Research on Weak Signal Detection Method Based on Duffing Oscillator in Narrowband Noise

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Abstract. One of the most important issues in communication is how to effectively detect signals. Being able to correctly detect the required signal is the basis for the partner to correctly implement the signal reception, and is also the basis for non-cooperative parties to implement information countermeasures and signal interference. The nonlinear signal detection method makes full use of the characteristics of the nonlinear system, and can detect the low SNR signal by converting the change of the signal into the change of the system state. Chaos theory is one of the nonlinear signal detection algorithms, and Duffing oscillator is the most typical. In this paper, the basic theory of Duffing oscillator is studied firstly, and the weak signal detection method based on Duffing oscillator is analyzed. In order to achieve signal frequency detection under narrowband noise conditions, narrowband noise is introduced into the Duffing oscillator to create a new Duffing oscillator model. Then analyzes the model by Melnikov equation, and the state form of the Duffing oscillator different from the traditional theory is obtained, that is, the probability period state of the oscillator under narrowband noise. A new method for weak signal detection using Duffing oscillator under narrow-band noise conditions is proposed. The state of the oscillator is judged by the period state time ratio (PSTR) method. Subsequently, using MATLAB to establish a weak signal detection platform based on PSTR method, the feasibility of detecting weak signals based on PSTR method under narrow-band noise conditions is verified.

Keywords: Signal detection · Duffing oscillator · Narrowband noise

1 Introduction

Signal detection is an important part of signal processing. With the changes in signal detection requirements and the advancement of signal detection theory and technology, researchers have proposed different signal detection algorithms. The increasing update of detection methods also makes the detection performance of the algorithm continuously improved and the scope of application is widened. With the proposed new signal

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detection method, a lot of signal detection problem is constantly overcome. At this stage, the goal of signal detection methods is how to use less prior knowledge to achieve detection of lower signal-to-noise ratios and more signals. According to the different signal processing systems, the signal detection methods are divided into linear signal detection and nonlinear signal detection (Fig. 1).

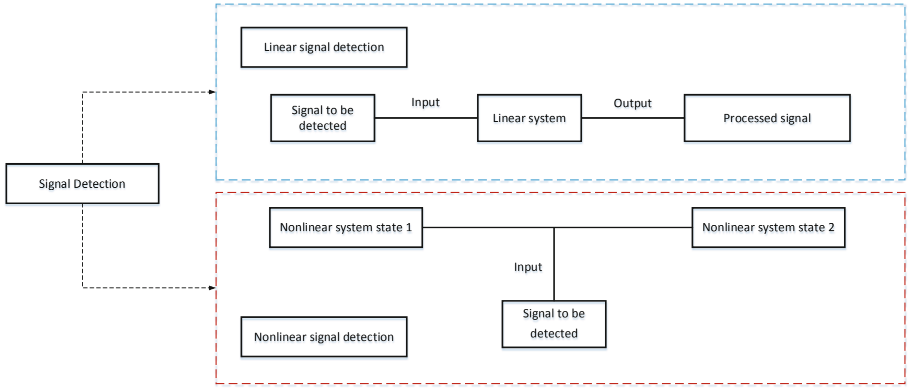


Fig. 1. Signal detection ideas and classification

The linear method is the most mature method in the current technology, but since the signal and the noise experience the same linear change, the linear method cannot detect the signal with low SNR. The nonlinear system uses a new idea to achieve signal detection. The nonlinear method inputs the signal to be tested into the nonlinear system. Due to the system characteristics of the nonlinear system, the useful signal in the input signal can be separated from the noise. The essence of nonlinear theory is that when the signal under test contains a signal that needs to be detected, the nonlinear system will change from one state to another, transforming the detection of the signal into a change in the state of the system. This shift of the signal achieved by amplifying, enabling the detection of low SNR signals. Among them, the oscillator in chaos theory has the characteristics of initial value sensitivity and noise immunity, which can transform weak signal changes into system state changes and realize the amplification of weak signals. At the same time, since the oscillator is immune to noise, the oscillator state is not affected by noise. It is possible to detect low SNR signals that are submerged under noise.

At present, the use of oscillators for signal detection is the direct use of the basic characteristics of the oscillator and theoretical analysis results, in theory to achieve the detection of weak communication signals. Many scholars and researchers have carried out a large number of analysis and simulation to prove the accuracy of the theoretical results. However, the existing simulation and analysis results are obtained under Gaussian white noise conditions, and can only be used for theoretical analysis. In the actual signal, the noise of the signal to be tested cannot be Gaussian white noise.

In this paper, the Duffing oscillator is used as the basic model of weak signal detection, and the narrowband Gaussian noise is used as the noise model to study the influence of narrowband noise on the state of the oscillator. Using the Melnikov equation to theoretically analyze and simulate the state of the oscillator under narrowband noise, and a new method based on oscillator for signal detection under narrowband noise is proposed. Then establish a signal detection model and analyze the weak signal detection performance of the new method.

2 Related Research Work

In the 1960s and 1970s, the idea of chaotic systems was proposed. In 1979, when Holmes studied chaotic motion, he found that there are special attractors in nonlinear oscillator differential equations. Therefore Duffing oscillators with fixed parameters will produce statistical properties similar to random processes [1]. In 1992, Birx D. L. and Pipenberg S. J. first realized weak signal detection with Gaussian white noise with a signal-to-noise ratio of −12 dB. All of their methods were Duffing oscillator combined with neural network method [2]. In 1995, SIMON H. and XIAO B. L first combined chaotic systems with neural networks to detect noise-containing signals in radar detection projects [3]. In 2001, Liu W. Y. and Zhu W. Q. used Duffing oscillator as the research model, and introduced bounded noise into the oscillator to obtain the stochastic process in the mean square sense of the oscillator Melnikov under noise excitation [4]. In 2011, Yi W. S. introduced the noise of non-Gaussian white noise into the Duffing oscillator and theoretically analyzed the Melnikov process of the oscillator at this time [5]. In 2012, Haykin S proposed a signal detection model based on phase space reconstruction [6]. In 2014, Zapateiro M et al. realized the frequency estimation of signals based on Duffing oscillators [7]. In 2015, Luo Z et al. proposed a signal frequency measurement method based on extended Duffing oscillators. It can improve the accuracy of frequency measurement and realize the fundamental frequency detection of power system based on extended Duffing [8]. In the same year, Zeng Z Z et al. proposed a new method for detecting weak pulse signals based on extended oscillators, which can further expand the detection range and application field of weak signals [9]. In 2016, Li N. proposed a method for adaptively adjusting the oscillator threshold parameters [10]. In 2017, Han D. Y. et al. proposed a weak signal feature extraction algorithm based on double-coupled duffing oscillator stochastic resonance to achieve effective extraction of weak signal fault feature information [11]. As one of the typical chaotic systems, Duffing oscillators have made great progress in signal detection and have lower signal-to-noise ratio thresholds.

3 Duffing Oscillator Weak Signal Detection Theory

3.1 Basic Theory of Duffing Oscillator

In 1918, Duffing used the standardized Duffing equation to describe forced motion with nonlinear restoring forces in the form of:

$$\ddot{x} + k\dot{x} + f(x) = g(x) \tag{1}$$

Where A is a nonlinear restoring force, including a nonlinear function of the x cube term, B is the oscillator cycle strategy and is a periodic function.

Based on the Duffing equation, Holmes proposed the Holmes-type Duffing equation in 1978:

$$\ddot{x} + k\dot{x} - x + x^3 = \gamma \cos(\omega t) \tag{2}$$

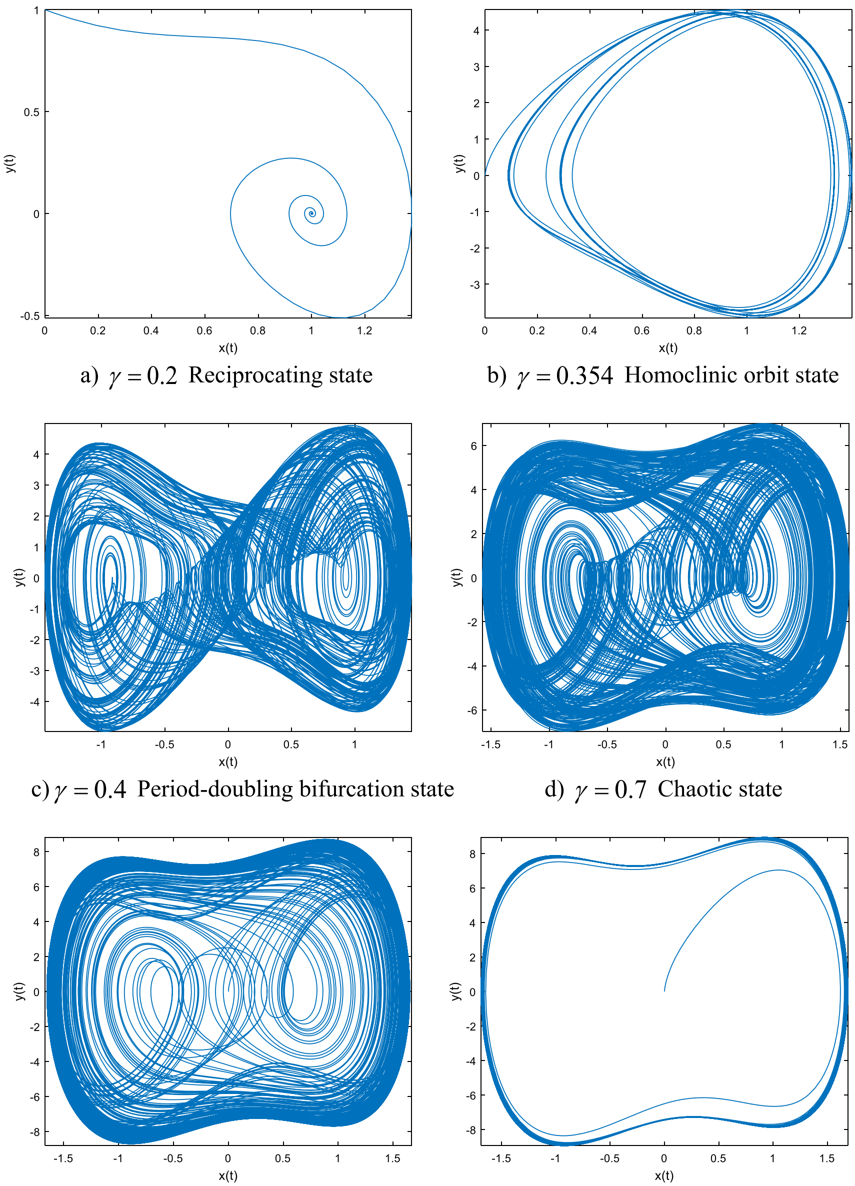
In Eq. (2 and 3), k is the damping ratio, $\gamma \cos(wt)$ is the periodic strategy, x is the state value of the system, and w is the angular frequency of the periodic strategy. When w takes 1 rad/s, the Duffing equation can be written as:

$$\ddot{x} + k\dot{x} - x + x^3 = \gamma \cos(t) \tag{3}$$

Its state equation is:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -ky + x - x^2 + \gamma \cos(t) \end{cases} \tag{4}$$

By analyzing the state equation of the Duffing oscillator, we can see that when the system cycle power amplitude is in a small range, the oscillator will finally perform a reciprocating cycle around the saddle point. Continue to increase the cycle power of the system, the system will present the state of the same orbit. Continue to increase the system cycle power amplitude until $\gamma = 0.4$, the system has a phase trajectory of periodic bifurcation, and chaotic state. When the system power amplitude $\gamma = 0.8260$, the oscillator will be in a critical state; continue to increase the oscillator power to $\gamma = 0.83$, the system will appear a large-scale cycle state, as shown in Fig. 2.



a) $\gamma = 0.2$ Reciprocating state

b) $\gamma = 0.354$ Homoclinic orbit state

c) $\gamma = 0.4$ Period-doubling bifurcation state

d) $\gamma = 0.7$ Chaotic state

e) $\gamma = 0.8260$ Critical state

f) $\gamma = 0.83$ Large scale periodic state

Fig. 2. The state of the Duffing oscillator system when γ is a different value

From the above state analysis of the Duffing system, it can be known that the system state is affected by the initial value of the periodic policy dynamic amplitude and has initial value sensitivity. Using the initial value sensitivity of the Duffing oscillator and the immunity of the noise, the weak signal to be measured is added to the oscillator in the form of an initial value, causing the state of the system to change. At the same time, the corresponding oscillator state judgment method is adopted to complete the weak signal detection based on the Duffing oscillator.

3.2 Using Duffing Oscillator to Detect Communication Signals

After the Duffing oscillator weak signal detection method is proposed, it is first used for the detection of low frequency signals such as sea clutter signals and rail signals. The signal to be detected in the communication system is the medium and high frequency modulated signal received by the receiver. Therefore, when applying the Duffing oscillator to the communication signal detection, it is necessary to perform corresponding scale transformation on the oscillator.

In formula (2 and 3), let $t = \omega\tau$, then find:

$$\ddot{x}(t) = d\left(\frac{1}{\omega} \bullet \dot{x}(\omega\tau)\right)/d(\omega\tau) = \frac{1}{\omega^2} \bullet \ddot{x}(\omega\tau) \tag{5}$$

Substituting formulas (4), (5), (6), can get:

$$\frac{1}{\omega^2} \bullet \ddot{x}(\omega\tau) + \frac{k}{\omega} \bullet \dot{x}(\omega\tau) - x(\omega\tau) + x^3(\omega\tau) = \gamma\cos(\omega\tau) \tag{6}$$

Formula (7) is an equation with time scale τ as an independent variable, and its equation of state can be written as:

$$\begin{cases} \dot{x} = \omega y \\ \dot{y} = \omega(-ky + x - x^3 + \gamma\cos(\omega\tau)) \end{cases} \tag{7}$$

Then get:

$$\ddot{x} = -\omega k\dot{x} + \omega^2\left(x - x^3 + \gamma\cos(\omega\tau)\right) \tag{8}$$

Based on the above analysis and derivation, it can be seen that the parameter w has no effect on the Hamilton equation and the Melnikov judgment method. The critical threshold of the Duffing oscillator phase trajectory from chaotic to large-cycle state does not change with the change of the system dynamic angular frequency w. Therefore, even if the frequency of the signal to be tested changes, it is only necessary to adjust the value of w to adapt to different signal frequencies to be tested, without having to solve the system critical threshold again.

4 Weak Signal Frequency Detection Method Under Narrowband Noise

In a communication system, the receiver performs band pass filtering on the received modulated signal. When performing signal detection, the signal to be tested is a mixed signal of the communication signal and the narrowband noise after being filtered by the receiver. The Gaussian white noise passing through the ideal rectangular band pass filter is called band pass white noise in communication. If the band B of the band pass filter is larger than fc at this time, it is called narrow band Gaussian white noise. The expression of narrowband noise is as follows:

$$n(t) = n_c(t)\cos(w_c t) - n_s(t)\sin(w_c t) \tag{9}$$

Narrowband Gaussian noise can be expressed as envelope and phase:

$$\begin{aligned} n(t) &= R(t)\cos(w_c t + \varphi(t)) \\ R(t) &= \sqrt{n_c^2(t) + n_s^2(t)} \\ \varphi(t) &= \tan^{-1}(n_s(t)/n_c(t)) \end{aligned} \tag{10}$$

The verification model for the influence of the narrowband Gaussian white noise on the system state can be established as shown in Fig. 3. The narrowband noise in the system is the narrowband Gaussian white noise after Gaussian white noise passes through a band pass filter. Set the band pass filter to FIR filter with a filter pass-band of 45–55 MHz and a filter bandwidth of 10 MHz. The Duffing oscillator period power frequency is 50 MHz, and the filter impulse response is shown in Fig. 4.

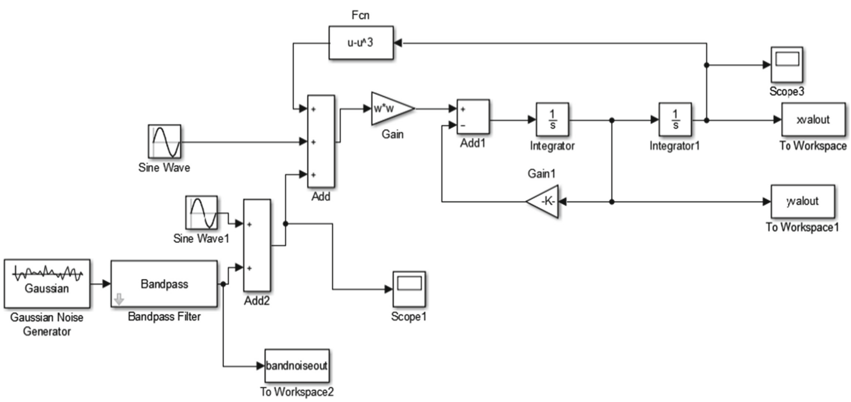


Fig. 3. Verification model for the influence of narrow-band noise on Duffing oscillator

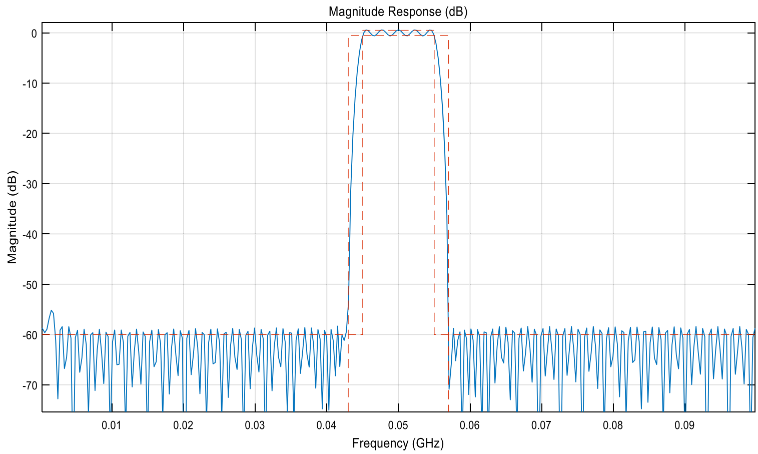
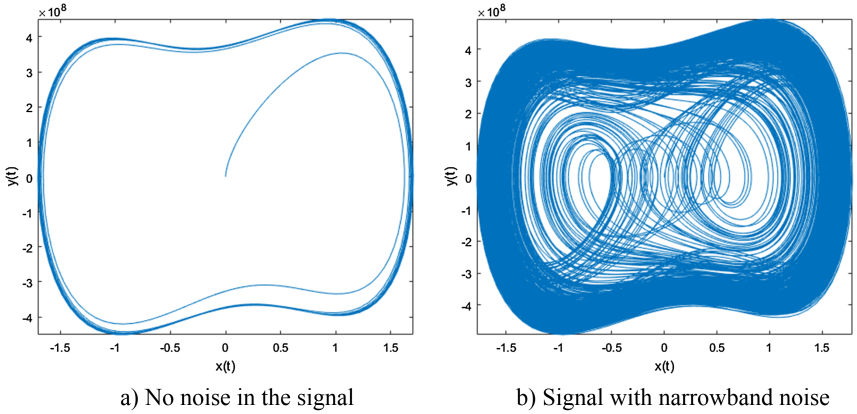


Fig. 4. Impact response function of FIR bandpass filter

Set the system strategy power amplitude to $\gamma = 0.8260$, and the power frequency to f_0 = 50 MHz. At this time, the oscillator is in a critical state. Assume that the signal to be tested contains a sinusoidal signal with a frequency of f_0 = 50 MHz, and set the damping ratio to 0.5. The simulation results are shown in Fig. 5, and the system is in a large-scale cycle state.



a) No noise in the signal b) Signal with narrowband noise

Fig. 5. The state of the Duffing oscillator system

Add narrowband noise into the signal to be tested and set the signal-to-noise ratio to −10 dB. The simulation results are shown in Fig. 6. The current state of the oscillator that entered the periodic state will be almost identical to the chaotic state and difficult to distinguish.

It can be seen that the narrow-band noise will cause the oscillator that should enter the cycle to enter chaos. At this time, the oscillator state judgment according to the conventional method cannot obtain the correct result. Therefore, in the case of narrowband noise, it is necessary to find a new oscillator state judging method.

When there is a periodic force and damping ratio in the oscillator, the state equation of the Duffing oscillator is as shown in formula (5), and the Melnikov method is used to judge the oscillator state of the oscillator equation. The Melnikov equation corresponding to formula (5) is:

$$\begin{aligned} M(t_0) &= \int_{-\infty}^{\infty} \gamma\cos(t+t_0)y_0(t)dt - \int_{-\infty}^{\infty} k \times y_0^2(t) \\ &= -\sqrt{2}\pi\gamma \sec h\left(\frac{\pi}{2}\right)\sin(t_0) - \frac{4}{3}k \end{aligned} \tag{11}$$

It follows that when $\left|\sqrt{2}\pi\gamma \sec h(\pi/2)\right| = -4/3k$, the oscillator is in a critical state, and can get the critical amplitude γ_d. When $\left|\sqrt{2}\pi\gamma \sec h(\pi/2)\right| > -4/3k$, there is t_0 such that $M(t_0) = 0$, the oscillator will be able to enter the periodic state.

When noise is added into the oscillator, the oscillator equation becomes:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -ky + x - x^3 + \gamma\cos(t) + n(t) \end{cases} \tag{12}$$

The corresponding Melnikov function can be expressed as:

$$\begin{aligned} M(t_0) &= \int_{-\infty}^{+\infty} (\gamma\cos(t+t_0) + n(t+t_0))y_0(t)dt - \int_{-\infty}^{+\infty} k \cdot y_0^2(t)dt \\ &= -\sqrt{2}\pi\,\gamma \sec h\left(\frac{\pi}{2}\right) sin(t_0) - \int_{-\infty}^{+\infty} n(t+t_0)y_0(t)dt - \frac{4}{3}k \\ &= -\sqrt{2}\pi\,\gamma \sec h\left(\frac{\pi}{2}\right) sin(t_0) - \int_{-\infty}^{+\infty} n(t+t_0)\frac{e^t - e^{-t}}{(e^t + e^{-t})^2}dt - \frac{4}{3}k \\ &= M_1(t_0) + M_2(t_0) + M_3 \end{aligned} \tag{13}$$

The first part M_1 is a function of γ and t_0, which can be regarded as a sine function whose amplitude change satisfies $|M_1(t_0)| = \sqrt{2}\pi\gamma \sec h(\pi/2)$, and its amplitude function is a one-time increasing function about γ. The third part M_3 is a positive number. As can be seen from the expression of the second part M_2, due to the existence of noise n(t), M_2 also becomes a random variable related to the noise distribution. Since the noise has a value after zero time, M_2 can be written as:

$$M_2(t_0) = -\int_{-t_0}^{+\infty} n(t+t_0)\frac{e^t - e^{-t}}{(e^t + e^{-t})^2}dt = \int_{-t_0}^{+\infty} n(t+t_0)f(t)dt \tag{14}$$

$$f(t) = \frac{e^t - e^{-t}}{(e^t + e^{-t})^2}$$

It can be seen that t has the same sign as $f(t)$. So when $t \in [-t_0, 0]$, $M_2(t_0) > 0$ M_2 will prevent the oscillator from entering the cycle state. When $t \in [0, +\infty]$ M_2 will promote the oscillator into the cycle state.

So, M_2 becomes a random process related to noise. For a certain period of the engine power amplitude γ, $M(t_0)$ becomes a function of t_0, by calculating whether t_0 exists so that $M(t_0) = 0$ is established, and whether the oscillator can be in a periodic state. At this time, the probability that the oscillator appears in the periodic state can be expressed as:

$$P_{cycle} = \lim_{N \to \infty} \left[\oint_{\sigma} f_{n(t)}(n) dn \right] \tag{15}$$

Where σ is the set of all n(t) that $M(t_0) = 0$ can have solutions, and $f_{n(t)}(n)$ is the N-dimensional joint probability density of noise. In order to solve the equation, the Melnikov equation is written as:

$$\begin{aligned} M(t_0) = M_1(t_0) + M_2(t_0) + M_3 = 0 \\ \Leftrightarrow M_2(t_0) = -M_1(t_0) - M_3 \end{aligned} \tag{16}$$

Since $\lim_{t \to \infty} f(t) = 0$ and $\lim_{t \to \infty} M_2(t_0) = 0$, then the formula (17) can be solved equivalent to the existence of t_0 such that the formula (18) holds:

$$M_2(t_0) > \min(-M_1(t_0) - M_3) = -|M_1(t_0)| - M_3 = M(\gamma) \tag{17}$$

Each P_{cycle} corresponding to t_0 is actually the probability of $M_2(t_0, n(t)) > M(\gamma)$ which can be expressed as:

$$P_{cycle}(t_0) = \lim_{N \to \infty} \int_{M(\gamma)}^{\infty} p(x) dx \tag{18}$$

$$\begin{aligned} P_{cycle} &= \int_{-\infty}^{+\infty} P_{cycle}(t_0) dt_0 = \lim_{M \to \infty} \frac{\int_{-M}^{+M} P_{cycle}(t_0) dt_0}{2M} \\ &= \lim_{M \to \infty} \lim_{N \to \infty} \frac{\int_{M(\gamma)}^{\infty} \int_{-M}^{+M} p(x) dx dt_0}{2M} \end{aligned} \tag{19}$$

$$
\begin{aligned}
\frac{dP_{cycle}}{d\gamma} &= \lim_{M\to\infty}\lim_{N\to\infty}\frac{d\int_{M(\gamma)}^{\infty}\int_{-M}^{+M}p(x)dxdt_0}{2M\times d\gamma} \\
&= \lim_{M\to\infty}\frac{1}{2M}\lim_{N\to\infty}\int_{-M}^{+M}\frac{d\int_{M(\gamma)}^{\infty}p(x)dx}{d\gamma}dt_0 \\
&= \lim_{M\to\infty}\frac{1}{2M}\lim_{N\to\infty}\left[-\int_{-M}^{+M}p(M(\gamma))\times\frac{dM(\gamma)}{d\gamma}dt_0\right] \\
&= \frac{\sqrt{2}}{2}\pi\sec h\left(\frac{\pi}{2}\right)\lim_{M\to\infty}\frac{1}{M}\lim_{N\to\infty}\int_{-M}^{+M}p(M(\gamma))dt_0 > 0
\end{aligned}
\tag{20}
$$

Therefore, under the action of narrowband noise, the Duffing oscillator will enter the periodic state according to a certain probability. It can be seen from formula (21) that the probability of entering the periodic state is an increasing function of the oscillator's power amplitude γ.

According to the previous analysis and derivation, when the noise contained in the signal to be tested is not Gaussian white noise, the Duffing oscillator no longer has complete noise immunity. The narrow-band noise mixed in the signal to be tested causes the oscillator that should enter the chaotic state to appear in a periodic state, showing a state in which the period and chaos alternately appear. This is similar to the existing intermittent chaotic state, but the difference is that the oscillator is in a periodic state or a chaotic state is subject to a certain probability distribution.

Based on the derivation and analysis, this paper proposes a method for judging the state of the oscillator by counting the period state time rate (PSTR) of the Duffing oscillator. The definition of PSTR is as follows:

$$
PSTR = \frac{T_{period}}{T_{total}} \tag{21}
$$

T_{period} represents the proportion of the time of the cycle state in a simulation time, and T_{total} is the total simulation time. Express it as a discrete sampling point as follows:

$$
PSTR = \frac{n_{period}}{n_{total}} \tag{22}
$$

Using MATLAB/Simulink to establish a statistical model of the oscillator PSTR under narrowband noise. In order to verify the correctness of the proposed method, the PSTR value of the Duffing oscillator under different cycle dynamics is simulated and analyzed. The simulation results are shown in Fig. 6.

As can be seen from Fig. 6. Under the influence of narrow-band noise, the oscillator does not exhibit stable periodic and chaotic states under classical theory. At this

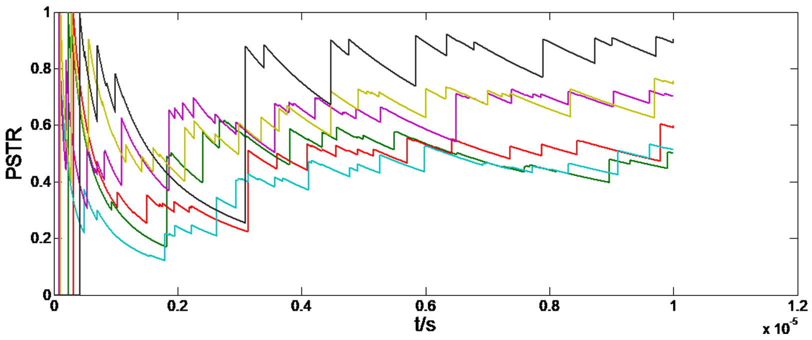


Fig. 6. PSTR of the Duffing oscillator with different initial values of the dynamic amplitude

time, the Duffing oscillator will appear alternating between the period and the chaotic state, and the probability of the periodic state appearing as the PSTR value is different. As the simulation time increases, the PSTR of the oscillator with different periodic power amplitudes will tend to a stable value. And the PSTR at the time of stabilization increases with the increase of the periodic power amplitude, which is the same as the analysis result.

5 Conclusion

As a typical chaotic system, Duffing oscillator is one of the most researching non-linear detection methods for low SNR signal detection. In this paper, the narrow-band Gaussian white noise is used as the noise of the signal to be tested, and the Duffing oscillator model under narrow-band noise conditions is established. Through the Melnikov function analysis, the conclusion that the periodic state of the Duffing oscillator exhibits a probability distribution under narrow-band noise is obtained. Based on the analysis, an oscillator state judgment method based on Duffing oscillator period state time ratio (PSTR) is proposed. The weak signal detection platform is built by MATLAB. The correctness of the method is verified by simulation analysis.

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