





# DOA Estimation Based on Intelligent FMCW Radar with Triangle Array Antenna

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**Abstract.** In this paper, we propose a target Direction of Arrival (DOA) estimation algorithm through the triangular array antenna to improve the perception accuracy of intelligent vehicular millimeter-wave radar. Firstly, we utilize Total Least Square- Estimation of Signal Parameters to obtain the solution set of target azimuth with the geometrical advantages of the triangular array. Subsequently, the preliminary optimal target azimuth can be derived based on the power spectrum function of the improved Multiple Signal Classification algorithm, which is used to estimate the DOA solution set. Finally, we could estimate the optimal azimuth parameters accurately through searching the preliminary optimal target azimuth. The proposed algorithm avoids a wide range of power spectrum search, and reduces the error of azimuth estimation. Compared with both TLS-ESPRIT and MMUSIC algorithm, the spatial resolution is further improved. The evaluation results indicate that the proposed algorithm reduces the root mean square error and the computational complexity, and meanwhile improves the target azimuth estimation accuracy.

**Keywords:** Millimeter wave radar · Array antenna · Array signal processing · Spatial resolution · DOA estimation

## 1 Introduction

The radar should detect the position of the target in front and behind the car and the adjacent lane when the car starts to change lane. Currently, the radar detection methods which applied in car collision prevention mainly include millimeter wave, laser, ultrasonic, infrared, etc., among which millimeter wave radar has the best performance. Frequency modulation Continuous Wave (FMCW) radar is used to measure the distance and speed of the target, it shows the better performance under complex weather conditions, such as fog. At present, it has been used in adaptive cruise, lane change assistance, blind spot detection and forward collision warning system [1, 2].

In some vehicle applications, FMCW radar is required to detect the azimuth of the target. At present, DOA estimation algorithms [3, 4] are divided into two part which include namely traditional methods and subspace methods. Bartlett and capon are part of the method of transmission. The traditional method needs to determine the DOA by searching the peaks in the power spectrum. This method depends on the physical size of the array antenna aperture, which will cause the radar signal resolution to be reduced. Therefore, the high-resolution subspace method is widely used to determine the DOA. Such as the Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotation Invariance Techniques (ESPRIT), they provide a more accurate signal estimation direction, independent of the physical size of the array antenna aperture.

This paper proposes a low complexity DOA estimation algorithm based on triangular array antenna. The algorithm can be applied to the intelligent car millimeter wave radar which can identify the multi-target in front of the smart car, the complexity of the algorithm is reduced while improving the spatial resolution of the target azimuth, the angle of estimation is also more precise, and overcomes the problem of detecting data storage and large computation. The contributions of this paper are as follows:

1. We proposed a DOA estimation algorithm based on triangle array antenna using the geometric advantage of triangle array and the total least square rotation vector invariance technology, the complexity of the algorithm is reduced, and the azimuth of the target can be determined quickly.
2. Improved ESPRIT algorithm is proposed in this paper, which could increase the accuracy of the algorithm in the case of low SNR.

## 2 Related Work

Currently, MUSIC and ESPRIT are two classical array signal processing algorithms. Schmidt republished the Multiple Signal Classification (MUSIC) algorithm that he proposed in 1979 in 1986 [5]. The covariance matrix is solved by MUSIC, and then the Eigen-decomposition is carried out to obtain the noise subspace orthogonal to the signal component and the signal subspace corresponding to the signal component. The orthogonality of the two Spaces is used to estimate DOA parameters. Since MUSIC algorithm [6] needs to conduct extensive search of power spectrum, it is more complex and requires higher data storage requirements than ESPRIT algorithm [7]. Roy et al. proposed a rotation-invariant technical algorithm for estimating signal parameters in 1986 [8]. ESPRIT does not need to know the exact information of array manifold guide vector, so it does not need array calibration. Through the rotation invariance of matrix, ESPRIT can overcome the problem of large data storage and calculation.

ESPRIT algorithm estimates DOA parameters differently from MUSIC. ESPRIT algorithm uses the rotation invariance between two uniform subarrays to solve the invariant matrix and estimate the DOA parameters of the target. There are several improved versions of ESPRIT algorithm, such as least-squares ESPRIT (LS-ESPRIT) and overall least-squares ESPRIT (TLS-ESPRIT) [9]. TLS-ESPRIT algorithm is improved on the basis of ESPRIT algorithm. At low SNR TLS-ESPRIT algorithm has

better effect, but the angular resolution is lower than the modified MUSIC (MMUSIC) algorithm. MMUSIC algorithm [10, 11] is an improvement of MUSIC algorithm, which reduces the coherence between signals and can estimate the azimuth Angle of coherent and non-coherent signals, improving the angular spatial resolution, but the computing speed is far lower than TLS-ESPRIT.

Due to the high computational load of the 2D-esprit [12] and 2D-music [13] algorithms, the target azimuth cannot be measured in real time in real scenes. The 2D-music algorithm has been improved in literature [14]. Although the complexity is reduced, the computation is still relatively high compared with ESPRIT.

The array antenna includes linear array antenna, circular array antenna, L array antenna, etc., among which the detection range of linear array antenna is front ( $0^\circ$ – $180^\circ$ ). It's the maximum of the beam gain for the target in the vertical direction of the array antenna. As the beam gain decreases to both sides, the radar detection range will be limited; The detection range of L array antenna is not only the azimuth information of the horizontal plane, but also the height parameters of the target, but it is limited by the detection range like linear array antenna; The circular array antenna detects the surrounding  $360^\circ$  environment, which is not limited by the detection range, but the algorithm is complex.

Considering the position of the FMCW smart car radar which placed on the car, and should detect the target in the forward ( $0^\circ$ – $180^\circ$ ) range, considering that the parameters of one-dimensional plane can meet the requirements. Based on the line array antenna, this paper adopts the triangular array antenna that overcomes the disadvantage of decreasing beam gain to both sides in the detection range, and enlarges the detection range of radar. Because the MMUSIC algorithm needs a wide range of power spectrum search, it has the disadvantage of large computation and low spatial resolution of TLS-ESPRIT algorithm, in order to accurately detect the azimuth between the intelligent car millimeter wave radar and the target, overcome the shortcomings of the TLS-ESPRIT algorithm and MMUSIC algorithm.

### 3 Triangular Array Signal Model

In this paper, we adopt the method of sending more than receiving. Mm-wave radar uses FM continuous wave radar signal, in which waveform is serrated wave, the cycle is 10 ms, the radar waveform has 24 GHz, and the FM bandwidth is 250 MHz. The triangle array is composed of M-elements evenly, in which the spacing of elements is  $d = 1/2 * \lambda$  (lambda for wavelength). The noise and signal received by the array elements are independent of each other.

The time-domain emission signal of FMCW MMW radar is expressed as follow:

$$P(t) = A \cos \left\{ 2\pi \left[ \left( f_0 - \frac{B}{2} \right) t + \frac{1}{2} \mu t^2 \right] + \varphi_P \right\} \quad (1)$$

And  $\mu = B/T$  is the modulation slope,  $f_0 = 24$  GHz,  $B = 250$  MHz,  $\varphi_0$  is the initial phase,  $T = 10$  ms is the period of the modulation waveform, and  $A$  is the amplitude of the signal.

The target moves uniformly with respect to the radar, so  $R_1$  is the relative distance between the target and MMW radar, the speed is  $V$  and  $c$  is the speed of light, the wavelength is  $\lambda = c/f_0$ , and the Doppler frequency is  $f_d = 2v/\lambda$ . The relationship between the distance of the target relative to MMW radar and time is as follows:  $R(t) = R_1 - Vt$ . The delay formula of the signal received by the radar antenna relative to the transmitted signal is as follow:

$$\tau(t) = \frac{2R(t)}{c} \quad (2)$$

The signal expression reflected back by the radar transmit signal is:

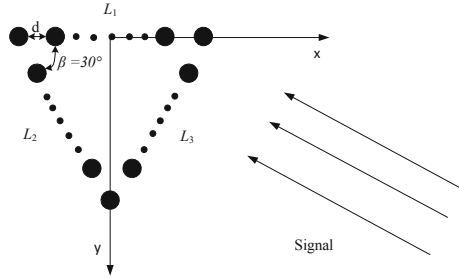
$$Y(t) = A_r \cos \left\{ 2p \left[ \left( f_0 - \frac{B}{2} \right) (t - t(t)) + \frac{1}{2} m (t - t(t))^2 \right] + j_Y \right\} \quad (3)$$

Beat signal could be obtained by means of mixing frequency both the radar's reflected and transmit signals:

$$S(t) = P(t) * Y(t) \quad (4)$$

The signal expression on the k-receiving antenna array element is derived by Eq. (5) as:

$$S_k(t) = \frac{1}{2} K_r A^2 \cos \left[ 2\pi \left( \frac{2BR(t)}{TC} - \frac{2V}{\lambda} \right) t + \varphi_0 \right] \quad (5)$$



**Fig. 1.** Triangle array model

Assuming that the DOA estimated by the  $L_1$  array is the direction of the target relative to the antenna, according to the geometric characteristics of the triangle, the  $Y$ -axis represents the direction of the smart car's motion, the  $X$ -axis represents the  $L_1$  array, and sets the midpoint as the origin, establishes a right-angle coordinate system, and the elements are arranged in a triangular state, as shown in Fig. 1.

For  $L_1$  array antenna, the assumed signal scenario has  $P$  ( $P < L_1$ ) FMCW MMW radar signal sources which are located in the far field, that is to say, the signal is a plane wave before it reaches the array element, and where  $\theta_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  is the azimuth Angle of reaching  $L_1$  element ( $i = 1, 2 \dots P$ ). If the first element of  $L_1$  array is set as the reference element, then the KTH snapshot output signal of  $L_1$  array is:

$$X(k) = AS(k) + N(k) \quad k = 1, 2 \dots Q \quad (6)$$

Where  $X(k) = [x_1(k), x_2(k), \dots, x_{L_1}(k)]^T$  is the output vector of array elements, and  $A$  is the array direction matrix,  $A = [a(\theta_1) \ a(\theta_2) \ \dots \ a(\theta_P)]$ . The  $k$ th snapshot array stream is:

$$a(\theta_k) = \left[ 1, e^{-\frac{j2\pi d \sin \theta_k}{\lambda}}, \dots, e^{-\frac{j2\pi d(L_1-1) \sin \theta_k}{\lambda}} \right]^T \quad (7)$$

$S(k) = [s_1(k) \ s_2(k) \ \dots \ s_P(k)]^T$  denotes the vector of incoming wave signals, and  $N(k) = [n_1(k) \ n_2(k) \ \dots \ n_{L_1}(k)]^T$  denotes the vector of noise signals, where  $n_i(k)$  represents the white noise which mean is zero and variance value is  $\sigma^2$ . Similarly, the signal model of  $L_2$  and  $L_3$  array antennas can be established.

The estimated DOA parameters of  $L_2$  and  $L_3$  array antennas need to be converted into the angle of relative radar direction. It is necessary to convert the corresponding DOA into DOA in the relative direction of MMW radar through formula (8). The direction of  $L_1$  antenna is the same as that of radar.  $\theta_{Vehicle}$  is the optimal DOA finally estimated by the algorithm in this paper.

$$\theta_{Vehicle} = \begin{cases} \theta_i + \beta, & \text{if } \theta_i \text{ comes from antenna 2} \\ \theta_i - \beta, & \text{if } \theta_i \text{ comes from antenna 3} \\ \theta_i, & \text{if } \theta_i \text{ comes from antenna 1} \end{cases} \quad (8)$$

The geometric advantage of triangle is used to increase the wide Angle of radar detection target. Since the triangular array antenna is composed of three groups of antennas, the target reflection wave is a vertical antenna, in other word, it coincides with the middle perpendicular line of the antenna, and the beam gain is the strongest. The further away from the mid-perpendicular the smaller the beam gain. Therefore, triangular array antenna is better than linear array antenna when radar detects the same range. The triangle array antenna receives the incoming wave signal reflected by the target, and TEMM algorithm processes three groups of incoming wave signals to find the optimal Angle of the target.

## 4 DOA Estimation of Triangular Array Antenna Based on TEMM Algorithm

In this paper, the triangular array antenna model is adopted to receive the radar signal reflected by the target, which increases the transverse range of the detected target and improves the beamforming gain of the target. TLS-ESPRIT algorithm and MMUSIC algorithm combined with the triangle array antenna proposed in this paper to estimate DOA, referred to as TEMM (TLS-ESPRIT-MMUSIC) algorithm. Compared with MMUSIC algorithm and 2D-MUSIC algorithm, EMM algorithm reduces the complex spectrum search range, obviously reduces the computational complexity of the algorithm, and solves the storage problem of detection data. Compared with TLS-ESPRIT algorithm and ESPRIT algorithm, the power spectrum function is utilized. The near-optimal solution is searched to find the final optimal solution of the target, which improves the spatial resolution of the target. By using the advantages of the triangular array antenna, the target estimation accuracy with relatively large relative azimuth of the radar can be improved.

### 4.1 TEMM Algorithm Estimation in Triangular Array

First, TEMM algorithm uses triangle array antenna to obtain incoming signals reflected by three groups of targets, then the rotation invariance between matrices in TLS-ESPRIT algorithm is used to solve the target azimuth. Due to the coherence between the signals, the influence of noise  $\Delta U_X$  and  $\Delta U_Y$  is reduced by using the idea of global least squares, and the DOA solution set corresponding to the target is finally solved.

TLS-ESPRIT algorithm assumes that there are two identical matrices, the front ( $M-2$ ) element of  $L_1$  array is set as  $X$  submatrix, and the rear ( $M-2$ ) element is set as  $Y$  submatrix. The formula is shown below:

$$\begin{aligned} X(k) &= [x_1(k) \quad x_2(k) \dots x_{L_1-2}(k)]^T \\ &= [a(\theta_1) \dots a(\theta_{L_1-2})]S(k) + N_X(k) \\ &= A_X S(k) + N_X(k) \end{aligned} \quad (9)$$

$$\begin{aligned} Y(k) &= [x_2(k) \quad x_3(k) \dots x_{L_1-1}(k)]^T \\ &= [a(\theta_2)e^{j\phi_1} \dots a(\theta_{L_1-1})e^{j\phi_p}]S(k) + N_Y(k) \\ &= A_X \Phi S(k) + N_Y(k) \end{aligned} \quad (10)$$

Let  $A_X = A$ ,  $A_Y = A\Phi$ , so there's  $A_Y = A_X\Phi$ , and  $\Phi = \text{diag}[e^{-j\frac{w_0 \sin \theta_1}{c}}, \dots, e^{-j\frac{w_0 \sin \theta_p}{c}}]$ . The model of the two submatrices is combined as follows:

$$X = \begin{bmatrix} X_X \\ X_Y \end{bmatrix} = \begin{bmatrix} A_X \\ A_X \Phi \end{bmatrix} S + N = \bar{A}S + N \quad (11)$$

The formula of  $X$ 's covariance matrix  $R$  is shown as follows:

$$R = E[X(k)X^H(k)] = AR_S A^H + R_N \quad (12)$$

The Eigen decomposition formula of covariance matrix is shown below:

$$R = U_S \Lambda_S U_S^H + U_N \Lambda_N U_N^H \quad (13)$$

$U_S$  represents the signal subspace of covariance matrix  $R$ , and  $U_N$  represents the noise subspace of covariance matrix  $R$ . At this point,  $\text{span}\{U_S\} = \text{span}(A(\theta))$ , and there is a unique non-singular matrix  $T$ , which decomposes the signal subspace  $U_S$  into a submatrix, so that:

$$U_S = \begin{bmatrix} U_X \\ U_Y \end{bmatrix} = \begin{bmatrix} A_X T \\ A_X \Phi T \end{bmatrix} \quad (14)$$

Define  $U_X \varphi = U_Y$  and substitute it into formula (14) to obtain:

$$\varphi = T^{-1} \Phi T \quad (15)$$

Where  $\Phi$  is the eigenvalue of  $\varphi$ , only  $\varphi$  is required, and DOA of the target can be solved through formula (16).

$$\varphi_k = (2\pi d \sin \theta_k) / \lambda \quad (16)$$

Considering that  $U_X$  and  $U_Y$  are affected by noise, in order to minimize the 2-Norm of noise  $\Delta U_X$  and  $\Delta U_Y$ ,  $U_X$  and  $U_Y$  are corrected, which is the idea of using the global least square method (TLS) and obtained from  $U_X \varphi = U_Y$ :

$$(U_X + \Delta U_X) \varphi = U_Y + \Delta U_Y \quad (17)$$

From Eq. (17), it can be concluded that:

$$\begin{cases} \min \|\Delta U\|^2 \\ (U + \Delta U)z = 0 \end{cases} \quad (18)$$

Among them  $z = \begin{bmatrix} 1 \\ \varphi \end{bmatrix}$ ,  $U = [-U_Y \ U_X]$ ,  $\Delta U = [-\Delta U_Y \ \Delta U_X]$ . Let  $U_S F = 0$  be established, where  $F$  is the unitary matrix, making  $F$  and  $U_S$  orthogonal. According to formula (14),  $U_S = [U_X \ U_Y]^T$ , it is eigen decomposed:

$$U_S^H U_S = E \Sigma E^H \quad (19)$$

The eigenvector matrix  $E$  is obtained:

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \quad (20)$$

$[E_{12} \ E_{22}]^T$  is the eigenvector matrix with zero eigenvalue, at same time it is the noise subspace.  $U_S F = 0$  holds As long as we set  $F = [E_{12} \ E_{22}]^T$ . So we can get:

$$ATF_1 + A\Phi TF_2 = 0 \quad (21)$$

Where  $F = [F_1 \ F_2]^T$ .  $\varphi = -F_1 F_2^{-1}$  and put it into formula (21):

$$T^{-1}\Phi T = \varphi \quad (22)$$

Formula (22) is the same as formula (15), and the azimuth of the target can be obtained.  $\varphi_{TLS}$  can be obtained by formula (18):

$$\varphi_{TLS} = -E_{12}E_{22}^{-1} \quad (23)$$

The direction of signals received by  $L_1$  array antenna can be obtained by TLS-ESPRIT algorithm. Similarly, we can also obtain the azimuth angle of signals received by  $L_2$  and  $L_3$  array antenna respectively. It is obtained what the solution set  $(a_i^{L1}, a_i^{L2}, c_i^{L3}), i = 1, 2, \dots, p$  of three sets of azimuth angles corresponding to the target, and then the initial optimal DOA is obtained from the solution set by TEMM algorithm.

## 4.2 TEMM Algorithm Obtains Preliminary Optimal DOA

MUSIC algorithm is suitable for high resolution spectrum estimation, but its disadvantage is that it requires eigenvalue decomposition and complex spectral search process. Therefore, compared with TLS-ESPRIT algorithm, its calculation time is long and the calculation speed is slow. MMUSIC algorithm is improved on the basis of MUSIC algorithm and has the same characteristics. Through the power spectrum function of MMUSIC algorithm, the azimuth Angle of the maximum signal power is estimated from three groups of DOA. Because it has an effect on the accuracy of DOA estimation by the geometric position of subarray, the estimation error of Angle is different with different subarray. Through TEMM algorithm to estimate the wave direction of  $P$  signals, the DOA solution set  $Z$  is obtained as follows:

$$Z_i = (a_i^{L1}, a_i^{L2}, c_i^{L3}) \quad i = 1, 2, \dots, P \quad (24)$$

Before the power spectrum function of MMUSIC algorithm estimates the azimuth angles of the three groups, the three groups of DOA sets are respectively sorted in ascending order. The radar waveform reflected has three estimated azimuth angles by each target, and the power spectrum of azimuth Angle of the  $i$ -th target can be estimated through formula (25).

$$G(i) = M(Z_i) \quad (25)$$

$M$  is the classical MUSIC power spectrum search function, and its expression is:

$$M(\theta) = \frac{1}{a^H(\theta)E_n E_n^H a(\theta)} \quad (26)$$

By calculating the Angle power spectrum of each target pair, the Angle corresponding to the largest power spectrum is selected as the initial optimal solution of the target. On the basis of the initial optimal solution, TEMM algorithm is used to search near the initial optimal solution, it will calculate the final optimal DOA.

### 4.3 TEMM Algorithm Obtains the Optimal DOA

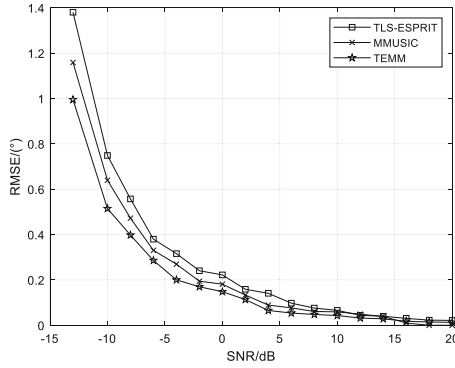
It is known from literature [4] that the conjugate reconstruction of the signal matrix  $X(k)$  output by the array antenna produces a new matrix  $Y(k)$ , and the covariance of which is respectively  $R_X$  and  $R_Y$ . It is obviously that  $R_X$ ,  $R_Y$  and  $R_Z$  have the same noise subspace for  $R_Z = R_X + R_Y$ . We can obtain the target azimuth angle  $\theta_i$  by substituting the noise subspace into Eq. (26) which is obtained by Eigen decomposition of covariance matrix  $R_Z$ . We define  $\theta_n=1^\circ$ , so the search range is  $(\theta_i - \theta_n, \theta_i + \theta_n)$ , and the precise target azimuth is obtained by searching near azimuth  $\theta_i$  through MMUSIC. Due to the different positions of the three groups of triangular array antennas, only the Angle conversion of  $L_2$  and  $L_3$  antennas can be used to realize the true orientation of the target. Through formula (8), the final optimal DOA of the target's relative millimeter wave radar is obtained.

DOA estimation algorithm for MMW radar based on triangular array is as follows:

- 1. The target reflected signal received through the triangular array, the signals  $S_1(t)$ ,  $S_2(t)$  and  $S_3(t)$  can be obtained from  $L_1$ ,  $L_2$  and  $L_3$  respectively;
- 2. Calculate submatrices  $X$  and  $Y$  according to Eq. (9) and (10) and combine submatrices  $X$  and  $Y$  to estimate covariance matrix  $R$  according to Eq. (12);
- 3. Get  $U_S$  matrix by Eigen decomposition of  $R$  according to Eq. (14);
- 4. The  $U_S$  matrix is Eigen decomposed to get matrix  $E$ , and  $E$  is decomposed;
- 5.  $\varphi_{TLS}$  is obtained from formula (22), and  $\varphi_{TLS}$  is eigen decomposed to obtain the solution of  $L_1$  antenna
- 6. Go to step 2, similarly, we can obtain the solutions of  $L_2$  and  $L_3$  antennas, and DOA solution set  $Z_i = (a_i^{L1}, a_i^{L2}, c_i^{L3})$  can be obtained too;
- 7. The three groups of DOA are arranged in ascending order;
- 8. Through formula (25), the maximum power general of the target is solved, and the corresponding angle is set as the initial optimal solution  $\theta_i$ ;
- 9. The final optimal DOA is searched within  $(\theta_i - \theta_n, \theta_i + \theta_n)$  range by the power spectrum function of TEMM algorithm.

## 5 Simulation and Analysis

Simulations are presented to verify the efficacy of the TEMM algorithm which uses geometric features of triangular array antenna to increase the target positioning range ( $-75^\circ$ – $75^\circ$ ). At the same time, the estimation accuracy of the target azimuth Angle is improved, and the calculation amount of MMUSIC algorithm is reduced under the same accuracy of DOA estimation. In DOA estimation of intelligent vehicle-mounted millimeter-wave radar, signal-to-noise ratio affects the accuracy of the estimated signal DOA, and the sampled fast beats cause errors to the estimation of DOA, and the estimated DOA sizes are different with different accuracy. The Root Mean Square Error (RMSE) is used to evaluate the positioning performance of TEMM algorithm under different SNR, azimuth and fast beats, and the spatial resolution of TEMM algorithm is analyzed with beamforming diagram. MATLAB simulation is conducted to verify the validity of TEMM algorithm in estimation accuracy and resolution.



**Fig. 2.** Performance comparison under different SNR

Case I: The array element of the triangular array was set as  $M = 26$  ( $L_1 = 12$ ,  $L_2 = 7$ ,  $L_3 = 7$ ) and the array element of the linear array antenna was set as 12, the array element spacing was set as half of the wavelength, the target azimuth Angle was set as  $50^\circ$ , 1024 times of fast beats were sampled, and 300 times of Monte Carlo experiments were conducted with the algorithm in this paper and the TLS-ESPRIT algorithm under different SNR. The simulation results were shown in Fig. 2, the three curves respectively showed the change of the root mean square error (RMSE) of DOA with the signal to noise ratio (SNR). With the increase of SNR, the RMS error became smaller and smaller, and the DOA estimation accuracy became higher and higher. When the signal-to-noise ratio was  $-13$  dB, the estimation errors of the three algorithms differ greatly. With the change of SNR from 20 dB–13 dB, it was clear that TEMM algorithm had higher DOA estimation accuracy than TLS-ESPRIT algorithm and MMUSIC algorithm.

Case II: The array element of the triangular array was set as  $M = 26$  ( $L_1 = 12$ ,  $L_2 = 7$ ,  $L_3 = 7$ ) and the array element of the linear array antenna was set as 12, the

array element spacing was set as half of the wavelength, the signal-to-noise ratio was set as 8 dB, and 1024 times of fast beats were sampled. Under different target azimuth angles, the algorithm in this paper and the TLS-ESPRIT algorithm were respectively used to conduct 300 times of Monte Carlo experiments. The simulation results were shown in Fig. 3. The two curves respectively showed with the Angle of the target the variation of the RMS error (RMSE) of DOA. As the target azimuth increases, the RMS error became larger and larger, and the DOA estimation accuracy became lower and lower. When the target azimuth Angle was  $80^\circ$ , it was obviously different in the estimation error of the two algorithms. It could be seen from the figure that the DOA estimation accuracy of TEMM algorithm was higher than that of TLS-ESPRIT algorithm with the increase of target azimuth Angle.

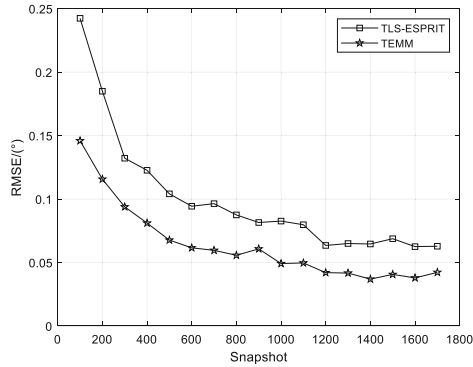


Fig. 3. Comparison of performance at different azimuths

Case III: The array elements of the triangular array were set to  $M = 26$  ( $L_1 = 12$ ,  $L_2 = 7$ ,  $L_3 = 7$ ) and the array element of the linear array antenna was set to 12, the array element spacing was set to half of the wavelength, and the target azimuth angle was set to  $50^\circ$ . The signal-to-noise ratio was 8 dB. Under the different sampling snapshots, 300 Monte Carlo experiments were performed using the algorithm and TLS-ESPRIT algorithm respectively. The simulation results were shown in Fig. 4. It could be seen that the azimuth rms error decreases with the increase of the number of sampling snapshots. When the number of snapshots increases to a certain extent, the root mean square error tends to be stable. In the 100–1700 snapshots, it could be seen that the root mean square error of the TEMM algorithm was small and had a high estimation accuracy.

Case IV: The array elements of the triangular array were set to  $M = 26$  ( $L_1 = 12$ ,  $L_2 = 7$ ,  $L_3 = 7$ ) and the array elements of the linear array antenna were set to 12, the array element spacing was set to half of the wavelength, and the target azimuth angle was set to  $60^\circ$ ,  $75^\circ$ , the signal-to-noise ratio was 8 dB, and the sampling snapshot was 1024. Independent experiments were performed using the algorithm and MMUSIC algorithm respectively. The simulation results were shown in Fig. 5.

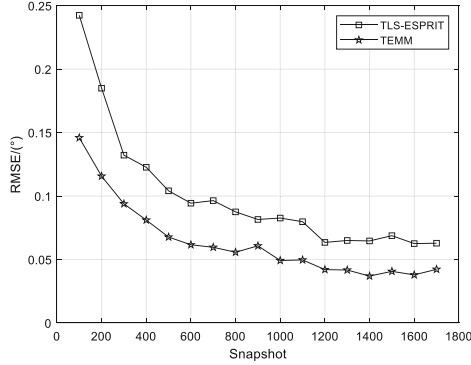


Fig. 4. Effect of different snapshots on root mean square error

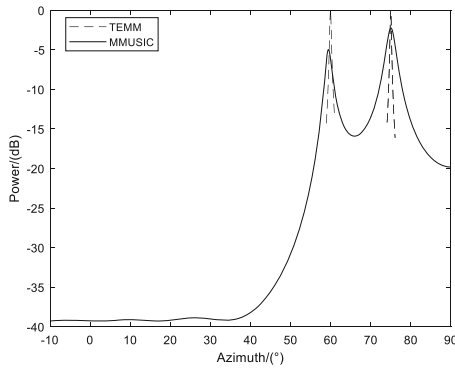


Fig. 5. Spectrum comparison between the algorithm and the MMUSIC algorithm

It could be seen from the Fig. 5 that MMUSIC requires a long time of spectrum search and a large amount of calculation. However, the algorithm in this paper avoids this defect and only needs to search within a short range to get an accurate DOA. Under the same conditions, the algorithm in this paper could more accurately distinguish DOA of adjacent signals, and the spatial resolution was improved. When the Angle reaches 75°, TEMM algorithm had a higher accuracy, and the power spectrum search range was smaller than MMUSIC algorithm.

## 6 Conclusions

In this paper, theoretical analysis and simulation of TEMM algorithm, MMUSIC algorithm and TLS-ESPRIT algorithm are carried out from the aspects of azimuth, signal-to-noise ratio and snapshot number. From the range of estimated azimuth Angle, it is concluded that the root mean square error of TEMM algorithm is smaller and the accuracy is higher. The TEMM algorithm has a higher accuracy when sampling the

same number of snapshots. From the beamforming diagram, the TEMM algorithm has a smaller power spectrum search range than the MMUSIC algorithm, which reduces the amount of computation. However, the target beam estimated by TEMM algorithm is sharper, which is easy to accurately identify the target Angle. In this paper, the improved TEMM algorithm based on TLS-ESPRIT and MMUSIC algorithm can accurately estimate the optimal DOA of the target. Therefore, TEMM algorithm increases the detection range, improves the spatial resolution and reduces the computation, which lays a foundation for the practice of engineering. The mentioned method is mainly aimed at mutually independent signals without considering the influence of coherent signals. However, in practical engineering applications, errors caused by coherent signals cannot be avoided. Therefore, in order to reflect the advantages of TEMM algorithm, coherent signals need to be processed.

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## References

1. Hyun, E., Jin, Y.S., Lee, J.H.: Moving and stationary target detection scheme using coherent integration and subtraction for automotive FMCW radar systems. In: Radar Conference. IEEE (2017)
2. Hyun, E., Jin, Y.S., Lee, J.H.: Design and development of automotive blind spot detection radar system based on ROI pre-processing scheme. *Int. J. Autom. Technol.* **18**(1), 165–177 (2017)
3. Cao, R., et al.: A low-complex one-snapshot DOA estimation algorithm with massive ULA. *IEEE Commun. Lett.* 1–1 (2017)
4. Li, J., Li, D., Jiang, D., et al.: Extended-aperture unitary root MUSIC-based DOA estimation for coprime array. *IEEE Commun. Lett.* **22**(4), 752–755 (2018)
5. Schmidt, R., Schmidt, R.O.: Multiple emitter location and signal parameters estimation. *IEEE Trans. Antennas Propag.* **34**(3), 276–280 (1986)
6. Abdalla, M.M., Abuitbel, M.B., Hassan, M.A.: Performance evaluation of direction of arrival estimation using MUSIC and ESPRIT algorithms for mobile communication systems. In: *Wireless and Mobile Networking Conference*, pp. 1–7. IEEE (2013)
7. Lavate, T.B., Kokate, V.K., Sapkal, A.M.: Performance analysis of MUSIC and ESPRIT DOA estimation algorithms for adaptive array smart antenna in mobile communication. In: *Second International Conference on Computer and Network Technology*, pp. 308–311 (2010)
8. Roy, R., Kailath, T.: ESPRIT-estimation of signal parameters via rotational invariance techniques. *IEEE Trans. Acoustic Speech Sig. Process.* **37**(7), 984–995 (1989)
9. Lina, Y.: Study on factors affecting DOA accuracy. *Modern Radar* **29**(6), 70–73 (2007)
10. Dzielski, J.E., Burkhardt, R.C., Kotanchek, M.E.: Modified MUSIC algorithm for estimating DOA of signals - Comment. *Sig. Process.* **55**(2), 253–254 (1996)
11. Lou, D., Xie, J., Wang, Y.: Comparison of the statistical characteristics of MUSIC and MMUSIC algorithms. *Space Electron. Technol.* **4**(3), 31–35 (2007)

12. Zoltowski, M.D., Wong, K.T.: ESPRIT-based 2-D direction finding with a sparse uniform array of electromagnetic vector sensors. *IEEE Trans. Sig. Process.* **48**(8), 2195–2204 (2000)
13. Belfiori, F., van Rossum, W., Hooeboom, P.: 2D-MUSIC technique applied to a coherent FMCW MIMO radar. In: *IET International Conference on Radar Systems (Radar 2012)*, Glasgow, UK, pp. 1–6 (2012)
14. Jun, X., Xin, Z., Haitao, H.: An improved two-dimensional music algorithm based on spatial rectangular array. *Digit. Manuf. Sci.* **69**(02), 5–9 (2018)