Propagation Properties of Optical Beams with Multi-OAM Modes: Effect of the Off-Axis Vortex

Ying Dang[1] and Wenrui Miao[2(✉)]

[1] No. 365 Institute of Northwestern Polytechnical University (Xi'an ASN Technology), Xi'an, China
snow365dy@163.com

[2] School of Electronics and Information, Northwestern Polytechnical University, Xi'an, China
wenruimiaonpu@163.com

Abstract. As one of the promising techniques to improve the spectral efficiency of the network in the Internet of Things (IoT), the orbital-angular-momentum (OAM) multiplexing optical wireless communication has been studied a lot. In the optical beams with multi-OAM modes, the vortices deviating from the beam center sometimes cannot be avoidable, and they will strongly influence the transverse patterns during the beam propagation. In this article, the expressions of optical beams with off-axis vortices are derived in a commonly used focusing system, and the effect of the vortices deviating from the beam center on the propagation properties of the optical beam is discussed. We find that the number of the bright spots in the transverse patterns of the superposition of two OAM modes with off-axis vortices is not always equal to the absolute value of the mode difference which is observed in the field with only on-axis vortices. The bright spot can also be found to rotate during the beam propagating, and as the number of the modes increases in the overlapping, the superposition patterns become more complicated and these patterns can be adjusted by the off-axis distance and the topological charge of the vortices. Our result will be helpful in developing the network of the IoT.

Keywords: Orbital angular momentum · Off-axis vortices · Multiplexing

1 Introduction

The term of Internet of Things (IoT) was first proposed in 1998 [1], after that, the IoT has been developing extensively in various areas, and will realize the goal of intelligent identifying, locating, tracking, monitoring, and managing things [2,3]. However, as the developing of the IoT, trillions of devices will be involved into

B. Li et al. (Eds.): IoTaaS 2019, LNICST 316, pp. 573–581, 2020.
https://doi.org/10.1007/978-3-030-44751-9_48

the IoT, therefore the network of the IoT will be congested. The orbital-angular-momentum (OAM) multiplexing optical wireless communication is a promising technique to overcome this problem [4,5].

In most of researches on the OAM mode multiplexing, only the vortices embedded in the beam center are considered [6,7] and many results are obtained based on the assumption of the axial vortices (i.e. located in the beam center) [8]. However, the off-axis vortices sometimes are inevitable in the generation of the beams with OAM modes, like in generating on-axis vortex with misalignment [9–11]. In another perspective, the vortices deviating from the beam center also have very interesting properties during the beam propagation, such as decreasing the total OAM of the beam [12,13], improving the acceleration of the beams [14], generating transverse focal shift [15] and having rotating trajectories in the focused fields [16–23], and also can lead to some special polarization structures [24,25]. In the OAM multiplexing optical wireless communication system, the off-axis vortices are also not avoidable in many circumstances, and these vortices will change the superposition patterns of multi-OAM modes, which will influence the efficiency at the receiving end. In this article, the expressions of the propagating optical beams with vortices off the beam center will be derived, and the influence of these vortices on the propagation properties of the beam with multi-OAM modes will be discussed.

2 Theory

In practical experiments or applications, in order to concentrate more energy in the propagation direction, a thin lens is usually used to focus the light before its propagation. Even if the thin lens is absent, the diffraction effect should also be considered in the propagation of the optical wave. The far field of the diffracted light is actually equivalent to the field at the focal plane of the wave going through a thin lens [26]. Generally an optical beam with OAM mode N means an optical vortex beam with topological charge N. In this article, we are considering the propagation of optical beams with multi-OAM modes, therefore, we first derive the field distribution along the propagation in the focal region of optical beams with different OAM mode.

Assume that there are N vortices with topological charge m_k located at $r = r_k$, $\phi = \phi_k$, embedded in a Gaussian beam. The amplitude distribution of the electric field V_0, according to [16] at the beam waist w_0 can be expressed as

$$V_0(r,\phi) = \prod_{k=1}^{N} e^{-r^2/w_0^2} \left(re^{\pm \mathrm{i}\phi} - r_k e^{\pm \mathrm{i}\phi_k} \right)^{|m_k|}, \tag{1}$$

where r denotes the radial distance and ϕ is the azimuthal angle. If m_k is positive, the sign of ϕ and ϕ_k is positive and vice versa.

Let us then consider a converging, monochromatic wave whose amplitude at the entrance plane can be expressed by Eq. (1), and this wave emerges from a circular aperture (which can be treated as passing through a thin lens) with its radius a. The geometrical focus is located at the origin of the Coordinates (see Fig. 1) and f is the focal length or the radius of a Gaussian reference sphere S. The electric field at point P in the focal region can be given by the expression [26]

$$E(P) = -\frac{\mathrm{i}}{\lambda}\frac{e^{-\mathrm{i}kf}}{f}\iint_S V_0 \frac{e^{\mathrm{i}ks}}{s}\mathrm{d}S, \tag{2}$$

where k is the wave-number. $k = 2\pi/\lambda$ and λ is the wavelength in free space.

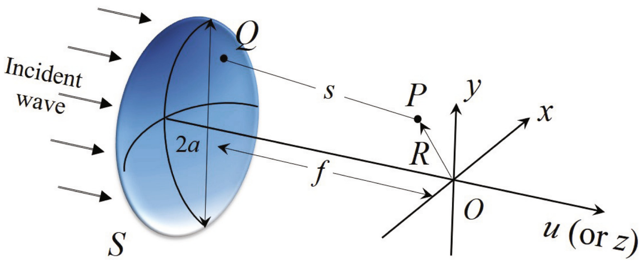


Fig. 1. Illustrating the notation.

When the vortex is located at $(a_1, 0)$ and its topological charge is $m = 1$ (i.e., the mode is 1), the complex amplitude at the entrance plane is expressed as

$$V_0(\rho, \phi) = e^{-(a\rho/w_0)^2}(a\rho e^{\mathrm{i}\phi} - a_1), \tag{3}$$

here $a\rho = r$. Substituting Eq. (3) into Eq. (2), we can get

$$E_1(P) = -\frac{\mathrm{i}}{\lambda}\frac{e^{-\mathrm{i}kf}}{f}\iint_S e^{-(a\rho/w_0)^2}(a\rho e^{\mathrm{i}\phi} - a_1)\frac{e^{\mathrm{i}ks}}{s}\mathrm{d}S, \tag{4}$$

where the subscript 1 of U_1 means that the mode is 1. In this article, we will adopt the dimensionless Lommel variables u and v instead of x, y, z, i.e.,

$$u = \frac{2\pi}{\lambda}\left(\frac{a}{f}\right)^2 z \tag{5}$$

$$v = \frac{2\pi a^2 \rho}{\lambda f} = \frac{2\pi}{\lambda}\frac{a}{f}\sqrt{x^2 + y^2}. \tag{6}$$

In order to get the explicit expression of Eq. (4), we will use some approximations. First the factor $1/s$ can be approximated as $1/f$, second the Debye approximation is also applied, i.e.,

$$s - f = -\boldsymbol{q} \cdot \boldsymbol{R}, \tag{7}$$

where $\boldsymbol{q}$ is a unit vector in the direction of OQ and $\boldsymbol{R}$ is the vector from the focus to the point P (see Fig. 1). Then Eq. (7) can be written as

$$k\boldsymbol{q} \cdot \boldsymbol{R} = v\rho\cos(\phi - \psi) - (f/a)^2 u + \frac{1}{2}u\rho^2, \tag{8}$$

with $\psi = \arctan(y/x)$. By applying Eq. (8) and $\mathrm{d}S = a^2\rho\mathrm{d}\rho\mathrm{d}\phi$, we can re-write Eq. (4) as

$$E_1(u,v,\psi) = -\frac{\mathrm{i}}{\lambda}\frac{a^2}{f^2}e^{\mathrm{i}\left(\frac{f}{a}\right)^2 u}\int_0^1\int_0^{2\pi} e^{-\mathrm{i}[v\rho\cos(\phi-\psi)+\frac{1}{2}u\rho^2]} \times e^{-(a\rho/w_0)^2}(a\rho e^{\mathrm{i}\phi} - a_1)\rho\mathrm{d}\rho\mathrm{d}\phi \tag{9}$$

Because of the following identity

$$\int_0^{2\pi} e^{\mathrm{i}[-v\rho\cos(\phi-\psi)]}e^{\mathrm{i}m\phi}\mathrm{d}\phi = J_m(-v\rho)\frac{2\pi}{\mathrm{i}^{-m}}e^{\mathrm{i}m\psi}, \tag{10}$$

where J_m is the first kind of Bessel function of order m, the Eq. (9) can be simplified as

$$E_1(u,v,\psi) = \frac{2\pi a^2}{\lambda f^2}e^{\mathrm{i}\left(\frac{f}{a}\right)^2 u}\left[e^{\mathrm{i}\psi}\int_0^1 -a\rho^2 J_1(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho + \mathrm{i}a_1\int_0^1 \rho J_0(v\rho)e^{-(a\rho/\omega_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho\right]. \tag{11}$$

Using the same steps, one can derive the expressions of the electric fields for arbitrary modes, and here the expressions for mode 2, 3 and -1 are presented, as

$$E_2(u,v,\psi) = \frac{2\pi a^2}{\lambda f^2}e^{\mathrm{i}\left(\frac{f}{a}\right)^2 u}\left[\mathrm{i}e^{\mathrm{i}2\psi}\int_0^1 a^2\rho^3 J_2(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho + 2a_2 e^{\mathrm{i}\psi}\int_0^1 a\rho^2 J_1(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d} - \mathrm{i}a_2^2\int_0^1 \rho J_0(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho\right], \tag{12}$$

$$E_3(u,v,\psi) = \frac{2\pi a^2}{\lambda f^2}e^{\mathrm{i}\left(\frac{f}{a}\right)^2 u}\left[e^{\mathrm{i}3\psi}\int_0^1 a^3\rho^4 J_3(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho - \mathrm{i}3a_3 e^{\mathrm{i}2\psi}\int_0^1 a^2\rho^3 J_2(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho - 3a_3^2 e^{\mathrm{i}\psi}\int_0^1 a\rho^2 J_1(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho + \mathrm{i}a_3^3\int_0^1 \rho J_0(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho\right], \tag{13}$$

$$E_{-1}(u,v,\psi) = \frac{2\pi a^2}{\lambda f^2}e^{\mathrm{i}\left(\frac{f}{a}\right)^2 u}\left[e^{-\mathrm{i}\psi}\int_0^1 -a\rho^2 J_1(v\rho)e^{-(a\rho/w_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho + \mathrm{i}a_{-1}\int_0^1 \rho J_0(v\rho)e^{-(a\rho/\omega_0)^2}e^{-\frac{\mathrm{i}}{2}u\rho^2}\mathrm{d}\rho\right], \tag{14}$$

where the subscript n of E_n $(n = 1, 2, 3, -1)$ indicates the mode of the beam (or the topological charge of the vortex), and a_n presents the off-axis distance of the vortex with charge n in the entrance plane.

3 Result and Discussion

In this section, we will use the expressions derived in the previous section to discuss the propagation properties of the focused optical beams with multi-OAM modes. In our research, only the case of perfect superimposed beams are considered, which means that the beams with different modes are overlapped with their beam centers and propagating directions coincident with each other.

First of all, the superposition of the mode $m = 1$ and mode $m = 2$ are considered. When the off-axis distances a_1 and a_2 are both equals to 0, i.e., the vortices are perfectly in the beam center, the superimposed pattern in the transverse planes during the propagation of the beam usually has one bright spot because of their phase distributions. It is shown in Fig. 2 that in the focal plane and in the transverse planes along the propagation direction there is only one diffraction spot (i.e. the bright spot), which is coincident with the result in [5]. Moreover, it also can be found that this bright spot rotates during its propagation.

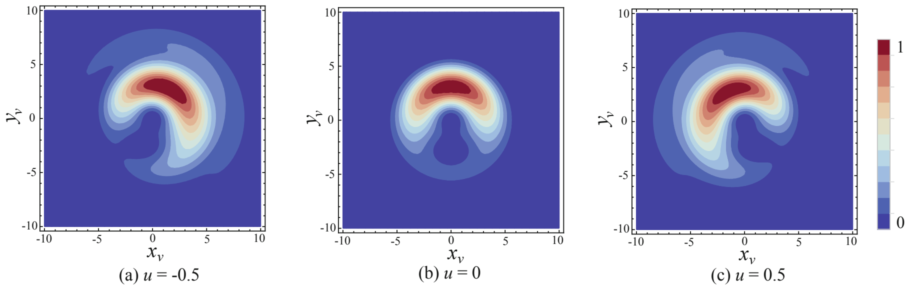


Fig. 2. Intensity distribution in the transverse planes along the propagation direction. Here $a = w_0$, $a/f = 10$, $a1 = a2 = 0$.

When there is one vortex moving away from the beam center, i.e., $a_N \neq 0$ $(N = 1, 2)$, the number of the bright spots in the superimposed pattern will not always equal $|N_1 - N_2|$ $(N_1, N_2$ are the numbers of the modes). It is shown in Fig. 3 that as the vortex of mode 2 is leaving off the beam center with $a_2 = 0.25w_0$, there appear two bright spots during the beam propagation and also these two spots rotate with the distance.

If the vortices in both two modes are placed at a very short distance from the beam center, i.e., $a_1 \neq 0$ and $a_2 \neq 0$, the superimposed field can also be quite different. In Fig. 4, it is clear to see that when $a_1 = a_2 = 0.25w_0$, although there are also two bright spots, the pattern in this case is perpendicular to that

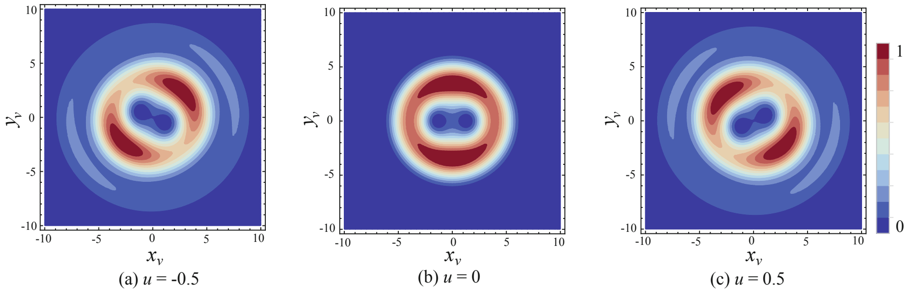


Fig. 3. Intensity distribution in the transverse planes along the propagation direction. Here $a = w_0$, $a/f = 10$, $a1 = 0$, $a2 = 0.25w_0$.

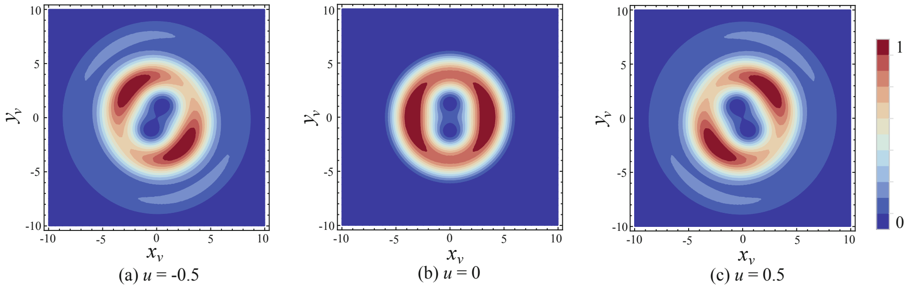


Fig. 4. Intensity distribution in the transverse planes along the propagation direction. Here $a = w_0$, $a/f = 10$, $a_1 = 0.25w_0$, $a_2 = 0.25w_0$.

in Fig. 3, for instance in the focal plane the bright spots are located along the y axis in Fig. 3, while they appear on the x axis in Fig. 4.

Secondly, the mode difference of two modes which is more than 1 is considered for superimposition. Here we choose two pairs of modes: mode 1 and 3, mode 1 and -1. It is displayed in Fig. 5 that since the mode difference in these two cases are both equal to 2, there exist two bright spots when $a_1 = a_3 = a_{-1} = 0$. When there is one vortex embedded away from the beam center, one can see that in the superposition of mode 1 and 3, the number of the bright spots becomes 1 [see plot (a) and (a') in Fig. 6], whereas in the case of mode 1 and -1, this number can be different in the focal plane and other transverse planes during the beam propagation [see plot (b) and (b') in Fig. 6].

At last, we will look at the superposition of three different modes. Here the mode 1, 2 and 3 are overlapped in one beam. There has not been any theoretical formula to describe the superposition pattern of the modes more than 2, thus it is hard to say how many bright spots will exist there. As it shows in Fig. 7, for the superposition of mode 1, 2 and 3, the number of the bright spots can be 1 or 2, and the position of the bright spot is dependent on the position of the vortices in the initial beams.

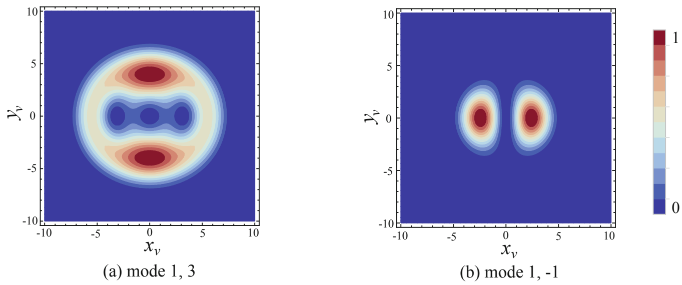


Fig. 5. Intensity distribution in the focal planes for (a) the superposition of mode 1 and 3, (b) the superposition of mode 1 and -1. Here $a = w_0$, $a/f = 10$, $a_1 = a_2 = 0$.

In summary, from the discussions in this section, we can see that if there exists any off-axis vortex in the beam with multi-OAM modes, it is hardly to define the superposition patterns during the propagation. When two different modes are superimposed, the number of the bright spots is not always equal to the absolute value of the mode difference. The positions of the bright spots also can be changed as the beam propagates. Even if there is no off-axis vortice, the position of the bright spots can rotate along its propagation direction. The

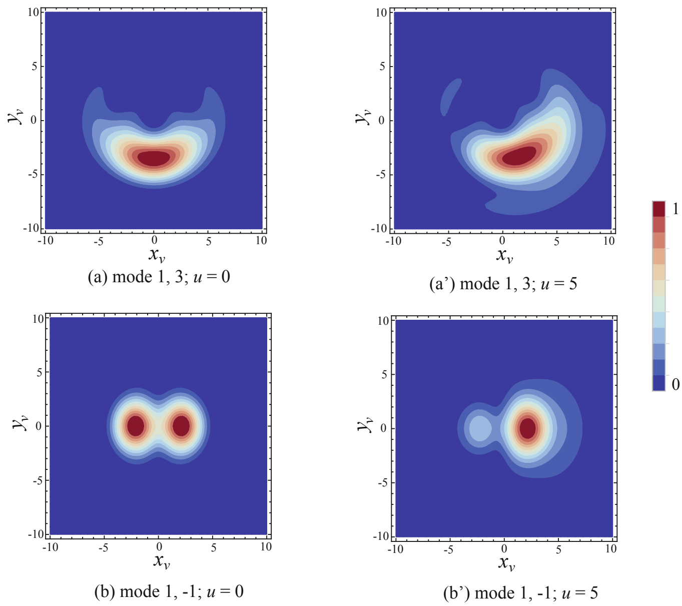
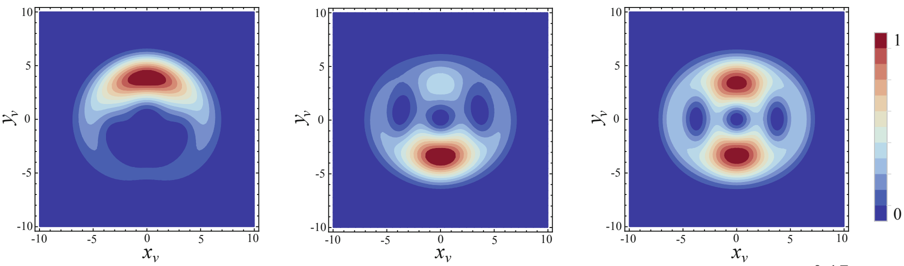


Fig. 6. Intensity distribution in the transverse planes. The superposition of mode 1 and 3 are shown in (a) $u = 0$ and (a') $u = 5$; The superposition of mode 1 and -1 are shown in (b) $u = 0$ and (a') $u = 5$; Here $a = w_0$, $a/f = 10$, $a_1 = 0$, $a_3 = 0.25w_0$, $a_{-1} = 0.50w_0$.



(a) $a_1 = a_2 = a_3 = 0$ (b) $a_1 = a_2 = 0$, $a_3 = 0.25w_0$ (c) $a_1 = 0$, $a_2 = -0.1w_0$, $a_3 = 0.17w_0$

Fig. 7. Intensity distribution in the focal planes for the superposition of mode 1, 2 and 3. In plot (b) $a_2 = -0.10w_0$ means that the vortex is embedded in the $-x$ axis with the distance $0.10w_0$. Here $a = w_0$ and $a/f = 10$.

superposition pattern will become more complicated as the number of modes increases. In this case the number and positions of the bright spots are also dependent on the topological charges, the off-axis distances of the vortices in the input field.

4 Conclusion

In this article, the influence of the off-axis vortices on the propagation properties of the optical beam with multi-OAM modes is discussed. The expressions for the optical beams with different OAM modes (including the on-axis/off-axis vortices) passing through a thin lens are derived. Based on the derivation, the superposition fields of different modes are analyzed. It is found that when there is any off-axis vortices, the number of the bright spots is not always equal to that in the superimposed field with only on axis vortices. The superposition patterns can rotate and the bright spots can split or combine during the beam propagation. The superposition patterns will become more complicated if the combined modes are increased. Our result shows that in a telecommunication system with multi-OAM modes the effect of the off-axis vortices should be taken into consideration seriously, and our finding will give implications in the receiving end of the telecommunication system with multi-OAM modes where an antenna needs to be adjusted according to the superposition pattern.

References

1. Ashton, Kevin, et al.: That Internet of Things of thing. RFID J. **22**(7), 97–114 (2009)
2. Stankovic, J.A.: Research directions for the Internet of Things. IEEE Internet Things J. **1**(1), 3–9 (2014)
3. Chen, S., Xu, H., Liu, D., Hu, B., Wang, H.: A vision of IoT: applications, challenges, and opportunities with china perspective. IEEE Internet Things J. **1**(4), 349–359 (2014)
4. Chen, C., Wang, W., Wu, J., et al.: Visible light communications for the implementation of internet-of-things. Opt. Eng. **55**(6), 060501 (2016)

5. Li, M.: Orbital-angular-momentum multiplexing optical wireless communications with adaptive modes adjustment in Internet of Things networks. IEEE Internet Things J. **6**(4), 6134–6139 (2018)
6. Wang, J., et al.: Terabit free-space data transmission employing orbital angular momentum multiplexing. Nat. Photonics **6**(7), 488 (2012)
7. Feng, Q., Xue, H., Liu, Y., Li, L.: Multiple orbital angular momentum vortex electromagnetic waves multiplex transmission and demultiplex reception analysis. In: 2018 IEEE International Conference on Computational Electromagnetics (ICCEM), pp. 1–3. IEEE (2018)
8. Ke, X., Pu, X.: Generation of orbital angular momentum superpositions and its test. Infrared Laser Eng. **47**(4), 56–61 (2018)
9. Almazov, A.A., Elfstrom, H., Turunen, J., Khonina, S.N., Soifer, V.A., Kotlyar, V.V.: Generation of phase singularity through diffracting a plane or gaussian beam by a spiral phase plate. J. Opt. Soc. Am. A **22**(5), 849–861 (2005)
10. Anzolin, G., Tamburini, F., Bianchini, A., Barbieri, C.: Method to measure off-axis displacements based on the analysis of the intensity distribution of a vortex beam. Phy. Rev. A **79**(3), 033845 (2009)
11. Bekshaev, A.Ya., Sviridova, S.V.: Effects of misalignments in the optical vortex transformation performed by holograms with embedded phase singularity. Opt. Commun. **283**(24), 4866–4876 (2010)
12. Oemrawsingh, S.S.R., Eliel, E.R., Nienhuis, G., Woerdman, J.P.: Intrinsic orbital angular momentum of paraxial beams with off-axis imprinted vortices. J. Opt. Soc. Am. A **21**(11), 2089–2096 (2004)
13. Kotlyar, V.V., Kovalev, A.A., Porfirev, A.P.: Asymmetric gaussian optical vortex. Opt. Lett. **42**(1), 139–142 (2017)
14. Zhu, W., She, W.: Improved nonparaxial accelerating beams due to additional off-axis spiral phases. J. Opt. Soc. Am. A **31**(11), 2365–2369 (2014)
15. Zhao, X., Pang, X., Zhang, J., Wan, G.: Transverse focal shift in vortex beams. IEEE Photonics J. **10**(1), 1–17 (2018)
16. Indebetouw, G.: Optical vortices and their propagation. J. Mod. Opt. **40**, 73–87 (1993)
17. Molina-Terriza, G., Torner, L., Wright, E.M., García-Ripoll, J.J., Pérez-García, V.M.: Vortex revivals with trapped light. Opt. lett. **26**, 1601–1603 (2001)
18. Molina-Terriza, G., Wright, E.M., Torner, L.: Propagation and control of noncanonical optical vortices. Opt. lett. **26**(3), 163–165 (2001)
19. Roux, F.S.: Canonical vortex dipole dynamics. J. Opt. Soc. Am. B **21**, 655–663 (2004)
20. Toda, Y., Honda, S., Morita, R.: Dynamics of a paired optical vortex generated by second-harmonic generation. Opt. Express **18**(17), 17796–17804 (2010)
21. Chen, Z., Pu, J., Zhao, D.: Tight focusing properties of linearly polarized gaussian beam with a pair of vortices. Phys. Lett. A **375**(32), 2958–2963 (2011)
22. Zhao, X., Zhang, J., Pang, X., Wan, G.: Properties of a strongly focused gaussian beam with an off-axis vortex. Opt. Commun. **389**, 275–282 (2017)
23. Li, J., Zhang, J., Li, J.: Optical twists and transverse focal shift in a strongly focused, circularly polarized vortex field. Opt. Commun. **439**, 284–289 (2019)
24. Pang, X., Zhang, J., Zhao, X.: Polarization dynamics on optical axis. Opt. Commun. **421**, 50–55 (2018)
25. Pang, X., Miao, W.: Spinning spin density vectors along the propagation direction. Opt. Lett. **43**(19), 4831–4834 (2018)
26. Born, M., Wolf, E.: Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light, 7th (expanded) edn. Cambridge University Press, Cambridge (1999)