



Joint Equalization and Raptor Decoding for Underwater Acoustic Communication

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Abstract. To improve the link reliability and solve the problem of long feedback delay, a joint equalization and Raptor decoding (JERD) algorithm is proposed for underwater acoustic communication. Compared with the existing approaches, the Raptor code is adopted. The Raptor code is consisted of LDPC code generated by Mackey-1A and weakened LT code, and Raptor decoding adopts the global-iteration algorithm. The detector is iteratively adapted by switching soft information between the equalization and Raptor decoding at the Turbo processing stage. Simulation results validate the feasibility and show the advantages of the proposed algorithm against the existing approaches.

Keywords: Underwater acoustic communication · Raptor codes · Joint equalization and Raptor decoding

1 Introduction

Recently, in the requirement of marine development and ocean exploration, underwater acoustic channel (UAC) has attracted increasing attention. The environment of UAC is extremely complex affected by various factors in the harsh underwater environment. The studies of UAC are confronted with the problem of the low signal-to-noise ratio and the time-space-varying channel parameters.

Considering the long feedback delay of the traditional mechanism such as automatic repeat request (ARQ) [1], a new class of sparse graph channel codes known as Fountain Codes (FCs) has been used for information transmission in the UAC. Fountain Codes can generate an infinite stream of encoded symbols from a given source message. Because the rate of the codes is not fixed a-priori [2], the Fountain Codes are rateless. There are two important classes of the Fountain Codes known as Luby Transform (LT) and Raptor codes. It has been shown that Raptor codes, which are constructed by serially concatenating LT codes with high-rate low-density parity-check (LDPC) codes, and outperform LT codes in the complexity of encoding and decoding process. Therefore, the Raptor codes can help to recover the input symbols that LT codes cannot recover [3]. Meanwhile, with the advent of turbo equalization [4, 5], there has been a wide interest in the application of turbo detection schemes for UAC communications. Compared with conventional one-time equalization, turbo equalization has a much more powerful detection capability [6]. Thus, to improve the performance of the receiver of the communication system using LT codes, an LT-Turbo

equalization method has been proposed, in which the adaptive linear equalization and the LT decoding are jointly optimized in the iterative process [7]. To our best knowledge, there is still lack of researches on the joint equalization and Raptor decoding algorithm.

In this paper, we propose a joint equalization and Raptor decoding (JERD) algorithm. The proposed scheme can jointly realize the adaptive equalization and the Raptor decoding. The rest of the paper is organized as follows. In Sect. 2, we review the soft decoding algorithm of Raptor codes known as global iterative Belief Propagation (BP) decoding. In Sect. 3, Joint Equalization and Raptor Decoding is introduced, where the adaptive equalization and the decoding of Raptor codes are jointly optimized in an iterative process. In Sect. 4, the performance results obtained by computer simulations of fixed-rate Raptor codes in the UAC are presented. And Sect. 5 presents the conclusions.

2 System Model

The Raptor codes in this paper concatenate weakened LT codes with LDPC codes as pre-codes that can patch the gaps in the LT code. The system model of the transmitter is shown in Fig. 1.

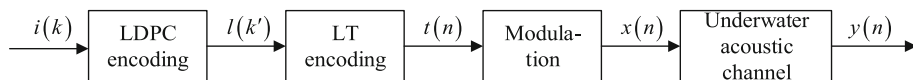


Fig. 1. The system model of the transmitter with Raptor codes.

The Tanner graph of the Raptor codes is shown in Fig. 2, it can be seen that the graph of Raptor codes consists of two component bi-partite tanner graphs (the LDPC codes graph and LT codes graph). The LDPC codes graph consists of source nodes and intermediate nodes (variable nodes), while the LT codes graph consists of intermediate nodes and encoded nodes.

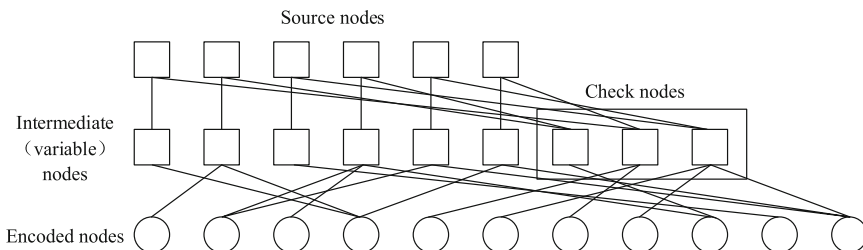


Fig. 2. The Tanner graph of Raptor codes.

2.1 LDPC Encoding

The LDPC codes can be described in terms of a sparse parity check matrix \mathbf{H} , which can satisfy $\mathbf{H}\mathbf{i} = 0$ for all source codes i . If each column of \mathbf{H} has the same weight, which means each column has the same number of non-zero elements, and the weight per row is also uniform, this class of LDPC codes is known as regular LDPC codes, otherwise, it is irregular LDPC codes. In this paper, \mathbf{H} is created by Mackay-1A as followed [8]:

A K' by K matrix (K' rows, K columns) is created at random with the same weight per column, and weight per row as uniform as possible. The overlap between any two columns is no greater than 1 (The overlap between two columns means their inner product).

According to \mathbf{H} , symbols can be encoded for a given rate.

2.2 LT Encoding

The length of symbols after LDPC encoding is denoted by K' and the length of encoded symbols is denoted by N . The method of LT encoding is as followed:

1. Choose the degree of each encoding symbol randomly according to a degree distribution $\rho(d)$, $d = 1, 2, 3, \dots, K'$.
2. Select d different input symbols randomly as neighbors of the encoded symbol and the encoded symbol is the result of the XOR of the d chosen neighbors. Tanner graph shown in Fig. 2 describes an example of the relationship between input symbols and output symbols of LT encoder.

Then the output of LT encoder is modulated with Binary Phase Shift Keying (BPSK). The sampled output of a multipath channel can be expressed as followed:

$$y(n) = \sum_{i=0}^M h_i x(n-i) + \omega(n) \quad (1)$$

where the $x(n)$ is a BPSK symbol sequence, and h_0, h_1, \dots, h_M are the tap coefficients of multipath channel impulse response, in this paper, the Bellhop model is adopted. $M+1$ is the number of channel tap coefficients and $\omega(n)$ is the additive white Gaussian noise sequence with variance σ^2 and mean zero.

3 Joint Equalization and Raptor Decoding

Previous studies have shown that joint equalization and decoding algorithm (Turbo equalization) can effectively improve the performance of equalizer in the UAC [6]. Thus, we proposed a joint equalization and Raptor decoding (JERD) algorithm, which could utilize the updated information from Raptor decoders. As shown in Fig. 3, a feedback loop is added between the equalizer and the Raptor decoder. The performance of detector can be improved by the Turbo iterative process.

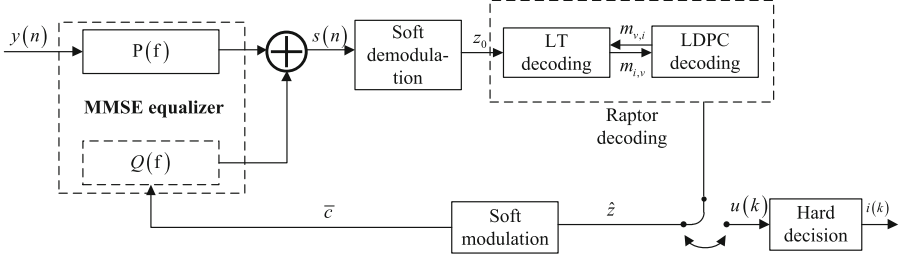


Fig. 3. The structure of JERD.

BP decoding is a common decoding algorithm for Raptor codes, in the iteration process, the message of information passes back and forth between different kinds of nodes. In this paper, we use a global-iteration algorithm that utilizes feedback between the LDPC and LT decoder in the Raptor code [9, 10].

The message of information in the decoder is denoted by m . After equalization and soft modulation, the message is received by the LT decoder. In the LT decoder, $m_{o,i}$ is the message sent from the encoded nodes o to the intermediate nodes i , and $m_{i,o}$ is the message sent from the intermediate nodes i to the encoded nodes o . In addition, the message sent from intermediate nodes i to its corresponding variable nodes v is denoted by $m_{i,v}$, from the variable nodes to the intermediate nodes oppositely by $m_{v,i}$. q is the global decoding iteration, while l_1 is the decoding iteration of LT decoder.

It has been proved that if $u = u_1 \oplus u_2$, and the log-likelihood ratio of u is defined as $L(u)$, then we can get the relation of them as [11]:

$$\begin{aligned} L(u) &= L(u_1 \oplus u_2) \\ &= 2 \tanh^{-1} \left(\tanh \left(\frac{L(x_1)}{2} \right) \tanh \left(\frac{L(x_2)}{2} \right) \right) \end{aligned} \quad (2)$$

According to the encoding of LT codes shown in Fig. 2 and (2), the LT codes are decoded as followed:

$$m_{o,i}^{(q,l_1)} = 2 \tanh^{-1} \left[\tanh \left(\frac{z_0}{2} \right) \prod_{i' \neq i} \tanh \left(\frac{m_{i',o}^{(q,l_1)}}{2} \right) \right] \quad (3)$$

$$m_{i,o}^{(q,l_1+1)} = \sum_{o' \neq o} m_{o',i}^{(q,l_1)} + m_{v,i}^{(q-1)} \quad (4)$$

where p_1 denotes iteration number of the LT decoder, l_1 represents the iterative number of LT decoding in progress, q expresses the q^{th} global decoding iteration. The soft outputs on the intermediate bits of LT decoder are provided as the channel LLRs of the variable nodes of the pre-code decoder (since the intermediate bits are the variable nodes of the pre-code), as followed:

$$m_{i,v}^{(q)} = \sum_o m_{o,i}^{(q,p_1)} \quad (5)$$

Similarly, we proceed by decoding the LDPC codes as followed and l_2 is the decoding iteration of LT decoder, $m_{v,c}$ can be initialized by

$$m_{v,c}^{(q,1)} = m_{i,v}^{(q)} \quad (6)$$

$$m_{c,v}^{(q,l_2)} = 2 \tanh^{-1} \left[\prod_{v' \neq v} \tanh \left(\frac{m_{v',c}^{(q,l_2)}}{2} \right) \right] \quad (7)$$

$$m_{v,c}^{(q,l_2+1)} = \sum_{c' \neq c} m_{c',v}^{(q,l_2)} + m_{i,v}^{(q)} \quad (8)$$

Via p_2 iterations of the LDPC decoder, the extrinsic information of the variable nodes in the LDPC decoder is provided as the a-priori information for intermediate bits in the LT decoder in the next global iteration as:

$$m_{v,i}^{(q)} = \sum_c m_{c,v}^{(q,p_2)} \quad (9)$$

After g iterations of the global decoder, the messages to variable nodes corresponding to the information bits are:

$$d(s) = \sum_c m_{c,v}^{(g,p_2)} + m_{i,v}^{(g)} \quad (10)$$

Then, a hard decision on every information bit is made to recover the source message. The soft information of the input data can be updated by the process of Raptor decoding as [7]:

$$\hat{z} = 2 \tan^{-1} \left[\prod_i \left(\frac{m_{i,o}^{(g,p_1)}}{2} \right) \right] + z_0 \quad (11)$$

Then, we try to estimate the interfering symbols to cancel the residual ISI by modulating the soft information of those symbols from the last time of Raptor decoding iteration. Thus, the symbols are estimated as followed [12]:

$$\bar{c} = E[c | \hat{z}] = \tanh \left(\frac{\hat{z}}{2} \right) \quad (12)$$

To minimize the mean-squared error (MSE) between the equalizer output and the transmitted sequence, after all the information of the symbols from the decoder is obtained, the coefficients of equalizer can be updated as followed:

$$s_n = \mathbf{P}_n \mathbf{Y}_n - \mathbf{Q}_n \bar{\mathbf{C}}_n \quad (13)$$

$$\mathbf{P}_{n+1} = \mathbf{P}_n - \mu \mathbf{Y}_n \times (s_n - \bar{s}_n) \quad (14)$$

$$\mathbf{Q}_{n+1} = \mathbf{Q}_n + \mu \bar{\mathbf{C}}_n \times (s_n - \bar{s}_n) \quad (15)$$

where \mathbf{P}_n and \mathbf{Q}_n are the coefficients of the equalization, \mathbf{Y}_n is a sequence received by the equalizer from the channel and $\bar{\mathbf{C}}_n$ is the estimates of symbols provided by Raptor decoder. The normalized output of the MMSE linear equalizer is denoted by s_n , while \bar{s}_n is the hard-decision symbols of the output information.

In the process of Turbo equalization, the equalizer can utilize the information from the decoder to adaptively track the channel to mitigate the Inter-Symbol Inference (ISI). Meanwhile, as the iteration of Turbo equalization proceeds, the updated output information of equalizer can improve the performance of Raptor decoder.

4 Simulation Results and Discussion

In this section, we evaluate the performance of Raptor codes of 1/2 in the UAC, which is generated by the Bellhop model. The setting parameters of Bellhop model are given in Table 1. For each of the code rates, 950 information bits are encoded by a rate 0.95 LDPC code of left degree three to produce 1000 intermediate bits. In the global Raptor decoder, we perform 2 global iterations, with 7 LT decoding iterations and 3 LDPC decoding iterations per global iteration.

Table 1. Parameters Setting of Bellhop Model

Parameter	Value
Sound frequency	15 kHz
Communication distance	800 m
Sea depth	45 m
The depth of source and receiver	10 m
Sound speed	1460–1480 m/s
The number of beams	10
The launch angle of ray	-15° – $+15^\circ$

The effect of iteration on the BER performance for Raptor codes with different signal to noise ratio (SNR) is shown in Fig. 4. It can be seen from Fig. 4 that with the increase of iteration number, the BER gradually decreases, and eventually tends to be flat. For SNR = 5 dB, the effect of iteration number is not obvious, the BER is only improved a little. When the SNR is set to 6 dB and 7 dB, a noticeable correlation between the iteration number and the BER performance has appeared. As the iteration number increases, the BER performance is improved significantly in the thirteen iterations and then tends to be stable. Thus, in the following simulation, we set the iteration number as thirteen.

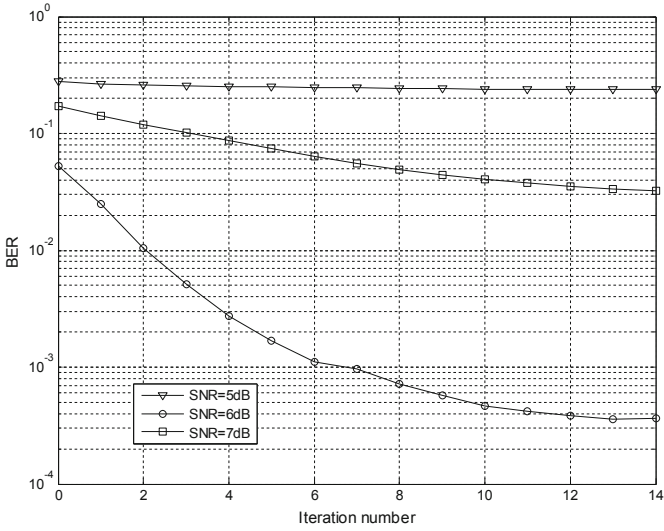


Fig. 4. The BER performance of the JERD in different iterations

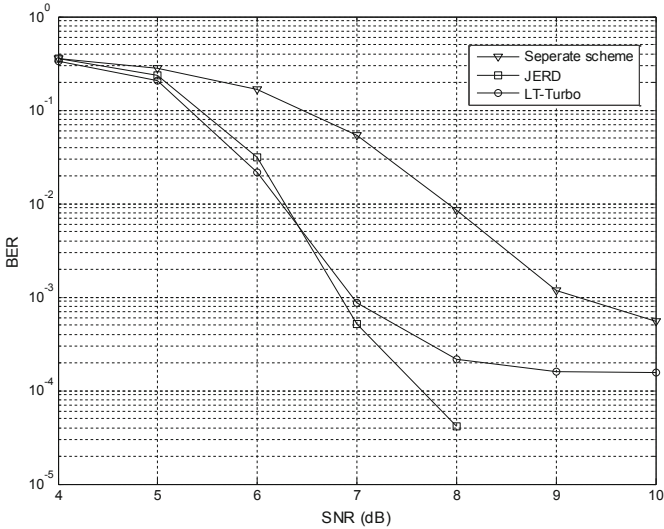


Fig. 5. The BER performance comparison of different schemes

Figure 5 gives the BER comparison of different schemes including the LT-Turbo [7], the separate scheme and the proposed JERD. In the separate scheme, the iteration process is not be used, the equalization and Raptor decoding are separately implemented. It is observed that the JERD achieves better BER performance than the separate scheme (at BER = 10^{-3} , about 2.5 dB gain). This is because the iteration process

can improve the correctness of feedback information. It is also observed that when the SNR is greater than 7 dB, the BER performance of JERD is significantly better than that of LT-Turbo. This is because that the Raptor decoding outperforms the LT decoding, in the iterative process, the feedback soft information is more accurate, with the iterative number increasing, better BER performance can be achieved.

Figure 6 shows the relationship between the decoding success rate (That means the ratio of the number of complete recovery to the total number of experiments) and code redundancy. To illustrate the relationship between the decoding success rate and redundancy of the joint equalization decoding scheme based on rateless code, in the simulation, the curve in Fig. 6 is obtained by averaging on 1000 independent experiments.

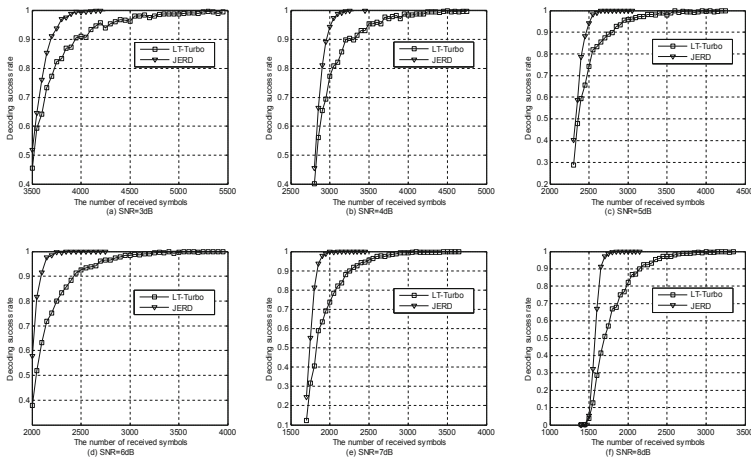


Fig. 6. The relationship between decoding success probability and redundancy

It can be seen from Fig. 6 that under different signal-to-noise ratios, for successfully decoding, the number of symbols required by JERD is less than that by LT-Turbo, this means that the redundancy required for successful decoding with JERD is smaller. It can also be observed from the Fig. 6 that as the signal-to-noise ratio increases, the redundancy required for successful decoding gradually decreases. When the decoding success rate is higher than 99%, the specific number of the received symbols is shown in Table 2.

In Table 2, the number of received symbols is given when the decoding success rate reaches 99%. In the simulation, the SNR varies from 3 dB to 8 dB. It can be seen that compared to the LT-Turbo, JERD uses fewer encoded symbols to recover the original information, which means that the redundancy rate of JERD is obviously lower.

Table 2. Simulation results for the JERD and LT-Turbo

SNR (dB)	The number of original symbols	JERD: the number of received symbols and redundancy rate	LT-Turbo: redundancy rate
3	1000	3950, 3.95	5.05
4	1000	3150, 3.15	4.25
5	1000	2600, 2.60	3.55
6	1000	2250, 2.25	3.15
7	1000	2000, 2.00	2.90
8	1000	1800, 1.80	2.75

5 Conclusions

In this paper, a joint equalization and Raptor decoding algorithm is proposed for underwater acoustic communication. In the algorithm, the adaptive equalization and Raptor decoding are jointly realized. By exchanging the soft information between equalization and Raptor decoding, an iterative process similar to Turbo equalization is implemented, the BER performance can be further improved. Simulation results demonstrate that the proposed JERD can achieve better performance than existing methods. Simulation results also show that compared with LT-Turbo, the redundancy rate of JERD is reduced.

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