Comparison of Thermal Load Models for MILP-Based Demand Response Planning

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Abstract. Demand response has the potential to reduce end-users electricity costs by promoting judicious use of existing power system infrastructure. This is most often assumed to require the adoption of time-varying electricity prices which can make load scheduling and energy resource management difficult to carry out in a time-effective and comfortable way without computational assistance and automated control. Automated home energy management systems can facilitate this process including by providing users with optimised plans. Creating these plans requires optimisation tools operating on mathematical models of the underlying problem. Mixed-integer linear programming (MILP) has been used extensively for this purpose though increasing complexity and time resolution can render this approach impractical. In this paper, we describe and compare MILP formulations of the same demand response problems using alternative thermal load models. The results, obtained using a state-of-the-art solver, can be summarised as follows: (1) the elimination of continuous temperature variables in one thermal load submodel increased the computation time in 99% of cases and by 981% on average; (2) two new discrete control formulations leading to a 40% reduction in the number of binary variables relative to the standard formulation were found to decrease the computation time in approximately 63% of cases and by 38–40% on average. Efforts are ongoing to evaluate these techniques under more diverse scenarios.

Keywords: Demand response · Thermal load model · Mixed-integer linear programming

1 Introduction

1.1 Context and Motivation

Demand response (DR) programs are often advanced as a way to ensure the existing power system infrastructure is used rationally and efficiently, and also to integrate increasing shares of variable renewable energy resources into power grids [1,2]. This potential implies the dissemination of intelligible information

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about the power system for a given period – most often in the form of time-differentiated electricity prices and ad-hoc demand reduction requests – so as to indirectly influence end-users' aggregate electricity consumption. Once in possession of this information, end-users are expected to decide when and how to adjust their actions to meet their objectives. For residential end-users, this implies scheduling and configuring domestic appliances as well managing local energy resources, particularly storage units, if there are any. However, for end-users to minimise their electricity bill under these circumstances while meeting their needs requires a level of effort most if not all end-users will want to avoid without computer assistance and control, arguably due to the task's participative, repetitive, schedule-constraining and computationally-demanding nature [3,4]. As such, automated home energy management systems have been proposed to not only carry out part of these actions but also to support the decision-making process by providing users with optimised reference plans [5].

Generating these plans requires the use of optimisation tools operating on mathematical models of the underlying DR problem and should be sufficiently time-effective to avoid protracted interactions with end-users. One common approach is to model DR problems using mixed-integer linear programming (MILP) and produce optimised solutions via MILP solvers. A wide range of DR problems have been modelled using MILP including those featuring thermal loads with significant DR potential, such as domestic hot water (DHW) vessels and air-conditioned spaces, since these allow loads to be deferred to an extent that allows for significant flexibility and possibly lower costs. Modelling thermal loads and respective actuators using MILP can thus be ascribed some importance. This paper describes a study comparing the computational performance of MILP formulations of DR problems using different yet equivalent thermal load models.

1.2 Literature Review

The thermal load MILP models surveyed in the literature cover loads such as hot water vessels, air-conditioned spaces, radiators and refrigerated compartments, and are mostly physical phenomena-inspired as opposed to physics-agnostic models. Most commonly, thermal loads are modelled as single-node temperature models that can be derived from first order ordinary linear differential equations, and often rely on continuous decision variables to hold temperature values [3,6–12]. One-dimensional multi-node models can also be found in the literature, particularly to model thermal stratification in storage vessels [13,14] and heat transfer across building envelopes [15,16]. In a few cases, thermal loads have also been modelled as time-shiftable or interruptible loads [17,18].

The models reviewed encompass combinations of temperature, power, and control constraints. Temperature constraints have generally been used to ensure safety and comfort standards [3,8,12], either constant or changing over time due to occupancy or demand, rather than the model's own accuracy (e.g., due to phase changes). Power constraints have been used to comply with individual equipment's power ratings [8,9,11,19] or those defined by fuses or the utility grid [3,17,20]. Control constraints have been used to implement discrete part-load operation, 2-level hysteresis control and minimum operation and inactivity cycle durations [20,21], and to ensure some modes are mutually-exclusive (e.g., heating or cooling) [8,22]. Accommodating these features may be a necessity but these also increase the computational burden, particularly for short time steps, due to the additional binary variables needed. Ultimately, models of a strongly-combinatorial nature (due to the number of binary variables) can be too difficult to solve within an acceptable time-frame (e.g., close to real-time) using affordable and low power computational resources [6,8,12,18–22], which limits the attractiveness of MILP models for in-house residential DR planning.

1.3 Objectives and Approach

The study described here set out to explore the effect of two modelling techniques on the ability to efficiently solve MILP-formulated residential demand response problems featuring thermal loads. The techniques evaluated sought to reduce the number of continuous and binary variables necessary to reproduce the same DR problem. The first technique concerns the elimination of continuous load temperature variables. The second technique relates to modelling multi-level discrete control of appliances interacting with thermal loads: three equivalent formulations were compared, namely the traditional one relying on one binary variable per level and time interval plus constraints to ensure that only one can be used at any given time interval, and two others potentially relying on a reduced set of binary variables and constraints. The main thrust was thus to understand how these techniques influence the computational performance of MILP-formulated DR problems involving thermal loads and under time-differentiated electricity prices. For this purpose, a state-of-the-art solver was used to produce optimised solutions to the same DR problem formulated differently and the computation times compared. A diverse set of case studies was defined and employed to reduce the possibility of bias influencing the conclusions. This effort is presented over 5 additional sections: Sect. 2 describes the DR problem under consideration; Sect. 3 details the models used to reproduce the problem; Sect. 4 defines the case studies considered; Sect. 5 presents and analyses the optimisation results; Sect. 6 summarises this endeavour's main conclusions.

2 Problem Description

The DR problem addressed in this study is centred around a grid-connected single-family household where the end-users' electricity consumption during a 36-hour period is charged according to a discretely-increasing (power) demand rate and time-differentiated energy prices. The automated home energy management system is tasked with preparing optimised plans and managing the operation of domestic appliances and energy resources to maximize profits while maintaining safety and comfort standards, and without interfering with the non-controllable demand (NCD). The other demand component is due to an electric water heater (EWH), a refrigerator (REF), a reversible speed-controlled heat pump (HP) as well as single dishwasher (DW), laundry machine (LM) and tumble dryer (TD) cycles observing user-designated appliance-specific comfort periods. Temperatures limits have to be observed in the refrigerated compartments, the air-conditioned room, and the water vessel. A photovoltaics-based behind-the-meter local electricity generation system is also present to allow for reduced energy costs via load matching and/or revenue by selling excess electricity to the utility grid as long as utility-specified power limits are observed.

3 Mathematical Model

A customisable MILP formulation of the single-household DR problem described in the previous chapter was developed to carry out this study. The MILP model relies on generic submodels for time-shiftable and single-node thermal loads, which were instantiated three times each to accommodate all appliances under consideration: the dishwasher, laundry machine, tumble dryer cycles were modelled as time-shiftable loads while the EWH, the refrigerator and the heat pump were assumed to interact with uniform temperature thermal loads using three different controls: on/off, proportional and multi-level discrete controls, respectively. A demand rate submodel is also used to relate power needs into costs and feed-in constraints. More information about the model is provided next.

3.1 Objective Function

The objective for this model is to maximise profits over the entire planning period duration (PPD) by simultaneously minimising energy and power costs and maximising revenue while meeting safety and comfort standards. Revenue is obtained by selling locally-generated excess electricity to the utility whereas costs are due to the NCD and L additional electrical loads, consisting of N_{SL} time-shiftable loads and N_{TL} thermal load-related electrical loads, and cost penalties prompted by excessive temperatures in selected thermal loads. Loads are identified through the set $S_L = \{1, ..., L\}$ and subsets $S_{SL} \subseteq S_L$ and $S_{TL} \subseteq S_L$ for time-shiftable and thermal load-related electrical loads, respectively, such that: $S_{SL} \cap S_{TL} = \emptyset; S_{SL} \cup S_{TL} = S_L$. Additional subsets include $S_{TL,\Delta t} \subseteq S_{TL}$ and $S_{TL,PPD} \subseteq S_{TL}$ to identify thermal loads capable of inducing costs due to excessive temperatures during each time interval and temperature reductions between planning periods, respectively. The objective function is thus given by (1), where $P^{ELEC}_{IMP,k}$ and $P^{ELEC}_{EXP,k}$ stand for the mean power drawn from and fed to the utility grid, respectively, during time interval k of the planning period (out of $K \in \mathbb{N}$, Δt-long intervals: $K = PPD/\Delta t$), $y_{U,p}$ indicates whether the nonnegative power level $P_{U,p}$ was exceeded during the planning period ($P_{U,0} = 0$ W, for islanded operation), $\Delta\theta_{l,PPD}$ is thermal load l's ($\forall l \in S_{TL,PPD}$) temperature decrease, if any, between planning periods, and $\Delta\theta^{UPPER}_{l,k}$ represents thermal load l's ($\forall l \in S_{TL,\Delta t}$) temperature increase, if any, above the reference temperature ($\theta^{UPPER}_{l,REF}$) during time interval k. With regard to the objective function coefficients, $p^{ELEC}_{IMP,k}$ and $p^{ELEC}_{EXP,k}$ are respectively the prices charged and offered to the end-user for electricity consumed from and delivered to the grid during time interval k, $c_{U,p}$ is the cost of exceeding the power level $P_{U,p}$, $c_{l,PPD}$ is the cost of increasing load l's ($\forall l \in S_{TL,PPD}$) temperature by one degree after the planning period, and $c_{l,k}^{UPPER}$ is the cost of load l's ($\forall l \in S_{TL,\Delta t}$) temperature exceeding $\theta_{l,REF}^{UPPER}$ by one degree during time step k.

$$\min \sum_{l \in S_{TL,\Delta t}} \sum_{k=1}^{K} c_{l,k}^{UPPER} \Delta\theta_{l,k}^{UPPER} + \sum_{l \in S_{TL,PPD}} c_{l,PPD} \Delta\theta_{l,PPD} + \sum_{p=0}^{P-1} c_{U,p} y_{U,p} + \Delta t \sum_{k=1}^{K} (p_{IMP,k}^{ELEC} P_{IMP,k}^{ELEC} - p_{EXP,k}^{ELEC} P_{EXP,k}^{ELEC}) \quad (1)$$

$$P_{IMP,k}^{ELEC} \geq 0, \;\; k = 1, ..., K; \; P_{EXP,k}^{ELEC} \geq 0, \;\; k = 1, ..., K \quad (2)$$

$$y_{U,p} \in \{0, 1\}, \;\; p = 0, ..., P - 1 \quad (3)$$

$$\Delta\theta_{l,PPD} \geq 0, \;\; l \in S_{TL,PPD} \quad (4)$$

$$\Delta\theta_{l,k}^{UPPER} \geq 0, \;\; k = 1, ..., K; \;\; l \in S_{TL,\Delta t} \quad (5)$$

3.2 Power Balances

Both $P_{IMP,k}^{ELEC}$ and $P_{EXP,k}^{ELEC}$ are defined using the power balance equations in (6), where $P_{l,k}$ is the mean power demand due to load l ($\forall l \in S_L$) during time interval k in accordance with (7), $P_{NCD,k}$ is the mean power for the NCD during time interval k, and $P_{LEG,k}$ is the mean power supplied by the local electricity generation system during time interval k.

$$\sum_{l \in S_L} P_{l,k} + P_{NCD,k} - P_{LEG,k} = P_{IMP,k}^{ELEC} - P_{EXP,k}^{ELEC}, \;\; k = 1, ..., K \quad (6)$$

$$P_{l,k} \geq 0, \;\; l \in S_L, \;\; k = 1, ..., K \quad (7)$$

3.3 Demand Rate

The demand rate submodel consists of (8)–(11). The peak mean net power demanded from the utility grid during the planning period (P_{PEAK}) is constrained by (8) and (9), and used to determine which power levels have been exceeded in (10), where $P_{U,p}$ is the peak power afforded by the demand rate level p (for P positive levels: $p = 0, ..., P$). Simultaneously, (11) ensures compliance between $P_{EXP,k}^{ELEC}$ and the maximum power the system is allowed to deliver to the grid, defined as a fixed percentage (λ_{EXP}) of the demand rate-defined peak power.

$$P_{IMP,k}^{ELEC} - P_{EXP,k}^{ELEC} - P_{PEAK} \leq 0, \;\; k = 1, ..., K \quad (8)$$

$$0 \leq P_{PEAK} \leq P_{U,P} \quad (9)$$

$$P_{PEAK} - P_{U,P}y_{U,p} \leq P_{U,p}, \;\; p = 0, ..., P-1 \tag{10}$$

$$P^{ELEC}_{EXP,k} - \lambda_{EXP} \sum_{p=0}^{P-1} y_{U,p}(P_{U,p+1} - P_{U,p}) \leq 0, \;\; k = 1, ..., K \tag{11}$$

3.4 Time-shiftable Loads

The behaviour of time-shiftable loads was reproduced using the model proposed in [21,23]. This model assumes such loads are characterised by non-interruptible cycles, each defined by an ordered sequence of stages with specific power demand levels, whose scheduling must conform to predefined comfort periods.

3.5 Thermal Loads

Thermal loads were generically modelled after uniform temperature bodies controlled through appliances in accordance with (12), where $\theta_l(t)$ is the effective temperature for load l ($\forall l \in S_{TL}$) at time $t \in \mathbb{R}$, $x_{l,m}(t)$ is the actuator control signal component m ($\forall m \in S_{M,l} = \{1, ..., M_l\}$) at time t, while $a_l(t)$, $b_l(t)$, $c_l(t)$ and $d_{l,m}(t)$ are potentially time-varying load- and context-specific coefficients.

$$a_l(t) \cdot \theta'_l(t) = b_l(t) \cdot \theta_l(t) + c_l(t) + \sum_{m \in S_{M,l}} d_{l,m}(t) x_{l,m}(t), \;\; l \in S_{TL} \tag{12}$$

The solution to (12) was approximated by assuming the coefficients are constant during one time step (zero-order hold). In doing so, a closed-form solution can be produced and the process repeated for multiple time steps. The corresponding MILP formulation is given in (13) for a given load l ($\forall l \in S_{TL}$) and time interval k, where $\theta_{l,k}$ is its temperature at the start of time interval k, $x_{l,m,k}$ is the decision variable for mode m, and $a_{l,k}$, $b_{l,k}$, $c_{l,k}$ and $d_{l,m,k}$ are coefficients.

$$\theta_{l,k+1} - \theta_{l,k} \exp(\frac{b_{l,k}}{a_{l,k}} \Delta t) - b^{-1}_{l,k}(c_{l,k} + \sum_{m \in S_{M,l}} d_{l,m,k} x_{l,m,k})(\exp(\frac{b_{l,k}}{a_{l,k}} \Delta t) - 1) = 0, \tag{13}$$

$$k = 1, ..., K, \;\; l \in S_{TL}$$

The load temperatures determined by (13) have to comply with minimum ($\theta_{MIN,l,k}$) and maximum ($\theta_{MAX,l,k}$) temperature limits during each time interval in accordance with (14)–(15) while separate ones constrain $\Delta\theta_{l,\Delta t}$ and $\Delta\theta^{UPPER}_{l,k}$ using (16) and (17), where $\theta_{l,1}$ is the initial load temperature.

$$\theta_{l,k} \geq \theta_{MIN,l,k}, \;\; k = 2, ..., K+1, \;\; l \in S_{TL} \tag{14}$$

$$\theta_{l,k} \leq \theta_{MAX,l,k}, \;\; k = 2, ..., K+1, \;\; l \in S_{TL} \tag{15}$$

$$\theta_{l,K+1} - \theta_{l,1} \geq -\Delta\theta_{l,PPD}, \;\; l \in S_{TL,PPD} \tag{16}$$

$$\theta_{l,k} - \theta_{l,REF}^{UPPER} \leq \Delta\theta_{l,k}^{UPPER}, \;\; k = 2, ..., K+1, \;\; l \in S_{TL,\Delta t} \tag{17}$$

Appliance Control. One of three actuator control types was considered for each thermal load: on/off, proportional and multi-level discrete controls. Thermal loads modelled using these controls can be respectively identified through the subsets $S_{TL,ON-OFF}$, $S_{TL,PROP}$ and $S_{TL,MULTI}$ ($\subseteq S_{TL}$) such that:

$$S_{TL,ON-OFF} \cap S_{TL,PROP} \cap S_{TL,MULTI} = \emptyset \tag{18}$$

$$S_{TL,ON-OFF} \cup S_{TL,PROP} \cup S_{TL,MULTI} = S_{TL} \tag{19}$$

The choice of actuator control ultimately defines the number of signal components (M_l) needed for each load, which can include one or two active heat transfer directions (i.e., heating and cooling). Consequently, it also defines the decision variable ($x_{l,m,k}$) types and associated constraints. Among the choices, on/off control is the simplest and requires one binary variable per time interval and heat transfer direction ($M_l = 1$ or 2) in accordance with (20).

$$x_{l,m,k} \in \{0,1\}, \;\; k = 1, ..., K, \;\; m \in S_{M,l}, \;\; l \in S_{TL,ON-OFF} \tag{20}$$

Proportional control, defined as a semi-continuous monotonically-increasing piecewise linear function above a minimum positive level, requires two control signal components per time interval and heat transfer direction ($M_l = 2$ or 4) in accordance with (21) and (22): one binary variable sets the minimum actuator level and a continuous variable sets the actuator level above the mininum level.

$$\begin{aligned} x_{l,2n-1,k} \in \{0,1\}, \;\; x_{l,2n,k} \in [0,1], \;\; l \in S_{TL,PROP} \\ k = 1, ..., K, \;\; n = 1, ..., M_l/2 \end{aligned} \tag{21}$$

$$\begin{aligned} x_{l,2n-1,k} \geq x_{l,2n,k}, \;\; k = 1, ..., K, \;\; l \in S_{TL,PROP}, \\ n = 1, ..., M_l/2 \end{aligned} \tag{22}$$

Multi-level discrete control is primarily intended to reproduce full- and part-load operation of appliances controlling thermal load temperatures and can be formulated in three alternative ways, all of which exclusively rely on binary variables in accordance with (23). The standard one (STD) uses one binary variable per positive load level and time interval for each heat transfer direction and prevents more than one of those corresponding to the same time interval (k) from being positive in accordance with (24).

$$x_{l,m,k} \in \{0,1\}, \;\; k = 1, ..., K, \;\; m \in S_{M,l}, \;\; l \in S_{TL,MULTI} \tag{23}$$

$$\sum_{m \in S_{M,l}} x_{l,m,k} \leq 1, \;\; k = 1, ..., K, \;\; l \in S_{TL,MULTI} \tag{24}$$

If the appliances can be assumed to operate with power-invariant COPs and the normalised load levels are separated at regular steps between 0 (no load) and 1 (full load), then two other formulations can be used. Both reproduce a specified set of load levels ($T_l = \{1, ..., N_l\}$) using combinations of those levels to achieve the same outcome using a subset of the original levels ($T_l' \subseteq T_l$), in accordance with (25), where $e_{l,m,k}$ and $e_{l,n,k}$ are the surrogates for $d_{l,m,k}$ needed to employ T_l and T_l', respectively, as surrogates for $S_{M,l}$. In doing so, these formulations have the potential to dispense with all the constraints and a part of the variables needed using the standard formulation, depending on the number of levels and heat transfer directions. For example, no reduction of binary variables is possible if 2 levels per heat transfer direction are considered but at 3, 5 and 10 levels, reductions of 33, 40 and 60% are possible. Additional constraints are not necessary if only one heat transfer direction is considered – otherwise (24) is necessary to prevent simultaneous heating and cooling modes – and if any potential combination only reproduces the original levels.

$$e_{l,m,k} = \sum_{n \in T_l'} e_{l,n,k} w_{l,n,k}, \;\; w_{l,n,k} \in \{0,1\}, \;\; k = 1, ..., K, \;\; m \in T_l, \;\; l \in S_{TL,MULTI} \tag{25}$$

The differences between the two non-standard formulations concern the approach by which to rule out solutions reproducing load levels not found in the original set, specifically those that exceed load ratings – since intermediate levels are implicitly excluded. The first of these more specific formulations (SPC1) does this by enforcing upper and lower load level limits for heating and cooling modes, respectively, in accordance with (26)–(27), if applicable.

$$\sum_{n \in T_l'} d_{l,n,k} x_{l,n,k} \leq \max_{m \in T_l} d_{l,m,k}, \;\; k = 1, ..., K, \;\; l \in S_{TL,MULTI} \tag{26}$$

$$\sum_{n \in T_l'} d_{l,n,k} x_{l,n,k} \geq \min_{m \in T_l} d_{l,m,k}, \;\; k = 1, ..., K, \;\; l \in S_{TL,MULTI} \tag{27}$$

The second formulation (SPC2) uses the linearised 0–1 polynomial constraints given in (28)–(29) to rule out any binary decision variable combinations that allow those limits to be exceeded, where V is the number of binary combinations to exclude and $S_{l,0,v}$ and $S_{l,1,v}$ are sets ($\forall v = 1, ..., V$; $S_{l,0,v}, S_{l,1,v} \subseteq S_{M,l}$; $S_{l,0,v} \cap S_{l,1,v} = \emptyset$) containing the indexes for the decision variables that equal nought and one, respectively, in the binary combination v.

$$\sum_{r \in S_{l,1,v}} x_{l,r,k} - \sum_{s \in S_{l,0,v}} x_{l,s,k} \le |S_{l,1,v}| - 1, \;\; k = 1, ..., K, \;\; v = 1, ..., V, \;\; l \in S_{TL,MULTI} \tag{28}$$

$$- \sum_{r \in S_{l,1,v}} x_{l,r,k} + \sum_{s \in S_{l,0,v}} x_{l,s,k} \le |S_{l,0,v}|, \;\; k = 1, ..., K, \;\; v = 1, ..., V, \;\; l \in S_{TL,MULTI} \tag{29}$$

Elimination of Load Temperature Variables. Employing continuous load temperature variables $\theta_l = [\theta_{l,2}, ..., \theta_{l,K+1}]$ in the thermal load submodel is not strictly required since their use can be replaced by equivalent functions of actuator variables, specifically (30), where x_l is given by (31). To do this, a constraint i ($\forall i \in \{1, ..., I\}$) whose left- and right-hand sides can be represented by $[\,\phi_i, \psi_i]\,[\,\theta_l, x_l]^{\,T}$ and ξ_i would have to be converted into one with $f_i \cdot x_l^T$ and g_i, respectively, which can be shown to require adopting (32) and (33), where $f_{i,j}$ is the element at column j of the vector f_i and similarly for $\alpha_{l,k,j}$, $\phi_{i,k}$ and $\psi_{i,j}$.

$$\theta_{l,k+1} = \alpha_{l,k} \cdot x_l^T - \beta_{l,k}, \;\; k = 1, ..., K, \;\; l \in S_{TL} \tag{30}$$

$$x_l = [x_{l,1}, ..., x_{l,M_l}]\,, \;\; x_{l,m} = [x_{l,m,1}, ..., x_{l,m,K}]\,, \;\; m \in S_{M,l}, \;\; l \in S_{TL} \tag{31}$$

$$f_{i,j} = \psi_{i,j} + \sum_{k=1}^{K} \phi_{i,k} \alpha_{l,k,j}, \;\; i = 1, ..., I, \;\; j = 1, ..., J = M_l \cdot K, \;\; l \in S_{TL} \tag{32}$$

$$g_i = \xi_i + \sum_{k=1}^{K} \phi_{i,k} \beta_{l,k}, \;\; i = 1, ..., I, \;\; l \in S_{TL} \tag{33}$$

4 Case Studies

The modelling techniques addressed in this study were evaluated using a set of case studies. These consist of 108 DR problems defined by different data and time interval duration combinations. Time interval durations of 300, 600, 900 and 1800 s were considered as well as 3 indoor heat gain profiles, 3 DHW demand profiles and 3 NCD profiles – each of which resampled using the appropriate time interval duration. For each of these case studies, one problem was created per combination of the two modelling techniques considered: multi-level discrete control formulation and load temperature variable elimination. In total, 648 problems were created and optimised to provide 324 and 216 sets of comparisons to explore the effect of variable elimination and the different control formulations, respectively.

The two modelling techniques under consideration were applied solely to the thermal load submodel for the air-conditioned room. This model assumes a heat pump can be off or cooling the room using 20, 40, 60, 80 or 100% of its rated power capacity. These 5 levels were reproduced using the various discrete control formulations. In the case of the SPC1 and SPC2 formulations, 3 levels, namely 20, 40 and 60%, were used to reproduce the 5 original ones thus enabling a 40% reduction in the number of binary variables. On the other hand, the SPC1 and SPC2 formulations required additional constraints to prevent the heat pump's rated capacity from being exceeded. The two remaining thermal loads, namely the EWH and the refrigerator, were modelled as using on/off and proportional control, respectively. In the latter case, the type of control selected was meant to represent on/off operation during less than one full time step to facilitate obtaining feasible solutions to problems defined using a comparatively-long Δt.

4.1 Problem Data

The case studies were defined primarily using data from [24]. This included the shiftable load operation cycles and comfort periods, the demand rates and respective power levels, the electricity prices, the local electricity generation profile, the indoor heat gain profile, the NCD profile, the outdoor temperature profile, the indoor temperature profile (for the EWH and refrigerator models), the utility water temperature profile, and most thermal load data. The exceptions concerned: the air-conditioned room's lumped capacity, which was doubled; the heat pump's specifications [25]; the refrigerator's COP temperature dependence [26]; the EWH lower and upper reference temperatures (60 and 70 °C); $c_{l,k}^{UPPER}$, defined as the ratio between the maximum cost prompted by not shifting the EWH load and the difference between the maximum and upper reference temperatures; the refrigerator's minimum cycle duration (3 min); the daily DHW volume (200 L at 45 °C); the DHW profile, instead based on the RAND profile [27]; the occupancy vector, defined as the sign function of the DHW profile. Finally, the indoor heat gain, DHW demand and NCD profiles were created using stochastic functions of the reference ones.

4.2 Computational Resources and Solver Settings

The MILP problems were optimised using IBM's CPLEX 12.8.0.0 which ran on a shared machine featuring an Intel Xeon Gold 6138 CPU and 320 GB of RAM. The solver was invoked from MATLAB using the official CPLEX class API. Standard solver settings were used except the termination criteria which included optimality, a 1% relative gap and a 15-min computational budget.

4.3 Methodology

The methodology adopted relied on comparisons between the optimisation results obtained for the same problem but formulated using different models.

As such, investigating the effect of load temperature variables required comparing solutions to problems whose models differ only on whether those variables are used or not. The same holds for the effect of different multi-level discrete control formulations but in that case three comparisons are possible per problem: SPC1 vs STD; SPC2 vs STD; SCP1 vs SCP2. For reasons of brevity, this study mainly focuses on performance variations relative to the standard formulation. Finally, computational performance was measured via the deterministic computation time, as returned by CPLEX, which is a repeatable measure of effort involved in solving each problem rather than the actual time to obtain a solution.

5 Results and Analysis

Optimised solutions and best bounds were determined for the 648 MILP problems created. The optimised solution to one of these is represented in Fig. 1.

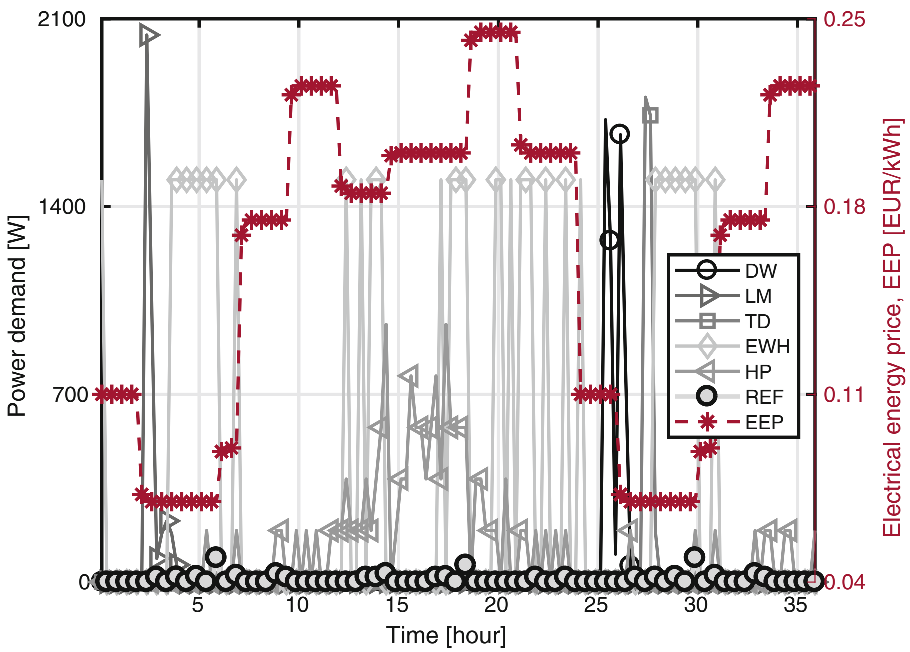

Fig. 1. Optimisation results for one DR problem (900 s time interval; reference input data) using load temperature variables and the STD formulation: power demand from scheduled loads and electrical energy price (EEP) versus planning period time.

5.1 Model Equivalence

Objective function differences were used to examine whether the formulations can be regarded as equivalent or not. In general, the differences due to the effect of load temperature variables and the multi-level discrete control formulations were lower than 0.075 €, relative to objective function values in the range of 4.6–5.3 € (<1.5%). More importantly, the best bound differences obtained do not exceed 0.025 € (<0.5%) and the objective function differences reached with the best solutions do not exceed 0.029 € (<0.7%). In light of the errors observed and the potential for improvement (via a higher computational budget and a lower relative MIP gap), the formulations compared can be regarded as equivalent.

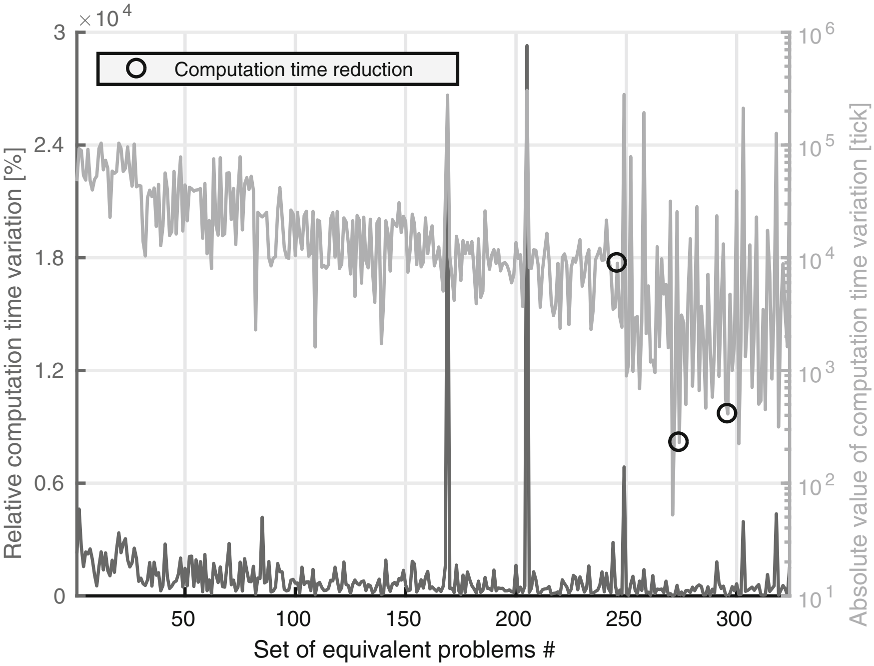

Fig. 2. Absolute value of and relative deterministic computation time variation (positive for increases) caused by the elimination of load temperature variables, for each set of equivalent problems. The circles point to the cases of computation time decreases.

5.2 Effect of Eliminating Load Temperature Variables

The comparisons undertaken show that eliminating the continuous load temperature variables led to computation time increases in almost all cases. More precisely, computation time increases were detected in 321 out of 324 cases (99%) and by as much as 29,292% (about 294 times higher) and 981% (11 times higher) on average, as illustrated in Fig. 2. In turn, the computation time decreased by as much as 32% (and 22% on average) in three cases – highlighted in Fig. 2 – none of which for the two shortest time intervals. These results show that inessential continuous variables can reduce the time needed to obtain practical solutions though a different inquiry is necessary to understand why this happens.

5.3 Effect of the Multi-level Discrete Control Formulation

The effect of the multi-level discrete control formulations on the computation time is summarised in Table 1. According to the results, in most cases (63%) the SPC1 and SPC2-based models required less time to be solved than the STD-based model. This advantage was also observed when considering each time interval duration separately, and found to be inversely correlated – to some extent. On the other hand, the differences between the SPC1- and SPC2-based models were far less significant, with the former outperforming the latter in 52% of cases though not for each time interval duration separately. The other factors (i.e., input data) did not produce recognisable patterns on the results.

Table 1. Number of times a formulation (SPC1, SPC2 or STD) was solved in less (deterministic computation) time by formulation pair and time interval duration.

Comparison	Formulation	Time interval duration				Total
		300	600	900	1800	
SPC1 vs STD	SPC1	40 (74%)	34 (63%)	29 (54%)	33 (61%)	136 (63%)
	STD	14 (26%)	20 (37%)	25 (46%)	21 (39%)	80 (37%)
SPC2 vs STD	SPC2	36 (67%)	33 (61%)	33 (61%)	35 (65%)	137 (63%)
	STD	18 (33%)	21 (39%)	21 (39%)	19 (35%)	79 (37%)
SPC1 vs SPC2	SPC1	27 (50%)	31 (57%)	25 (46%)	30 (56%)	113 (52%)
	SPC2	27 (50%)	23 (43%)	29 (54%)	24 (44%)	103 (48%)

Magnitude-wise, no formulation appears to have a meaningful or consistent advantage. On average, the SPC1 and SPC2 formulations were unable to reduce the computation time by more than half or increase it by more than twice (38–40% reductions vs 80–97% increases). However, the computation time reductions afforded by the SPC1 and SPC2 formulations over the STD formulation were, on average, one order of magnitude higher than the computation time increases. These results can be partially explained by the correlation between time interval duration and computation time (i.e., finer time-discretisation contributes to a more strongly-combinatorial model) and the fact that SPC1- and SPC2-based models tended to fare better at lower time interval durations – cf. Table 1.

6 Conclusions

The efforts described in this paper concern an investigation into the effect of two modelling techniques on the computational performance of MILP-formulated residential DR problems. The techniques concern new multi-level discrete control formulations and the elimination of continuous load temperature decision variables. These techniques were compared using a state-of-the-art solver to optimise a set of equivalent single-household DR problems.

This endeavour's main conclusions can be summarised as follows: (1) the elimination of load temperature variables was found to increase the computation time in 99% of cases and by 981% on average; (2) the new multi-level discrete control formulations required less time to be solved than the standard formulation in most cases (63%), achieving computation time reductions of 38–40% on average for a 40% reduction in the number of binary variables. These results indicate that additional continuous decision variables can be desirable to reduce computation times whereas using less binary variables does not necessarily lower the computation time. Future work will focus on causality, namely by reproducing more diverse conditions and examining the formulation strength and size [28].

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