Application of Vague Sets and TOPSIS Method in the Evaluation of Integrated Equipment System of Systems

Shangwei Luo, Yonggang Li(✉), and Yanyan Chen

School of Communication and Information Engineering,
Chongqing University of Posts and Telecommunications,
Chongqing 400065, People's Republic of China
1144919304@qq.com, liyg@cqupt.edu.cn

Abstract. There are many uncertain factors in the evaluation process of integrated equipment system of systems (IES), owing to lacking the effective evaluation method. Considering the expert evaluation process is often subjective, so take the combination of entropy weight method, the Gini coefficient weighting method and AHP method are used to calculate the weight of the combat capability index; the expert evaluation information is also vague, and the vague set theory can well describe the support, neutral and opposition information. Therefore, the combination of vague set and TOPSIS method is used to calculate the degree of closeness to measure the importance of IES; Given that the combat process, equipment may be failed. The fault function is introduced to evaluate the contribution of IES dynamically by defining the new fault function and the recurrent fault function. Finally, through the case analysis, it is proved that the proposed algorithm can more accurately evaluate the contribution of IES.

Keywords: Vague set · TOPSIS · IES · Combined weight · Fault function · Combat effectiveness

1 Introduction

IES is a heterogeneous weaponry system coupled upward from different functional nodes and subsystems in accordance with the overall requirements of building an information-based army and winning an information or war. Assessing the operational effectiveness of IES in a reasonable and effective, which can play an indispensable part in optimizing the system structure and accelerating the development of weaponry. It can be able to fight and win the war security. Many experts and scholars have conducted related research and have achieved many results. Literature [1] uses grey theory to analyze the contribution of radar anti-stealth capability. Literature [2] uses data envelopment method for evaluation. Cheng C H [3] appraises according to fuzzy set theory. Gong Y [4] proposed ADC-based assessment method. Shu J S. et al. [5]

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proposed an evaluation based on Bayesian networks. Liu P. et al. [6] introduced price parameters to appraise the contribution rate of the equipment system. There are also related scholars who carry out simulation-based evaluation methods ah, such as Huang Y Y [7] who proposed a process modeling-based approach. Metin D. et al. [8] proposed an evaluation method based on hierarchical analysis, and Xiao H H. et al. [9] proposed a research method based on the vague set.

In this paper, the contribution of IES is reviewed from a new perspective, using the ordering method TOPSIS [10] (Technique for Order Preference by Similarity to Ideal Solution). A ranking method approximates the ideal solution. The TOPSIS ranks the combat capability of equipped weapon, and the higher the ranking, the greater the contribution of the equipped weapon to IES. However, the expert appraisal information is subjective in the assessment process. From this perspective, this study uses a combination of entropy weighting, Gini coefficient method [11] and hierarchical analysis to determine the indicator weights of equipped weapon. At the same time, the expert evaluation information is ambiguous and the introduction of vague set theory can be a very effective solution to this problem. In addition, the contribution rate should be evaluated taking into account the failure of the weaponry, by constructing nascent and recurring failure functions to produce the failure function. Combined, they determine the contribution of a weapon to the overall system.

2 IES Structure Model

As shown in Fig. 1, IES presents a tree-like hierarchical structure from top to bottom, composed of functionally interconnected and performance complementary equipment weapons. Underlying equipment-level weapon nodes are up-coupled into platform-level equipment, based on their different operational capabilities and characteristics. Platform-level equipment is a subsystem composed of equipment-level weapons for a particular mission, whose function includes sub-team piloting and coordinated attack, such as tank, satellite, and UAV groups. System-level equipment plays a part in leading the entire system in the context of an integrated combat network, coupled from platform-level equipment into a giant complex system.

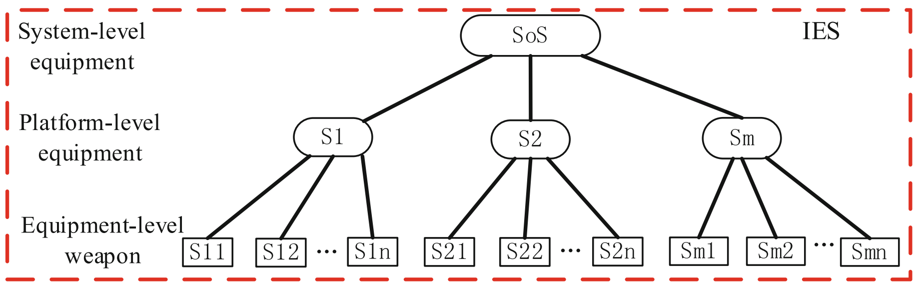

Fig. 1. IES hierarchy

IES is integrated. Locally, we can follow the OODA ring theory and view equipment-level weapons as sensor nodes, decision nodes, and influence nodes. Taken as a whole, IES can be viewed as a black box that performs a particular type of task. It can also be abstracted as a collection of three types of nodes: sensor nodes, decision nodes, and influence nodes. As illustrated in Fig. 2, IES exists to perform a series of dynamic tasks. System-level equipment divides the tasks received and assigns them to platform-level equipment. Platform-level equipment refines the tasks and assigns them to equipment-level weapons. Equipment-level weapons execute missions to detect or attack the target and feed information back to platform-level equipment. The platform level equipment forwards the received message to the system level equipment to finalize the task.

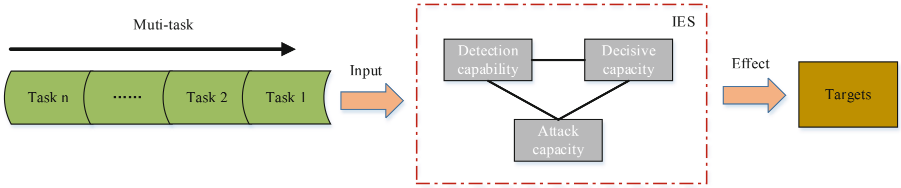

Fig. 2. IES completes tasks dynamically

3 IES Contribution Evaluation

The study of the contribution of weaponry to the system's combat capability should build the system of capability indicators for equipment. As shown in Fig. 3, the capability indicator system in this paper consists of three level 1 capability indicators, namely: detection capability, decisive capability and attack capability.

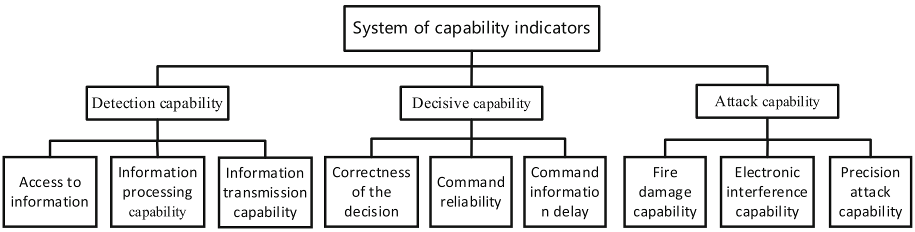

Fig. 3. System of combat capability indicators

Detective capability refers to the ability to acquire, process, and transmit information. It specifically includes the ability to access information, the ability to process information and the ability to transmit information.

Decision capability refers to the ability of aids equipment to make judgments. This includes correctness of decision-making, command reliability and command information delay.

Attack capability is the ability to attack an enemy target and incapacitate it. It specifically includes fire damage capability, electronic interference capability, and precision attack capability.

When evaluating the contribution of equipment in the IES, the characteristics of the IES should be considered. Moreover, the contribution of equipment-level weapons should be calculated through expert evaluation information. As shown in Fig. 4, this paper combines entropy weighting method, the Gini coefficient method and AHP to calculate the weights of combat capability indicators. Considering the uncertainty of expert evaluation information, thus the relative proximity is obtained by the vague set and TOPSIS method. Finally, the joint weaponry failure function collaboratively evaluates the contribution of the weaponry.

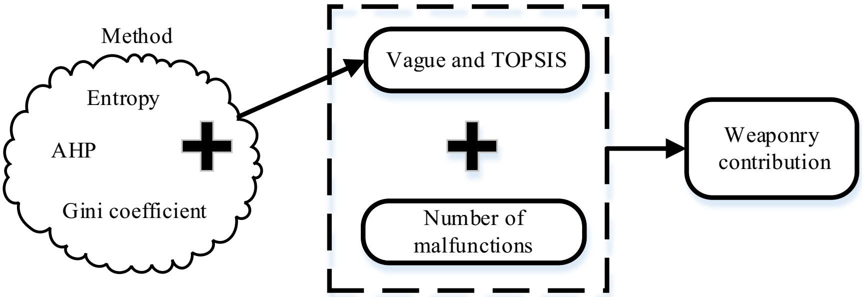

Fig. 4. Evaluation model of contribution degree of IES

3.1 Calculate the Weight of Combat Capability Indicator

The accuracy of the combat capability indicator has a direct impact on the assessment of weaponry. Therefore, it is especially important to discover a reasonable way to get the weight of the combat indicator. Considering that, IES is a complex system with many uncertainties in the battlefield; the expert assesses the combat indicators based on empirical judgment, which is also subjective. Information entropy is used to measure the amount of information contained in the indicator. It takes objective data as a landing point and talking in terms of data; The Gini coefficient reflects the accuracy of the objective data and adequately conveys the information of the objective data; to sum up, this paper uses combination of entropy weighting, Gini coefficient method and AHP to get weaponry weights. This approach takes into consideration subjectivity as well as expressing objectivity.

Due to in the process of obtaining the weapon combat capability indicator, the expert evaluation information is vague, so the evaluation indicator should be unified. Expert evaluation information is vague and needs to be measured by unified and standardized indicators. Based on previous research, this paper uses the method $1 \sim 9$ scale. It expresses the importance of current operational capability indicators relative to the higher level. As shown in Table 1.

Table 1. Relative importance judgment of the method 1–9-scale table

Relative importance	Extremely important	Very important	Important	Slightly important	Equally important
Quantified value	9	7	5	3	1

When evaluating the contribution of weaponry in the IES, there are a total of m combat indicators. It can be denoted by the set $V = (v_1, v_2, \cdots, v_n)$. During the evaluation process, n military experts evaluate m combat indicators. Since the metrics for each combat indicator are different, the evaluation indicators need to be standardized, which will result in an evaluation matrix B. where the set of experts $U = (u_1, u_2, \cdots, u_n)$. The evaluation matrix B is:

$$\boldsymbol{B} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1\mathrm{m}} \\ a_{21} & a_{22} & \cdots & a_{2\mathrm{m}} \\ \vdots & \vdots & \ddots & \vdots \\ a_{\mathrm{n}1} & a_{\mathrm{n}2} & \cdots & a_{\mathrm{nm}} \end{bmatrix} \tag{1}$$

Through expert analysis, the judgment matrix C is given:

$$\boldsymbol{C} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1\mathrm{m}} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2\mathrm{m}} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{\mathrm{n}1} & \beta_{\mathrm{n}2} & \cdots & \beta_{\mathrm{nm}} \end{bmatrix} \tag{2}$$

The Gini Coefficient Weighting Method

The Gini coefficient is a quantitative measure of the degree of income distribution disparity. It is widely used in the analysis of income distribution differences within the population. The formula is shown in the Eq. (3) [13]:

$$\Delta = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| Y_i - Y_j \right|, 0 < \Delta < 2\lambda \tag{3}$$

where Δ is the value of the Gini coefficient, n is the sample size, Y_i is the income level of group i, and λ is the income expectation.

Each column of evaluation matrix B represents a different combat capability indicator, and each row represents the expert's evaluation of the indicator in that row. When using the Gini coefficient method to obtain the weights of combat indicators, this study will treat the n rows of evaluation information corresponding to a column of the evaluation matrix as different income situations in order to calculate the weights of combat capability indicators. The steps of the solution are as follows.

Step1: calculation the Gini coefficient of the IES indicator.

$$G_k = \begin{cases} \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|Y_{ki} - Y_{kj}\right|}{2n^2\lambda_k}, & \lambda_k \neq 0 \\ \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|Y_{ki} - Y_{kj}\right|}{n^2 - n}, & \lambda_k = 0 \end{cases} \tag{4}$$

where G_k is the Gini coefficient for indicator k, n is the total number of data for the indicator, Y_{ki} is the i-th data for indicator k, and λ_k is the expectation for indicator k.

Step2: normalization of G_k obtains the weights of the k-th indicator $\omega_k^{''}$.

$$\omega_k^{''} = \frac{G_k}{\sum_{i=1}^{m} G_i} \tag{5}$$

Linear Weighting to obtain the Combat Indicator Weights

In this study, linear weighting will be used to obtain the weights of the combat capability indicators. The use of linear weighting can easily express the subjectivity and objectivity of expert evaluation information. Moreover, this can ensure the accuracy of the combat capability indicators. According to literature [12], we can obtain the entropy weight $W' = \left(\omega_1^{'}, \omega_2^{'}, \cdots, \omega_n^{'}\right)$. By Eq. (5), the Gini coefficient weight are obtained $W'' = \left(\omega_1'', \omega_2'', \cdots, \omega_n''\right)$. According to literature [14], we can apply the AHP to find the weight $W''' = \left(\omega_1^{'''}, \omega_2^{'''}, \cdots, \omega_n^{'''}\right)$. Finally, we obtain the weight of the weaponry operational capability indicator $W = 1/3(W' + W'' + W''')$.

3.2 Vague and TOPSIS Evaluate the Contribution of the Integrated Equipment System

Introduction to Vague

Vague sets [15] is an extension of fuzzy sets. Cao and Buehree proposed vague sets based on fuzzy sets theory [16]. They argue that the membership of each element can be divided into supporting and opposing sides, i.e., a truth-membership and a false-membership. From an objective point of view, a vague set provides evidence for and against. Through supporting and against arguments, neutral evidence can be derived. It can be seen that vague sets are more realistic and more graphic than fuzzy sets in describing the objectivity, and better describes the uncertainty of the data source. Using vague sets to represent the subjectivity, uncertainty and ambiguity of the expert assessment information is more precise than fuzzy sets and more flexible in treatment as they are widely used in solution selection [17].

A vague set A in $X = (x_1, x_2, \cdots, x_n)$ is characterized by a truth-membership function $t_A(x_i)$ and a false-membership function $f_A(x_i)$. $t_A(x_i)$ is a lower bound on the grade of membership of x_i derived from the evidence for x_i, and $f_A(x_i)$ is a lower bound on the negation of x_i derived from the evidence against x_i. $t_A(x_i)$ and $f_A(x_i)$ are interrelated and have some relationship, where $t_A(x_i) + f_A(x_i) \leq 1,\ x_i \in [0, 1]$. In other words, that is $t_A : X \rightarrow [0, 1]$ and $f_A : X \rightarrow [0, 1]$. This approach bounds the grade of membership of any variable $x_i \in \mathrm{X}$ to a subinterval $[t_A(x), 1 - f_A(x)]$ of [0, 1].

Evaluate the Contribution of Weaponry using Vague Set

There are many uncertainties in the sources of information used to assess the contribution of weaponry, and the experts' evaluation has both qualitative and quantitative indicators. At the same time, different experts will have different evaluations, and some will choose to abstain. Therefore, in order to ensure the accuracy of the assessment, the chosen method should be able to portray the three aspects of support, opposition and neutrality. This paper combines vague sets with the TOPSISS method, which aptly expresses these three aspects. The steps of the algorithm for assessing the contribution of weaponry using vague sets and the TOPSIS method are divided into the following main steps.

Step1: converting evaluation information into the vague value.

Differences in the outline and physical meaning of evaluation indicators should be taken into account in the process of converting evaluation information into the vague values. Based on previous studies, the evaluation indicators are usually divided into benefit, cost and fixed target indicators. As can be seen from Fig. 3, this paper deals only with benefits-based indicators.

S_{ij} is the degree to which the weaponry q_j, under the combat indicator v_i, fulfils a task. It is a benefit interval data indicator and can be expressed as $[\mathrm{x_{ij}}, \mathrm{y_{ij}}]$. It is converted into a vague value by Eqs. (14) and (15) [18].

$$t_{ij} = \frac{x_{ij}^p}{x_{j\max}^p}\left(1 + \frac{y_{ij}^p - x_{ij}^p}{x_{j\max}^p}\right) \tag{6}$$

$$f_{ij} = \left(1 - \frac{y_{\mathrm{ij}}^p}{x_{j\max}^p}\right)\left(1 + \frac{y_{\mathrm{ij}}^p - x_{ij}^p}{x_{j\max}^p}\right) \tag{7}$$

where $x_{j\max} = \max\left(x_{1j}, y_{1j}, x_{2j}, y_{2j}, \cdots, x_{mj}, y_{mj}\right), p \in N^+$. Moreover, in this paper, p equals two.

Step2: constructing the vague decision matrix M.

The vague set is formed based on expert evaluation scores and is represented by matrix $Q = \{\mathrm{Q_1}, \mathrm{Q_2}, \cdots, \mathrm{Q_m}\}$, where Q_i denotes the valuation of the weaponry q_i under the operational capability indicator v_i, expressed as a vague value:

$$Q_i = \left\{(v_1, T_{i1}), (v_2, T_{i2}), \cdots, \left(v_{\mathrm{n}}, T_{ij}\right)\right\} \tag{8}$$

where $T_{ij} = [t_{ij}, 1 - f_{ij}]$, t_{ij} is the grade of support of the operational indicator S_j for the weaponry. In addition, f_{ij} is the degree of negative reaction of the combat indicator S_j for the weaponry q_i.

From the formula $t_A(x_i) + f_A(x_i) \leq 1$, $x_i \in [0, 1]$, let $\lambda_{ij} = 1 - f_{ij}$. So, the formula (6) can be expressed by the following matrix M.

$$\boldsymbol{M} = \begin{bmatrix} [t_{11}, \lambda_{11}] & [t_{12}, \lambda_{12}] & \cdots & [t_{1n}, \lambda_{1n}] \\ [t_{21}, \lambda_{21}] & [t_{22}, \lambda_{22}] & \cdots & [t_{2n}, \lambda_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [t_{m1}, \lambda_{m1}] & [t_{m2}, \lambda_{m2}] & \cdots & [t_{mn}, \lambda_{mn}] \end{bmatrix} \tag{9}$$

Step 3: Identification of the ideal weaponries.

The TOPSIS method [19] is an ordering that approach the ideal solution. The TOPSIS value for each weaponry is obtained by determining the positive ideal solution (PIS) and negative ideal solution (NIS).

The ideal weaponry is selected based on the vague matrix M. The greater the similarity between the weaponry to be evaluated and the combat capability of the ideal weaponry, the greater the contribution of the weaponry to IES.

Vague sets contain three aspects of information, support, oppose and abstain. The matrix M is converted into a suitable matrix V for the combat indicator through Eq. (10). Use V_{ij} to represent the suitability of weaponry q_i for the combat indicators [20].

$$V_{ij} = (t_{ij} - f_{ij}) + (\partial_{ij} - \beta_{ij})\pi_{ij} \tag{10}$$

where ∂_{ij} and β_{ij} are the sort parameter, $\partial_{ij} \in [0, 1]$, $\beta_{ij} \in [0, 1]$. When ∂_{ij} is not equal to β_{ij}, let ∂_{ij} equal to t_{ij} and β_{ij} equal to f_{ij}. π_{ij} is the part that abstained.

$$\pi_{ij} = 1 - t_{ij} - f_{ij} \tag{11}$$

Positive ideal and negative ideal solutions are obtained from the fit matrix V, where $V_j^+ = \max\limits_{1 \leq i \leq m} V_{ij}, V_j^- = \min\limits_{1 \leq i \leq m} V_{ij}$, $1 \leq j \leq n$.

$$VPIS = (V_1^+, V_2^+, \cdots, V_n^+) \tag{12}$$

$$VNIS = (V_1^-, V_2^-, \cdots, V_n^-) \tag{13}$$

VPIS is the vague solution for the positive ideal weaponry corresponding to the decision matrix. VNIS is the vague solution for the negative ideal weaponry corresponding to the decision matrix.

Step 4: Combined vague and TOPSIS calculations to assess the distance to best-case solution D_i^+ and worst-case solution D_i^- of the weaponry q_i.

$$D_i^+ = 1 - \frac{1}{n}\sum_{j=1}^{n} \omega_j M([t_{ij}, 1 - f_{ij}], VPIS), i = 1, 2, \cdots, m \tag{14}$$

$$D_i^- = 1-\frac{1}{\mathrm{n}}\sum_{j=1}^{n}\omega_j M([t_{ij}, 1-f_{ij}], VNIS), i = 1, 2, \cdots, m \tag{15}$$

where $M(x, y)$ can be expressed by the following equation.

$$M(x,y) = 1-\frac{|t_x - t_y - f_x + f_y|}{8} - \frac{|t_x - t_y + f_x - f_y|}{4} - \frac{|t_x - t_y| + |f_x - f_y|}{8} \tag{16}$$

Step 5: Calculating the close degree of proximity of weaponry $S(q_i)$.

$$S(q_i) = \frac{(D_i^+)^2}{(D_i^-)^2 + (D_i^+)^2} \tag{17}$$

Step6: Rank $S(q_i)$. The higher the ranking, the greater the contribution to IES.

3.3 The Least-Squares Method to Derive the Number of Faults

In this paper, IES faults are divided into new and recurrent faults, and the probability density functions of new faults and recurrent faults are derived qualitatively by formulas.

$$g(t) = g_1(t) + g_2(t) \tag{18}$$

where t represents the cumulative number of hours worked on the weapon. $g(t)$, $g_1(t)$, $g_2(t)$ denote equipment failure rate, new failure rate, recurrence failure rate at time t, respectively.

The probability of exposure to a new fault per unit of time is called the probability of new failure and it is displayed using a multi-exponential function, as in Eq. (19)

$$\mathrm{g}_1(t) = A_1 e^{-B_1 t} + A_2 e^{-B_2 t} + \cdots + A_\mathrm{n} e^{-B_n t} = \sum_{n=1} A_\mathrm{n} e^{-B_n t}, \mathrm{n} \in N^+ \tag{19}$$

where A_n is the parameter strength,B_n is the shape parameter, and both parameters are greater than zero. $g_2(t)$ represents the probability of a failure occurring again within a unit of time after a failure.

$$g_2(t) = \int_0^t g_1(x)\varphi(t-x)dx \tag{20}$$

where $\varphi(t)$ can be represented by Eq. (21).

$$\varphi(t) = \sum_{\mathrm{j}=1} \partial_\mathrm{j} \mathrm{e}^{-\mathrm{b_j t}} \tag{21}$$

where j is a positive integer,∂_j is a weight parameter, b_j is a shape parameter, and both parameters are greater than zero.

Bringing Eqs. (25), (26) and (27) into Eq. (24) derives the total number of malfunctions that occur with the weaponry working for t hours.

$$G(\mathrm{t}) = \int_0^{\mathrm{t}} g(x)dx = \int_0^{\mathrm{t}} g_1(x)dx + \int_0^{\mathrm{t}} g_2(x)dx \tag{22}$$

Assuming the weaponry works at time t_k, a total of q failures occurs, and the corresponding time for each failure is $\tau_1, \tau_2, \cdots, \tau_q$. A total of p faults is exposed, and the time of occurrence of each fault is $t_1, t_2, \cdots, t_p$.

The value of the highest fit will be obtained by least squares. As shown in Eq. (23), we first take the discrepancy sum of the total number of the weaponry new failures. Then, make the partial derivatives of the parameter strength A_n and shape parameter B_n in p equal to 0.

$$\varphi = \sum_{k=1}^{p} (G_1(t_k - k))^2 \tag{23}$$

$$\begin{cases} \dfrac{\partial \varphi}{\partial A_i} = 0 \\ \dfrac{\partial \varphi}{\partial B_i} = 0 \end{cases} \tag{24}$$

A set of solutions to φ can be derived by Eq. (24), denoted by the vector as $\alpha = (A_1, A_2, \cdots, A_n)$. Similarly, the probability density function of the failure attenuation function is obtained by the least square's method, and finally the probability density function of the failure of the weaponry is obtained. The number of malfunctions of the weaponry working in the time interval $[t_a, t_b]$ is derived from Eq. (25).

$$Con_v = \int_{t_a}^{t_b} g(t)dt \tag{25}$$

3.4 Assessment Model of the Contribution Rate of the Weapon Equipment System

The close degree of proximity of weaponry $S(q_i)$ and the number of failures Con_v are derived from Sects. 3.2 and 3.3, respectively. Assuming that the cost of breakdown repair λ_i is proportional to the cost of weaponry δ_i (RMB).

$$\chi_i = \lambda_i \times \delta_i \times con_v \tag{26}$$

where χ_i is the cost of losses due to possible malfunction of weaponry during the mission. Since the costs are economic, the smaller the better, the contribution of weaponry is:

$$C_i = \beta \frac{S(q_i)}{\sum_{j=1}^{n} S(q_i)} + (1-\beta)\left(1 - \frac{\chi_i}{\sum_{j=1}^{n} \chi_i}\right) \tag{27}$$

where C_i represents the degree of contribution of weapon q_i to the completion of a mission in the IES. Since $S(q_i)$ and χ_i are different in the outline of indicators, β is used to adjust the corrections to ensure accuracy. Finally, it is normalized.

4 Case Analysis

4.1 Obtain the Weight of Combat Indicators

The system of the combat capability indicators for weaponry, shown in Fig. 3, contains three first-level combat capability indicators, namely, detection capability, decision capability and attack capability. The experts use the 1–9 scale method to evaluate the three first-level combat capability indicators. In addition, we can get the judgment matrix B by them.

$$\boldsymbol{B} = \begin{bmatrix} 1 & 3 & 1/5 \\ 1/3 & 1 & 1/7 \\ 5 & 7 & 1 \end{bmatrix}$$

Eight experts in the relevant fields were invited to rate the relative importance of the three first-level combat capability indicators to obtain the evaluation matrix C.

$$\boldsymbol{C} = \begin{bmatrix} \text{expert1} & 9.0 & 9.3 & 7.4 \\ \text{expert2} & 8.2 & 8.6 & 8.4 \\ \text{expert3} & 9.8 & 9.0 & 6.6 \\ \text{expert4} & 8.4 & 7.5 & 5.5 \\ \text{expert5} & 8.6 & 8.1 & 9.4 \\ \text{expert6} & 7.8 & 9.2 & 6.6 \\ \text{expert7} & 9.2 & 7.5 & 7.5 \\ \text{expert8} & 8.3 & 8.5 & 9.0 \end{bmatrix}$$

The Gini Coefficient Solution Weight

The Gini coefficients of the first-level combat capability are calculated from Eqs. (4) and (5) and, which are shown in Table 2.

Table 2. Gini coefficient weight assignment

First level combat capability indicator	Detective capability	Decision capability	Attack capability
Expectation value λ_k	8.6625	8.4625	7.5500
Gini coefficient G_k	0.0768	0.0812	0.1854
Weight W_k''	0.236	0.2365	0.5399

Linear Weighting to obtain the Combat Indicator Weights

Firstly, using the evaluation matrix B can derive the entropy weight [12] of combat capability indicators. Next, the hierarchical analysis weights [14] is derived by determining the matrix C. Finally, the linear weighting method is used to derive the weight of the first-level combat capability indicator, as shown in Table 3.

Table 3. Weight distribution table

Methods	Detective capability	Decision capability	Attack capability
The entropy weight	0.1209	0.1648	0.7143
The Gini weight	0.236	0.2365	0.5399
AHP	0.1884	0.0810	0.7306
Linear weighting	0.1818	0.1608	0.6574

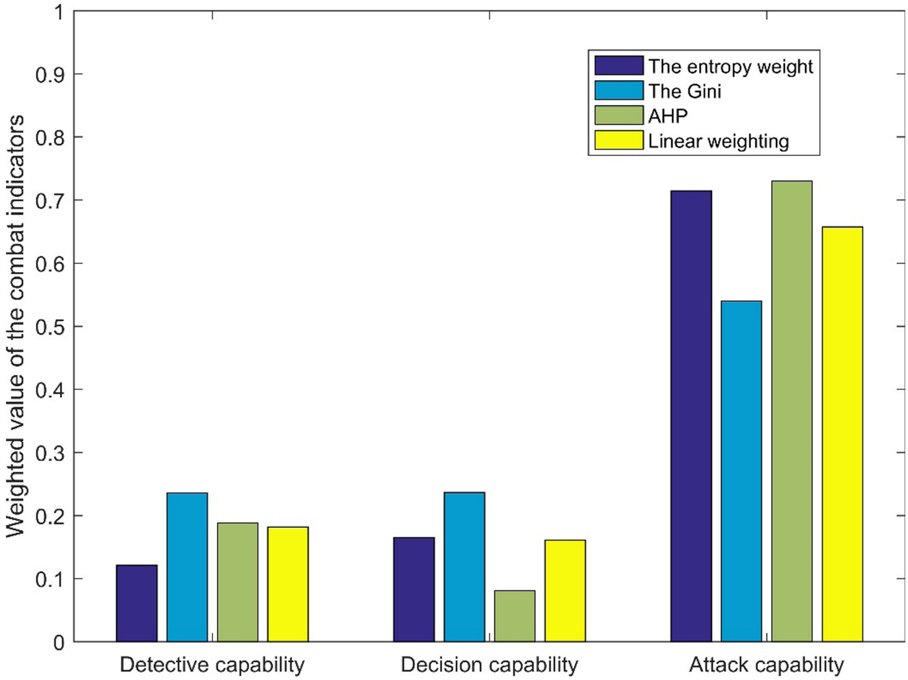

Fig. 5. Combat capability index weight

As shown in Fig. 5, it is unreasonable to obtain the weight of the combat capability indicator by AHP, ignoring the objectivity of the evaluation information. Linear weighting finds a balance between subjectivity and objectivity. The simulation graph shows that the linear weightings are always between subjectivity (AHP) and objectivity (the entropy weight and the Gini coefficient). This can prove linear weighting consider both subjectivity and objectivity. Moreover, it has high reliability. Given the weights of the secondary combat indicators, the combined weights are derived by multiplying them with the first-level combat capability indicators, as Table 4 shows.

Table 4. Table of the combat capability indicators

The first-level combat capability indicator	Weight	The second-level combat capability indicator	Weight	Combined weight
Detective capability	0.1608	Access to information	0.3	0.04824
		Information processing capability	0.5	0.0804
		Information transmission capability	0.2	0.03216
Decisive capability	0.6574	Correctness of the decision	0.35	0.23009
		Command reliability	0.25	0.16435
		Command information delay	0.4	0.26296
Attack capability	0.1818	Fire damage capability	0.55	0.09999
		Electronic interference capability	0.2	0.03636
		Precision attack capability	0.25	0.04545

4.2 Finding the Closeness of Vague Sets

Due to space limitations, this paper calculates the level of combat capability satisfaction in terms of the combat capability, as shown in Fig. 6.

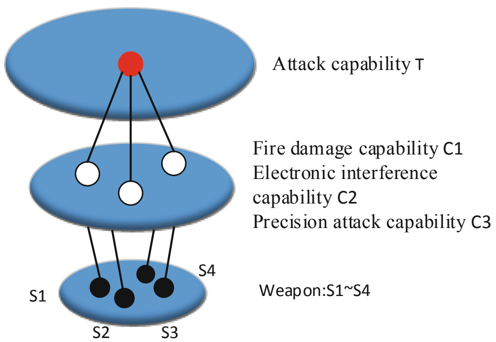

Fig. 6. Topological relationship of attack capability

In Fig. 6, T represents attack capability, C1, C2, C2 respectively represent different combat capability indicators, and S1, S2, S3, and S4 represent different weaponry.

The expert evaluates the satisfaction of the equipment in completing its mission based on experience. Looking at the basic information of the attack weaponry's status, we can get the cumulative working time (100 min) of the weapon and the amount of money spent on purchasing the weapon (millions of dollars). Table 5 shows the evaluation information and basic status of the attack weaponry.

Table 5. Table of evaluative information and basic status of weaponries

Weaponry	Attack capability			*Time*	*Cost*
	C1	C2	C3		
S1	70–75	81–90	84	5–6	2
S2	85	60–75	88	15–16	3
S3	84–92	82–87	82–90	25–16	4
S4	85–90	83–85	86–94	50–51	5

The expert evaluation is ambiguous and therefore supports interval scoring to better reflect and express this ambiguity. In this paper, using vague sets and TOPSIS method, it is possible to deal with evaluation values as single and interval values. The value of the fuzzy decision matrix M is obtained by Eqs. (6), (7), (9) and the result is shown in Table 6.

Table 6. Vague value of combat equipment

Weaponry	Attack capability		
	C1	C2	C3
S1	70–75	81–90	84
S2	85	60–75	88
S3	84–92	82–87	82–90
S4	85–90	83–85	86–94

From Table 4, it can be seen that the weight vector for the secondary combat capability indicator is $W = (0.35,0.25,0.4)$. The *VPIS* and *VNIS* are derived by Eqs. (12), (13). The results are *VPIS* = ([0.9723, 1.0], [0.9639, 1.0], [0.9503, 0.9757]) and *VNIS* = ([0.6285, 0.6358], [0.5556, 0.6181], [0.7986, 0.7986]).

Calculation of the distance between the striking equipment and the positive and negative ideals by Eqs. (18), (19), (20). Then according to the Eq. (21) to calculate the close degree of proximity of the weaponry $S(q_i)$, the results are shown in Table 7.

Table 7. Similarity and closeness of combat equipment

Weaponry	D_i^-	D_i^+	$S(q_i)$
S1	0.9490	0.9008	0.4740
S2	0.9446	0.9040	0.4780
S3	0.8701	0.9766	0.5575
S4	0.8679	0.9776	0.5592

Table 7 shows that S4 > S3 > S2 > S1 when damage to equipment is not taken into account. From the table you can get the best results for weaponry S4. Nevertheless, that is clearly, not how we measure the combat contribution of IES. Under resource-constrained conditions, we should take a comprehensive view of the problem, consider equipment failures, and choose the optimal strategy. In a war, the cost of equipment malfunction should be a secondary, but necessary. Number of malfunctions of the weaponry can be calculated by formula (25). Then Eq. (26) takes the cost of weaponry and finally the combined contribution is obtained through (27). When $\beta = 0.9$, the data results are shown in Table 8. The number of failures is *Con*, and the maintenance cost is *Cos* (10,000 Yuan).

Table 8. Consider the comprehensive contribution of the malfunction

Weaponry	*Cos*	*Con*	Ranking of $S(q_i)$	The comprehensive contribution
S1	0.9490	0.9008	0.4740	0.2255
S2	0.9446	0.9040	0.4780	0.2369
S3	0.8701	0.9766	0.5575	0.2661
S4	0.8679	0.9776	0.5592	0.2715

From the results in Table 8, it can be seen that the number of failures decreases over time as reliability grows, taking into account changes in the state of the technology. When $\beta = 0.90$, the overall contribution rate is ranked as S4 > S3 > S2 > S1. It is consistent with the ranking of the contribution of the weaponry to IES when failure is not considered.

Considering the effect of different values of β, assuming that the four strike equipment have equal cumulative working hours, and work in the same time period, and make *Con* = 1, so that the weaponry S1, S2, S3, and S4 have the battle losses of 20,000, 30,000, 40,000, and 50,000, respectively.

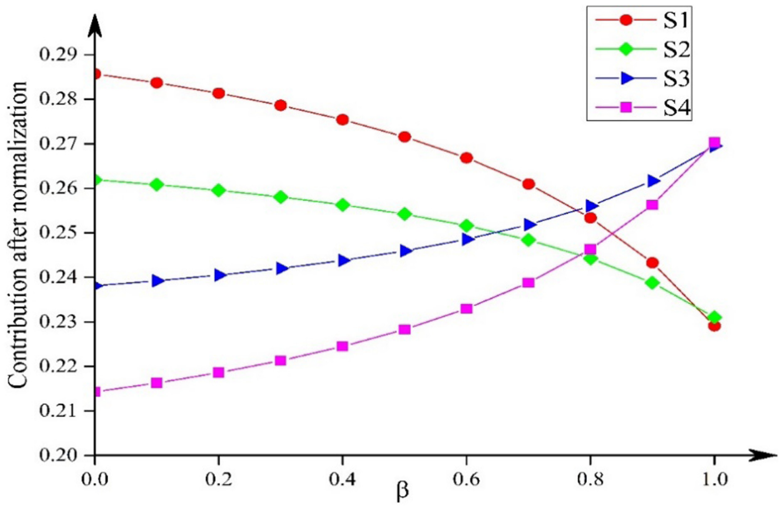

Fig. 7. Topological relationship of strike capability

As shown in Fig. 7, when the number of failures is constant, the contribution of the weaponry to the overall changes with the change of the β value. When $\beta = 0$, S1 has the best effect. As the β value rises, the impact of cost becomes smaller and smaller, and more attention is paid to the degree of weaponry completing the mission. When $\beta = 1$, it is equivalent not to considering the influence caused by the number of failures. It is only related to the closeness of the weaponry, which is consistent with the $S(q_i)$ normalized results in Table 7. As the closeness coefficients of weaponry S1 and S2 are relatively low, with the increase of β value, the impact of cost becomes weaker and weaker, its contribution to the entire system is also lower and lower, and the curve shows a downward trend. On the contrary, S3 and S4 show an upward trend.

5 Conclusion

As a complex system, the integrated equipment system has many uncertainties, and the expert assessment of IES is somewhat subjective. This article makes full use of the evaluation information and uses the combination of objective and subjective methods to comprehensively obtain the weight of combat indicators. The literature does not consider the impact of new and recurring failures on the equipment system when considering the contribution of equipment system effectiveness. Aiming at the problem of equipment body failure, this paper introduces new failures and recurring failures to measure the number of failures. Viewing that the expert evaluation score is vague, it has three forms: support, opposition, and abstention. The TOPSIS method and the vague set express these three forms. In view of this, this paper considers the contribution of IES using a combination of vague and TOPSIS when considering equipment failures. It provides new ideas for evaluating IES. Moreover, Simulation shows that the algorithm in this paper can be used to find the balance point between loss cost and combat capability, and provide a theoretical basis for decision makers to select suitable weapons and equipment and accelerate equipment development.

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