, and $\boldsymbol{N}_S = \boldsymbol{NVD}_K$. $\boldsymbol{S}_S$ remains the sparsity unchanged. It is the signal theoretically denoised from $\boldsymbol{S}$, then the solution to $\boldsymbol{S}$ can be simplified as the solution to $\boldsymbol{S}_S$. Compared to the common high sampling frequency under the practical condition, data dimension can decrease from T to K with the application of this algorithm, hence the calculation is significantly reduced. Simultaneously, SVD can be comprehended as a denoising process, which is helpful in DOA estimation under low SNR.

The algorithm steps are as follows:

(1) Conduct SVD to observation matrix $\boldsymbol{Y}$ to get $\boldsymbol{Y}_S$, and keep the right singular vector $\boldsymbol{U}_K$;
(2) Initialize residual matrix and index set, i.e. $\boldsymbol{R} = \boldsymbol{Y}$ and $\boldsymbol{\Lambda}_0 = \emptyset$;
(3) Take the inner product of each column in $\boldsymbol{U}_K$ and $\boldsymbol{\Phi}$, i.e. $\boldsymbol{G}^n = \left|\boldsymbol{\Phi}^H \boldsymbol{U}_{Kn-1}\right|$;
(4) Find out the row index value λ corresponding to the maximum norm of row vector q in $\boldsymbol{G}^n$, and update the index set $\boldsymbol{\Lambda}_n = \boldsymbol{\Lambda}_{n-1} \cup \{\lambda_n\}$ and its column vector set $\boldsymbol{\Psi}_n = \Psi_{n-1} \cup \{\boldsymbol{\Phi}_{\lambda_n}\}$;
(5) Obtain its approximate solution $\boldsymbol{S}_S^n = \left(\boldsymbol{\Psi}_n^H \boldsymbol{\Psi}_n\right)^{-1} \boldsymbol{\Psi}_n^H \boldsymbol{Y}_S$ with the least square method;
(6) Update the residual error $\boldsymbol{R}_n = \boldsymbol{Y}_S - \boldsymbol{\Psi S}_S^n$, where $n = n + 1$;
(7) Judge whether the end condition of the algorithm is reached. If so, the calculation will be terminated. Otherwise, skip to Step (2) and repeat.

3.3 Multi-task Processing

The improved algorithm applies the current separable array signal model, which is only suitable for single snapshot. To solve the problem of DOA estimation under the condition of multiple snapshots, this paper introduces the idea of multitasking Bayesian compressed sensing into the algorithm.

Define snapshot as K, solve the data of the i-th snapshot to obtain δ_φ and δ_θ,

$$\begin{aligned} \boldsymbol{Y}_i &= \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi i}\right)\boldsymbol{H} + \boldsymbol{N}_i \\ \boldsymbol{Y}_i &= \boldsymbol{G}(\boldsymbol{\Theta} + \boldsymbol{\Delta}\delta_{\theta i}\boldsymbol{C}) + \boldsymbol{N}_i \end{aligned} \tag{25}$$

Conduct sparsity constraint to $\boldsymbol{\Delta}\delta_{\varphi i}$ and $\boldsymbol{\Delta}\delta_{\theta i}$ with 2-norm to get the optimized problem:

$$\begin{aligned} \min\left\|\delta_{\varphi i}\right\|_2^2 + \left\|\boldsymbol{Y}_i - \left(\boldsymbol{\Psi} + \boldsymbol{B}\boldsymbol{\Delta}\delta_{\varphi i}\right)\boldsymbol{H}\right\|_F^2 \\ \min\|\delta_{\theta i}\|_2^2 + \|\boldsymbol{Y}_i - \boldsymbol{G}(\boldsymbol{\Theta} + \boldsymbol{\Delta}\delta_{\theta i}\boldsymbol{C})\|_F^2 \end{aligned} \tag{26}$$

Solve it with least-squares solution to get the analytical solution of $\delta_{\varphi i}$

$$\delta_{\varphi i} = (\boldsymbol{B}_{\Delta H})^+ \boldsymbol{Y}_{\Psi H i} \tag{27}$$

And the analytical solution of $\delta_{\theta i}$

$$\delta_{\theta i} = (\boldsymbol{G}_{\Delta C})^+ \boldsymbol{Y}_{G\Theta i} \tag{28}$$

To sum up, the solving steps of 2D multitasking compressed sensing off-grid algorithm based on SVD are summarized as follows:

(1) Set a pitch dimension $\boldsymbol{\Psi}$ and an azimuth dimension $\boldsymbol{\Theta}$, and conduct derivation to get matrixes $\boldsymbol{B}$ and $\boldsymbol{C}$.
(2) Obtain spectral matrix $\boldsymbol{S}$ by applying the OMP reconstruction method to matrixes $\boldsymbol{Y}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Theta}$.
(3) Take the sampling data of the i-th snapshot $i(i = 1, \cdots, K)$, and obtain $\delta_{\theta i}$ and $\delta_{\varphi i}$ according to (26) and (27) till the termination conditions are met.
(4) Work out the incident angles with $\left(\theta_p, \varphi_q\right)$, $\delta_{\theta i}$, and $\delta_{\varphi i}$ in $\boldsymbol{S}$ obtained in Step 2.

4 Performance Study

To verify the effectiveness of the modified algorithm, a supposed simulation model is a 8×8 matrix URA with $d = \lambda/2$. Two independent narrow-band signals are separately incident on the array with azimuth angles 9.9° and 18.4°, and pitch angles 3.3° and 24.2°. Their ranges of spatial discrete grids are both $[0°, 90°]$. The following experiments are conducted based on the above conditions.

Experiment 1

To test the feasibility of signal model with separable array manifold under the condition of single snapshot, the DOA estimation performance of MMV-OMP, BCS and off-grid algorithms under different SNR is simulated, with SNR increasing from −15 dB to 10 dB, the number of snapshots 1, and Monte-carlo simulation 500 times. Results are shown in Fig. 4.

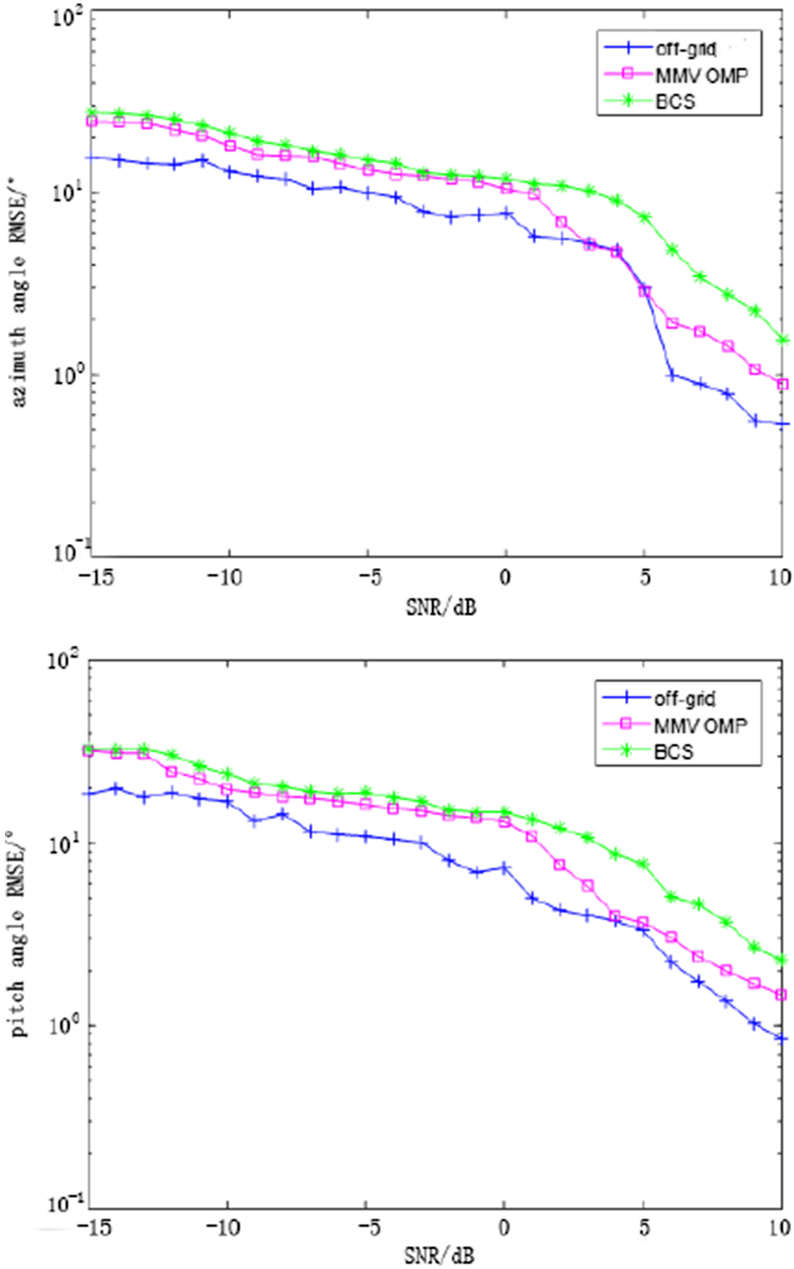
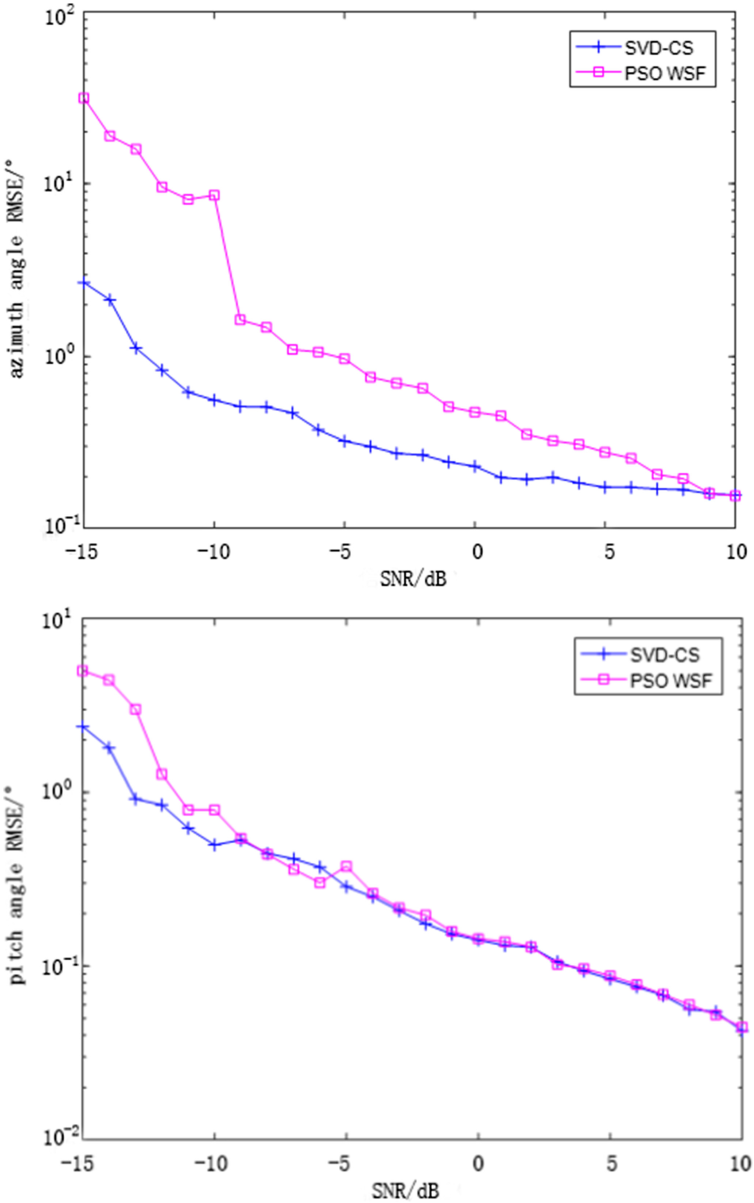
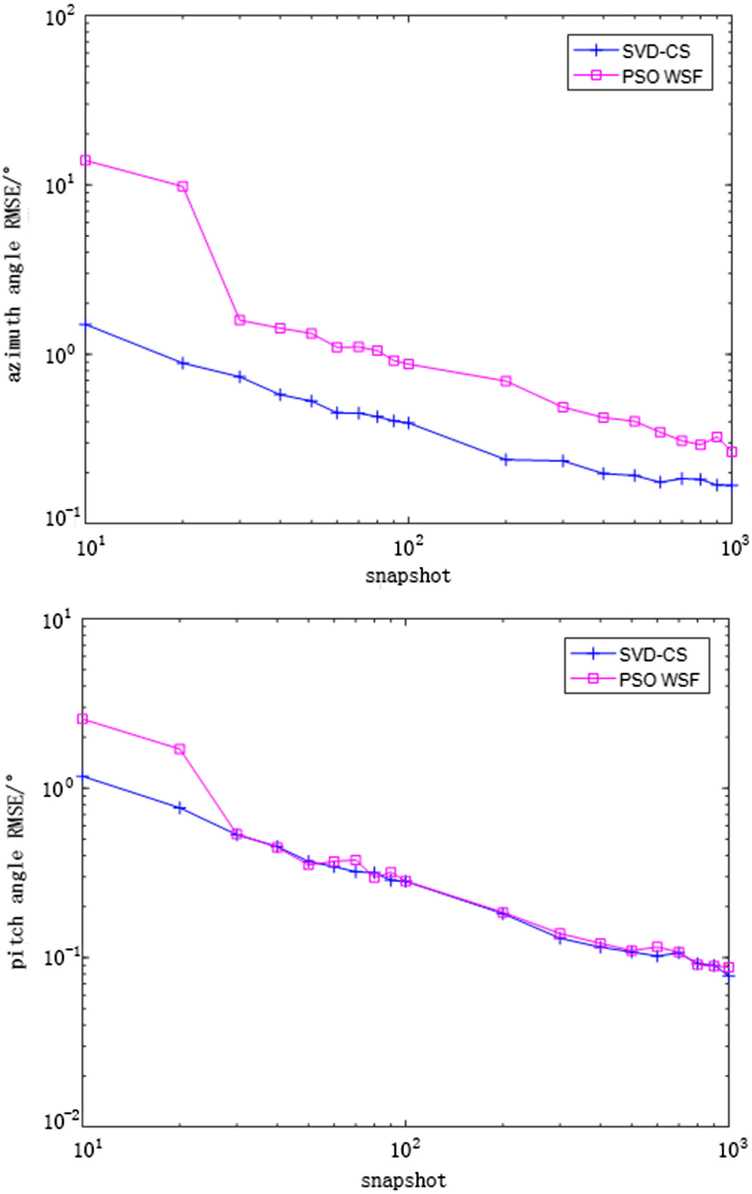


Fig. 4. Curves of mean square error



(a) snapshot 100

Fig. 5. Curves of mean square error



(b) SNR 0dB

Fig. 5. (*continued*)

Simulation results verify the effectiveness of the model and a higher estimation accuracy of the 2D off-grid algorithm based on the first-order Taylor expansion for the off-grid incident signal under the condition of single snapshot.

Experiment 2

To verify its effectiveness, the modified algorithm is compared with WSF algorithm based on particle swarm optimization (PSO) algorithm. The distance between two adjacent grids is 1°, with SNR increasing from -15 dB to 10 dB, snapshot from 10 to 1000, and Monte-carlo simulation 500 times. Results are shown in Fig. 5.

Simulation results show that the proposed algorithm can deal with the more snapshots problem, and improve the solution accuracy under off-grid conditions with better robustness. Compared with the traditional subspace DOA estimation algorithms, it has better accuracy and is less affected by SNR and snapshot number. It solves the off-grid problems better than the basic compressed sensing methods.

5 Concluding Remarks

This paper introduces the signal model with separable array manifold and its 2D off-grid algorithm based on the first-order Taylor expansion. The effectiveness of the algorithm in off-grid conditions is analyzed by simulation, while unsuitable in more snapshots. In order to solve this problem, a 2D multitasking compressed sensing off-grid algorithm based on SVD is proposed. Experimental results show this scheme can solve the problem of 2D off-grid DOA estimation in such cases, and its performance is significantly improved compared with the traditional subspace DOA estimation algorithms.

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