Modeling and Derivation of Small Signal Model for Grid-Connected Inverters

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Abstract. With the development of new energy power generation technology, more and more regions are increasing the construction of new energy power generation facilities. However, due to the unstable factors in the source of new energy, the traditional modeling methods require a lot of work. Modeling the impedance of the system in a d-q frames has become a new option, but there is no reference for the derivation process of the system impedance, control model and mathematical model. In order to reduce the amount of tasks of more complex impedance modeling, this paper correspond the control model and mathematical model of Grid-connected inverters in Matlab/Simulink. Among them, the most important part of the control model is the model building of PLL, which includes power control and current control.

Keywords: Grid-connected inverters · Small signal model · Impedance modeling

1 Introduction

Grid-connected inverters are the key components that deliver renewable energy to the grid [1, 2]. In the process of grid connection, harmonic pollution will occur, which pollutes the electricity. The frequency and phase of the output current of the grid-connected inverter are delayed from the frequency and phase of the system, so the phase-locked loop is needed to achieve synchronization [3]. Compared with state space equation method, impedance-based method has advantages that analysis of high-order equations can be avoided and model doesn't have to be rebuilt when system structure changes [4].

Recently, the impedance model-based analysis [5–7] has been proposed to provide insights into this instability issue.

Because the Grid-connected terminal is a three-phase AC system, it cannot be studied in the static coordinate system. The existing methods need to carry out Park transformation and Clark transformation, so as to realize from three-phase static coordinate system to two-phase static coordinate system or to d-q frames. Impedance model is easier to establish in this paper in d-q frames. The model equivalent input is given in this paper considering rotor current controllers and PLL. And use the small signal method to analyze the stability of the system [8–10].

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This paper is organized as follows. Section 2 is the topology, mathematical model and control model of Grid-connected inverters. Section 3 is the mathematical model and control model of PLL. Section 4 is the model with current control and power control. Section 5 is based on MATLAB/Simulink, which corresponds the simulation model to the control model and deduces the equivalent impedance. Section 6 concludes this paper.

2 Impedance Modeling of Grid-Connected Inverter

Figure 1 is the structure diagram of three-phase Grid-connected inverter, which can be divided into main power route control circuit. In the main circuit, U_a, U_b and U_c are the three-phase voltage measured with grid entry point; I_a, I_b and I_c are the three-phase current measured with grid entry point; u_d,u_q,0 are the d-axis q-axis grid entry point voltage output by the PLL in the controller coordinate system; i_d, i_{dref}, i_q and i_{qref} are the d-axis q-axis grid entry point current and its reference value; PQ are the active power and reactive power calculated by the measured data in the power control; P_{ref}, Q_{ref} are the active power reference quantity of power and reactive power [11].

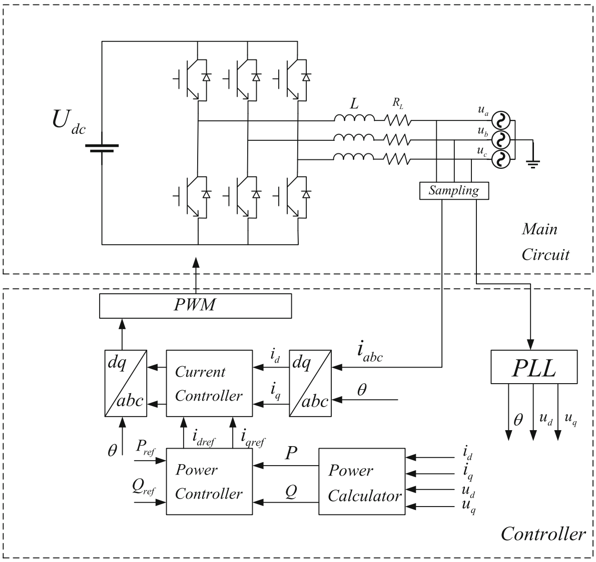

Fig. 1. The structure diagram of three-phase Grid-connected inverter

The small signal model in d-q frames is shown in Fig. 2. In the Fig. 2, the D, U and I respectively represent duty ratio, voltage and current in stable state, and d, u and i are the small signal disturbance values. Among them, the main circuit parameters are indicated by superscript s, and the subscript d and q corresponds to the d-axis q-axis respectively. $u_{dc} = U_{dc} + \tilde{u}_{dc}$; u_d, i_d, u_q, i_q represents the voltage disturbance value of d-axis parallel node, the current disturbance value of parallel node, the voltage disturbance value of q-axis parallel node and the current disturbance value of parallel node in the main circuit coordinate system respectively; D_d, D_q, d_d, d_q represents the duty ratio steady value of d-axis and q-axis and the small signal disturbance value respectively.

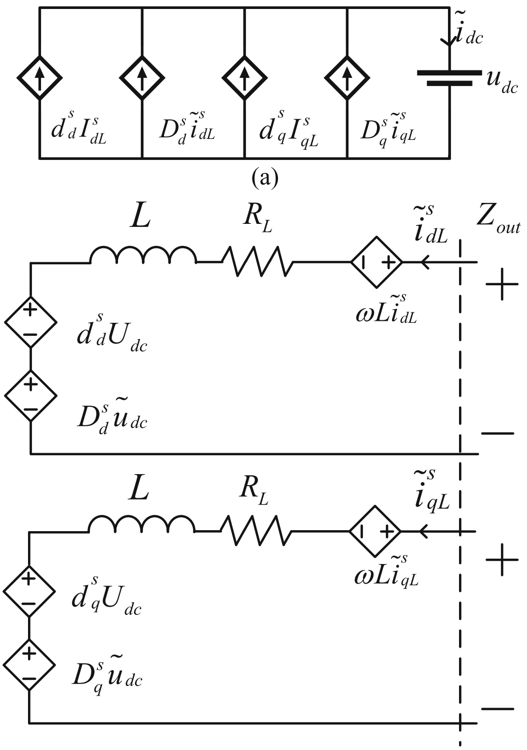

Fig. 2. Small signal model of Grid-connected inverter in d-q frames

From the small signal model in Fig. 2 (a),

$$u_{\mathrm{dc}} = \tilde{d}^s_d I^s_{dL} + D^s_d \tilde{i}^s_{dL} + \tilde{d}^s_q I^s_{qL} + D^s_q \tilde{i}^s_{qL} \tag{1}$$

Its matrix form is

$$\begin{bmatrix} u_{\mathrm{dc}} \\ 0 \end{bmatrix} = \begin{bmatrix} D^s_d & D^s_q \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{i}^s_{dL} \\ \tilde{i}^s_{qL} \end{bmatrix} + \begin{bmatrix} I^s_{dL} & I^s_{qL} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{d}^s_d \\ \tilde{d}^s_q \end{bmatrix} \tag{2}$$

From Fig. 2 (b), we can see the small signal model of AC measurement, and the small signal disturbance of grid entry point voltage is

$$\begin{bmatrix} \tilde{u}^s_d \\ \tilde{u}^s_q \end{bmatrix} = \begin{bmatrix} sL + R_L & -\omega L \\ \omega L & sL + R_L \end{bmatrix} \begin{bmatrix} \tilde{i}^s_{dL} \\ \tilde{i}^s_{qL} \end{bmatrix} + \begin{bmatrix} D^s_d & 0 \\ D^s_q & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}_{dc} \\ 0 \end{bmatrix} + \begin{bmatrix} U_{dc} & 0 \\ 0 & U_{dc} \end{bmatrix} \begin{bmatrix} \tilde{d}^s_d \\ \tilde{d}^s_q \end{bmatrix} \tag{3}$$

Based on (2) and (3)

$$\begin{bmatrix} \tilde{u}_d^s \\ \tilde{u}_q^s \end{bmatrix} = \begin{bmatrix} sL + R_L + D_d^{s2} & -\omega L + D_d^s D_q^s \\ \omega L + D_d^s D_q^s & sL + R_L + D_d^{s2} \end{bmatrix} \begin{bmatrix} \tilde{i}_{dL}^s \\ \tilde{i}_{qL}^s \end{bmatrix} + \begin{bmatrix} U_{dc} + D_d^s I_{dL}^s & I_{qL}^s D_d^s \\ I_{dL}^s D_d^s & U_{dc} + D_q^s I_{qL}^s \end{bmatrix} \begin{bmatrix} \tilde{d}_d^s \\ \tilde{d}_q^s \end{bmatrix} \tag{4}$$

The transfer function matrix of the voltage to the current at the grid entry point is

$$G_{iu} = \begin{bmatrix} sL + R_L + D_d^{s2} & -\omega L + D_d^s D_q^s \\ \omega L + D_d^s D_q^s & sL + R_L + D_d^{s2} \end{bmatrix} \tag{5}$$

The matrix of current transfer function from duty ratio to grid entry point is

$$G_{id} = \begin{bmatrix} sL + R_L + D_d^{s2} & -\omega L + D_d^s D_q^s \\ \omega L + D_d^s D_q^s & sL + R_L + D_d^{s2} \end{bmatrix}^{-1} \begin{bmatrix} U_{dc} - D_d^s I_{dL}^s & I_{qL}^s D_d^s \\ I_{dL}^s D_q^s & U_{dc} - D_q^s I_{qL}^s \end{bmatrix} \tag{6}$$

Figure 3 depicts the open-loop input admittance model.

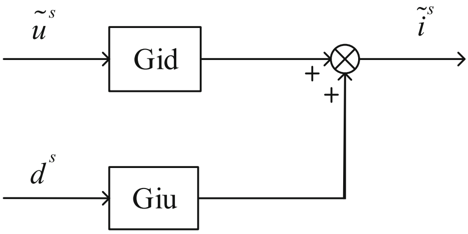

Fig. 3. Control program block diagram of main circuit

3 Control Model of PLL

From [12], the function of phase-locked loop is synchronize the control system with the main system and the control system, the PI control method is used in the phase-locked loop. A grid-connected converter usually needs PLL to synchronize with the grid.

The transfer function between two systems can be expressed as

$$T_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{7}$$

When the output small disturbance of the PLL between the main system and control system is small enough, $\cos\theta = 1$, $\sin\theta = \theta$, we can get

$$\vec{U}^c = \begin{bmatrix} 1 & \theta \\ -\theta & 1 \end{bmatrix} \vec{U}^s \tag{8}$$

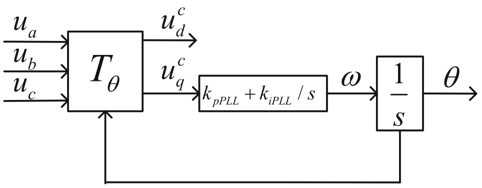

Fig. 4. Control block diagram of PLL in d-q frames

It can also be written as

$$\begin{bmatrix} U_d^c + \tilde{u}_d^c \\ U_q^c + \tilde{u}_q^c \end{bmatrix} = \begin{bmatrix} 1 & \theta \\ -\theta & 1 \end{bmatrix} \begin{bmatrix} U_d^s + \tilde{u}_d^s \\ U_q^s + \tilde{u}_q^s \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \tilde{u}_d^c \\ \tilde{u}_q^c \end{bmatrix} \approx \begin{bmatrix} U_d^s\theta + \tilde{u}_d^s \\ -U_q^s\theta + \tilde{u}_q^s \end{bmatrix} \tag{10}$$

From Fig. 4

$$\theta = \tilde{u}_q^c K_{PLL} \frac{1}{s} \tag{11}$$

Where $K_{PLL} = k_{pPLL} + \frac{k_{iPLL}}{s}$, so (11) can also be written as

$$\theta = \frac{K_{PLL}}{s + U_d^s K_{PLL}} \tilde{u}_q^c \tag{12}$$

Similarly,

$$G_{PLL}^i = \begin{bmatrix} 0 & I_q^s G_{PLL} \\ 0 & -I_d^s G_{PLL} \end{bmatrix} \tag{13}$$

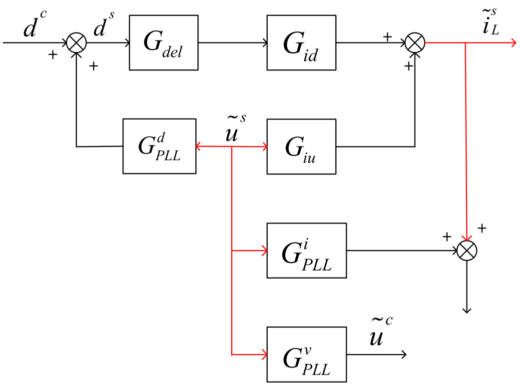

Fig. 5. Control block diagram with PLL

$$G_{PLL}^{d} = \begin{bmatrix} 0 & -D_q^s G_{PLL} \\ 0 & D_d^s G_{PLL} \end{bmatrix} \tag{14}$$

The control block diagram with PLL is as shown in the Fig. 5

4 Model with Current Control and Power Control

According to the main circuit structure, the PI current loop control mode is adopted and the current loop control matrix is

$$G_{ci} = \begin{bmatrix} k_{pi} + \frac{k_{ii}}{s} & 0 \\ 0 & k_{pi} + \frac{k_{ii}}{s} \end{bmatrix} \tag{15}$$

Since the current loop control contains coupling, the d-axis and q-axis are decoupled, the matrix form is

$$G_{dei} = \begin{bmatrix} 0 & -\frac{3\omega L}{U_{dc}} \\ \frac{3\omega L}{U_{dc}} & 0 \end{bmatrix} \tag{16}$$

After the current controller is added, the power controller is add to achieve more accurate control effect. The transfer function matrix of power control is

$$G_{cPQ} = \begin{bmatrix} k_{pPQ} + \frac{k_{iPQ}}{s} & 0 \\ 0 & k_{pPQ} + \frac{k_{iPQ}}{s} \end{bmatrix} \tag{17}$$

Through linearization, the transfer function of active power and reactive power in d-q frames can be obtained

$$G_{PQ}^{i} = \begin{bmatrix} U_d^s & U_q^s \\ -U_q^s & U_d^s \end{bmatrix} \tag{18}$$

$$G_{PQ}^{v} = \begin{bmatrix} I_d^s & I_q^s \\ I_q^s & -I_d^s \end{bmatrix} \tag{19}$$

The control block diagram including current control, power control and PLL is shown in Fig. 6.

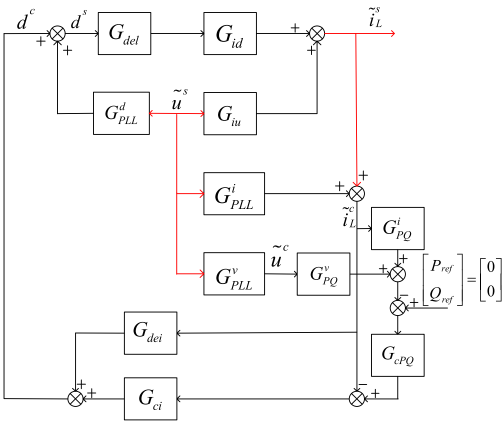

Fig. 6. Control block diagram including current control, power control and PLL

5 Matlab/Simulink Partial Simulation Model and Equivalent Impedance Derivation

From the control block diagram in Fig. 6, we can get

$$\begin{bmatrix} P \\ Q \end{bmatrix} = G_{PQ}^{v}\begin{bmatrix} u_d^c \\ u_q^c \end{bmatrix} + G_{PQ}^{i}\begin{bmatrix} i_{dL}^c \\ i_{qL}^c \end{bmatrix} \tag{20}$$

$$\tilde{i}_{Lref}^{c} = G_{cPQ}(PQ_{ref} - PQ) \tag{21}$$

$$\tilde{d}^{c} = G_{dei}\tilde{i}_L^c + G_{ci}(\tilde{i}_{Lref}^c - \tilde{i}_L^c) \tag{22}$$

Based on (15) to (19), the simulation model of duty ratio of control system can be obtained as follows

$$\begin{aligned} \tilde{d}^{c} = & \left[(P_{ref} - P)\left(k_{pPQ} + \frac{k_{iPQ}}{s}\right) - \tilde{i}_{dL}^{c}\right]\left(k_{pi} + \frac{k_{ii}}{s}\right) - \frac{3\omega_0 L}{U_{dc}}\tilde{i}_{qL}^{c} \\ & + \left[(Q_{ref} - Q)\left(k_{pPQ} + \frac{k_{iPQ}}{s}\right) - \tilde{i}_{qL}^{c}\right]\left(k_{pi} + \frac{k_{ii}}{s}\right) + \frac{3\omega_0 L}{U_{dc}}\tilde{i}_{dL}^{c} \end{aligned} \tag{23}$$

The simulation model in Matlab /Simulink is (Fig. 7).

Continue to derive the control model based on (21), (22)

$$\tilde{i}_L^c = \tilde{i}_L^s + G_{PLL}^{i}\tilde{u}^s \tag{24}$$

$$\tilde{i}_L^s = G_{del}G_{id}\tilde{d}^s + G_{iu}\tilde{u}^s \tag{25}$$

$$\tilde{d}^s = \tilde{d}^c + G_{PLL}^{d}\tilde{u}^s \tag{26}$$

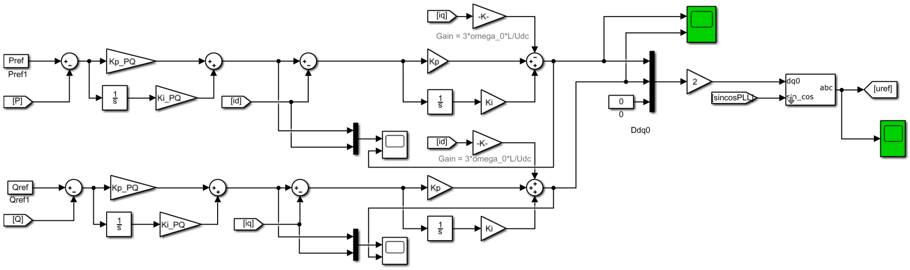

Fig. 7. Simulation model of duty ratio $\tilde{d}^c$ of control system in Matlab/Simulink

If the reference value of active power and reactive power is 0, then

$$\tilde{d}^c = \left(G_{dei} - G_{ci} - G_{ci}G_{cPQ}G^i_{PQ}\right)\tilde{i}^c_L - G_{ci}G_{cPQ}G^v_{PLL}G^v_{PQ}\tilde{u}^s \tag{27}$$

Substituting Eq. (24) into Eq. (27)

$$\tilde{d}^c = \left(G_{dei} - G_{ci} - G_{ci}G_{cPQ}G^i_{PQ}\right)\tilde{i}^s_L + \left(G_{dei}G^i_{PLL} - G_{ci}G^i_{PLL} - G_{cPQ}G^i_{PLL}G^i_{PQ} - G_{cPQ}G^v_{PLL}G^v_{PQ}\right)\tilde{u}^s \tag{28}$$

$$\begin{aligned}\tilde{d}^s = &\left(G_{dei} - G_{ci} - G_{ci}G_{cPQ}G^i_{PQ}\right)\tilde{i}^s_L \\ &+ \left(G_{dei}G^i_{PLL} - G_{ci}G^i_{PLL} - G_{ci}G_{cPQ}G^i_{PLL}G^i_{PQ} - G_{ci}G_{cPQ}G^v_{PLL}G^v_{PQ} + G^d_{PLL}\right)\tilde{u}^s\end{aligned} \tag{29}$$

Based on (29) and (25)

$$\begin{aligned}\tilde{i}^s_L = &G_{del}G_{id}\left(G_{dei} - G_{ci} - G_{ci}G_{cPQ}G^i_{PQ}\right)\tilde{i}^s_L \\ &+ \left[G_{iu} + G_{del}G_{id}\left(G_{dei}G^i_{PLL} - G_{ci}G^i_{PLL} - G_{ci}G_{cPQ}G^i_{PLL}G^i_{PQ} - G_{ci}G_{cPQ}G^v_{PLL}G^v_{PQ} + G^d_{PLL}\right)\right]\tilde{u}^s\end{aligned} \tag{30}$$

So the equivalent impedance is

$$Z_{out} = \frac{\left[E + G_{del}G_{id}\left(G_{ci} + G_{ci}G_{cPQ}G^i_{PQ} - G_{dei}\right)\right]}{\left[G_{iu} + G_{del}G_{id}\left(G_{dei}G^i_{PLL} - G_{ci}G^i_{PLL} - G_{ci}G_{cPQ}G^i_{PLL}G^i_{PQ} - G_{ci}G_{cPQ}G^v_{PLL}G^v_{PQ} + G^d_{PLL}\right)\right]} \tag{31}$$

Simulation verification is carried out in MATLAB/Simulink, the results are as follow Figure.

From the Fig. 8, the theoretical results of the model are in good agreement with the actual results of frequency sweep the reliability of the impedance model has been deduced.

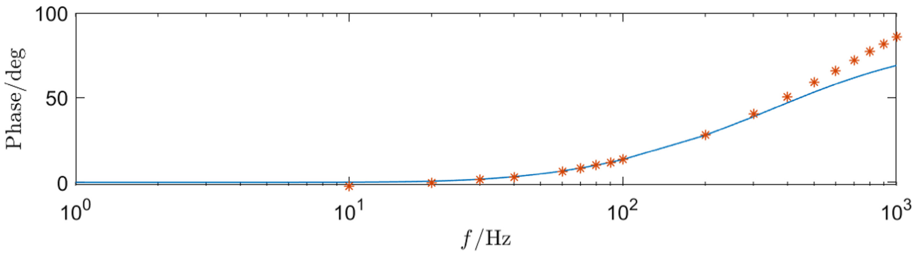

Fig. 8. Simulation result

6 Conclusion

In this paper, the solution process of the impedance model of the Grid-connected inverters in the d-q frames is deduced. According to the small signal model in the d-q frames, the transfer matrix of the main circuit with disturbance is derived firstly, then the influence of the phase-locked loop in the control system is considered, same as the influence of the current control and the power control. Finally, the output equivalent impedance is derived. It provides a bridge for the later establishment and research of impedance model, reduces the workload of model understanding, and lays foundation for the subsequent stability analysis in a longer term.

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