Distributed Scheduling in Wireless Multiple Decode-and-Forward Relay Networks

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Abstract. In this paper, we study the distributed DOS problem for wireless multiple relay networks. Formulating the problem as an extended three-level optimal stopping problem, an optimal strategy is proposed guiding distributed channel access for multiple source-to-destination communications under the help of multiple relays. The optimality of the strategy is rigorously proved, and abides by a tri-level structure of pure threshold. For network operation, easy implementation is presented of low complexity. The close-form expression of the maximal expected system throughput is also derived. Furthermore, numerical results are provided to demonstrate the correctness of our analytical expressions, and the effectiveness is verified.

Keywords: Distributed opportunistic scheduling · Multiple relays selection · Joint time and spatial diversity · Optimal stopping theory

1 Introduction

With explosively increasing demands on enhanced quality of service, conventional communication network faces serious challenges where the medium access control layer and the physical layer are independently designed. As the wireless medium is shared by multiple users and severe channel fading is experienced, a cross-layer design concept, channel-aware scheduling (DOS) is thus motivated. By letting the MAC layer aware of the physical layer information, users can make channel access decision depending on the channel quality, and users in good-quality link are scheduled.

Recently, existing researches have drawn much attentions on centralized scheduling [1,2] where a controller collects channel state information (CSI) of all users, and schedules those of best channel links to access. On the other hand, the research on distributed scheduling is still in its infancy. The difficulty lies in how a user decides when to access channel access. By means of optimal stopping, the problem in ad hoc network without relays was first addressed in [3], and

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an easy implementation is benefited from the proposed pure-threshold strategy. Extended from that, the problem of opportunistic channel access in interference channel allowing multiple nodes simultaneous transmission is investigated in [4], while the problem with time constraints for real-time service is studied in [5].

Under such distributed multi-source network system, cooperative communication has great potentials to enhance system performance and thus has been studied [6–9,11]. Utilizing cooperation diversity, network performance is improved, including reliability and capacity. Recognizing the benefits, increasing interests have been observed in cross-layer design between MAC and physical layer. To say a few, the DOS problem for amplify-and-forward (AF) relay networks is investigated in [12]. Based on bi-level optimal stopping theory, optimal opportunistic channel access strategies are derived for both cases where multiple relays are coordinated and not controlled by each winner source-destination pair. In exploration of observed information influence on the network performance, opportunistic channel access problem under single relay network is also studied in literature [13], where each source has only partial CSI.

This paper differs from existing efforts by jointly considering three following aspects. First, a direct link is used, and multiple relays participle transmission by opportunistic relaying. Second, all channels, including direct channels, channels from each source to all relays, and from all relays to each destination are not necessarily reciprocal. Third, follow-up transmission in two-hop channels is taken into accounts, for which both source and relay can transmit data for duration of channel coherence time. In terms of practicality, these aspects are usually faced in wireless networking.

These considerations lead to new challenges for finding DOS strategy for wireless multi-relay network, and distributed channel access by multiple sources, relays and destinations should be managed in an orderly manner. Moreover, under help of multiple relays, each source is endowed with more flexibility in the strategy design for first-hop channel access, and the broadcast rate at first hop determines the number of relays available for data forward at second hop. Under availability of multiple relays at second hop, more benefits are expected in terms of time and capacity. Furthermore, as a direct link is available, the problem of when direct transmission dominates and how it enhances two-hop transmissions, is worth to be investigated. Thus, optimal DOS strategy is much desired to explore and exploit joint diversity in terms of multiple relays and time between two hops.

To address the challenges, a problem of DOS for multi-relay networks is investigated, and our contribution are summarized as follows.

- Extended from the two-level optimal stopping theory, the DOS problem is formulated, with the goal to maximize average system throughput. Being the novelty, after each successful channel contention, each winner source should determine how to access channel through three-step decision, which are when sources to stop, how they stop, and when relays to stop.
- As a solution to our problem, an optimal strategy for DOS with multiple relays is proposed, maximizing the average system throughput. Different from the existing results, it is in two-threshold structure. Particularly, at first hop

the optimal rule of sources is pure-threshold, and an optimal broadcast rate is calculated through a sequential threshold comparison; at second hop, the optimal rule of relays is also pure-threshold.
- The implementation of our proposed strategy is illustrated, and the effectiveness is also validated in terms of system throughput enhancement.

The rest of this paper is organized as follows. System model and problem formulation is described in Sect. 2, and an optimal DOS strategy maximizing average system throughput is proposed. Performance evaluation is provided in Sect. 4, followed by concluding remarks in Sect. 5.

2 System Model and Problem Formulation

2.1 System Model

Consider a wireless multi-source DOS network with K source-destination pairs aided by L relays in decode-and-forward (DF) mode. In such network, a direct link between each source and destination is included, and for transmission from a source to its destination, several relays are used opportunistically aiding the transmission in a half-duplex mode. The transmission power of a source and a relay is P_s and P_r, respectively.

We denote the channel gain from the ith source to its destination as h_i, the channel gain from the ith source to the jth relay as f_{ij}, and the channel gain from the jth relay to the ith destination as g_{ji}. Typically, we assume channels gains $\sqrt{P_s}h_i$, $\sqrt{P_s}f_{ij}$ and $\sqrt{P_r}g_{ji}$ are Rayleigh faded, with $\sqrt{P_s}h_i \sim CN(0, \sigma_h^2)$, $\sqrt{P_s}f_{ij} \sim CN(0, \sigma_f^2)$ and $\sqrt{P_r}g_{ji} \sim CN(0, \sigma_g^2)$. Time-varying channel environment is considered, and channel coherence time is denoted as τ_d. Channel gains remain constantly within the duration. Notably, results from this research can be extended for general channel fading environment.

The opportunistic channel access protocol by multiple sources is operated as follows. At the beginning of a time slot with duration δ, each source independently contends for the channel by sending a request-to-send (RTS) packet with probability p_0. There are three possible outcomes:

- If there is no source transmitting RTS in the time slot (with probability $(1-p_0)^K$), then all the sources continue to contend in the next time slot;
- If there are two or more sources transmitting RTS (with probability $1-(1-p_0)^K - Kp_0(1-p_0)^{K-1}$), a collision happens, and then in the next time slot after the RTS transmission, all sources continue to contend;
- If there is only one source, say Source i, transmitting RTS (with probability $Kp_0(1-p_0)^{K-1}$), Source i is called *winner* of the contention. By reception of the RTS, each relay and Destination i can estimate CSI between Source i and itself. Then all relays send a RTS to Destination i in turn, in which CSI from the source to each relay is included. After reception of the RTSs, Destination i knows CSI from its source to itself, from the source to all relays and from all relays to itself. Then Destination i decides to *stop*, i.e. transmit data, or

continue, i.e. re-contend channel with other sources. And if stop, it requires to determine in what manner and data rate for transmission. There are three options.

- **End-to-end transmission**: Destination i sends a CTS to Source i and all relays, notifying them to transmit under optimal scheme. Upon reception of the CTS, Source i transmits to relays and destinations in the first hop, and then in the second hop a best relay is selected forwarding data to Destination i. After channel coherence time τ_d, the two-hop transmission finishes.
- **Sources contention**: Source i gives up its transmission opportunity, and other sources can detect an idle slot after the RTS and CTS exchanges among Source i, all relays, and Destination i (i.e., that idle slot tells other sources that Source i gives up its transmission opportunity). After that a new contention is started among all the source nodes.
- **First-hop broadcast**: Destination i sends a CTS to Source i, letting it transmit data at first hop within duration τ_d. Depending on the transmission rate, a subset of relays decode the data. The destination also receive the transmitted signals from the source. Then, the relays sequentially send a RTS to Destination i. After estimating the CSI of second-hop channels, Destination i has to decide whether *to stop* or not, and thus three choices are faced in the following.
    1. **Second-hop forward**: when there exists relays at good link, Destination i decide to stop by sending a CTS for selecting a single relay to forward its received data to Destination i. After that, the two-hop transmission is accomplished;
    2. **Relay termination**: when relays which can decode the first-hop transmission are all at bad link, Destination i decide to stop by sending a CTS to the relays and sources for telling them giving up the data forward;
    3. **Relays contention**: otherwise, Destination i decides to continue, and then channel coherence time τ_d is waited until the next observation.
- After Destination i stops, either second-hop forward or relay termination, new channel contention is started among all sources.

2.2 Problem Formulation

In this sub-section, we develop a decision-theoretic approach to DOS design for distributed multi-relay networks. Based on the optimal stopping theory, the DOS can be formulated as a variant of optimal stopping problem, namely two-stage optimal stopping problem (TSOSP), and the strategy correspondences to an extended stopping rule for the problem.

We illustrate the DOS dynamics of the distributed multi-relay network in Fig. 1 as specified in Sect. 2. A dynamic two-stage observation and decision model is formulated, including the main layer for sources and sub layer for relays.

Main Layer Decision Process: At beginning of each transmission between a source-to-destination pair, multiple sources contend the channel. We define

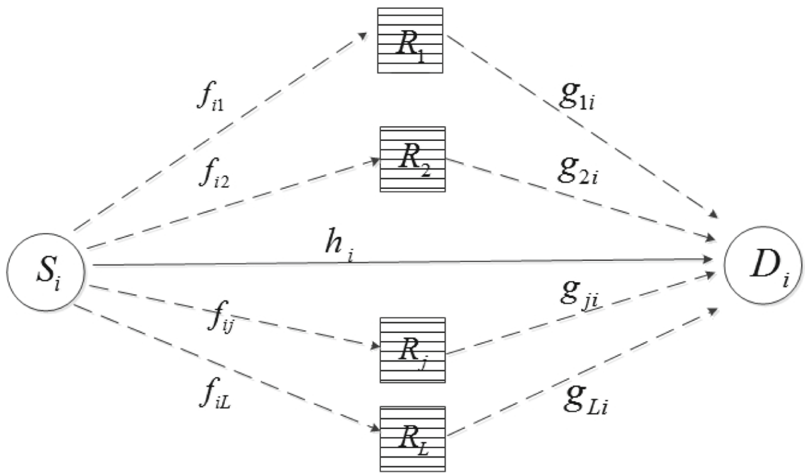

Fig. 1. System model

an *observation in the main layer* as the process of channel contention among the sources until a successful contention. In an observation, the number of contentions follows a geometric distribution with parameter $Kp_0(1-p_0)^{K-1}$. Among all the contentions in an observation, the last contention is successful, with total duration $(L+1)\cdot\tau_{RTS}+\tau_{CTS}$. The mean of an observation duration is thus given as $\tau_o=(L+1)\cdot\tau_{RTS}+\tau_{CTS}+\frac{(1-p_0)^K}{Kp_0(1-p_0)^{K-1}}\cdot\delta+\frac{1-(1-p_0)^K-Kp_0(1-p_0)^{K-1}}{Kp_0(1-p_0)^{K-1}}\cdot\tau_{RTS}$.

Given nth observation in the main layer, the winner source $s(n)$ observes information $\mathcal{F}_n=\{s(n),t_s(n),h_{s(n)}(n),f_{s(n)1}(n),...,f_{s(n)L}(n),g_{1s(n)}(n),...,g_{Ls(n)}(n)\}$, including those on CSI and channel contention. Based on information history until observation n, denoted by $\mathscr{F}_n:=\vee_{i=1}^n\mathcal{F}_i$, the winner source decides if to stop, i.e. $N=n$, i.e. sources stop channel contention by taking some actions. Furthermore, the destination chooses an action $\phi(n)\in\{1,2\}$. In particular, $\phi(n)=1$ means end-to-end transmission, and $\phi(n)=2$ means first-hop broadcast. To be further, if $\phi(n)=1$, the winner source finishes end-to-end transmission, and receives a reward $Y_{n,\phi(n)}$; if $\phi(n)=2$, the winner source further decides the broadcast rate R_n in first hop. After broadcast, channel access process of relays who can decode the broadcast data starts the process for data forwarding in second hop. Otherwise, new channel contention starts and the next observation is obtained.

Sub Layer Decision Process: When the main layer stops with action $\phi(n)=2$, a subset of relays, denoted as $\mathcal{J}_n$, start *observation in the sublayer*. In each slot of channel coherence time, says observation m, each relay obtains second-hop CSI from itself to Destination $s(n)$. We denote observation information as $\mathcal{G}'_{nm}=\left\{g_{js(n)}(m),j\in\mathcal{J}_n\right\}$, and the accumulated information until observation m is denoted as $\mathcal{G}_{nm}=\vee_{i=1}^m\mathcal{G}'_{ni}$. Based on the history information in sub layer, Destination $s(n)$ decides if to stop, i.e. $M=m$, which means that relays stop forwarding by taking some actions. Then, if relays stop, an action $\psi(m) \in \{1, 2\}$ is chosen. In particular, $\psi(m) = 1$ means second-hop forward, and $\psi(m) = 2$ means terminating relays. Furthermore, if $\psi(m) = 1$, relays finish the second-hop transmission, and receives a reward $Y_{n,m,\psi(m)}$; and if $\phi(n) = 2$, relays terminate the transmission and let sources re-contend the channel. Based on this formulation, a DOS strategy is fundamentally a policy of the TSOSP.

Reward and Objective Functions: For DOS problem with multiple relays, in accordance with network protocol as described in Subsect. 2.1, utility functions are defined for the main and sub layer decision process, and are in two parts. The reward function represents transmitted data traffic, and the cost function represents time spent.

In the definitions, upon observed information until nth observation at main layer, for sources to stop by $\phi(n) = 1$, the winner source obtains a reward $Y_{n,1} = \tau_d \max\{C_h(n), C_r(n)\}$, and a cost $T_{n,1} = \sum_{l=1}^{n} t_s(l) + \tau_d$. In details, rate $C_h(n)$ represents the maximal rate in direct link, which is calculated as that

$$C_h(n) = \log_2(1 + P_s|h_i(n)|^2). \tag{1}$$

The rate $C_r(n)$ represents the achievable rate through best single-relay selection, which is calculated as

$$\begin{aligned} C_r(n) =& \frac{1}{2} \max_{j\in\{1,2,\ldots,L\}} \big\{ \min\big(\log_2(1 + P_s|h_i|^2 + P_r|g_{ji}|^2), \\ & \log_2(1 + P_s|f_{ij}|^2)\big)\big\}. \end{aligned} \tag{2}$$

Moreover, for stop by $\phi(n) = 2$, a cost is received with $T_{n,2} = \sum_{l=1}^{n} t_s(l) + \tau_d/2$, and a broadcast rate R_n (i.e., $R_n = \log_2(1+r_n)$) is decided by the winner source. Subsequently, a relay set, denoted as $\mathcal{J}_n$, is determined, representing the relays who can decode the broadcast signal successfully.

Then, sub layer decision process of multiple relays begins, which will give a reward $Y_{n,m,\psi(m)}$ in future.

Upon observed information until mth observation at sub layer, if relays stop by $\psi(m) = 1$, a reward $Y_{n,m,1} = \tau_d R_n \mathbb{I}[g_{\mathcal{J}_n}(m) \geq r_n - P_s|h(n)|^2]$ is received as well as a cost $T_{n,m,1} = T_{n,2} + T_m$. T_m means the time spent in sub layer process, and is calculated as

$$\begin{aligned} T_m = (m-1)\tau_d &+ \mathbb{I}[g_{\mathcal{J}_n}(m) \geq r_n - P_s|h_{s(n)}(n)|^2]\tau_d \\ &+ \mathbb{I}[g_{\mathcal{J}_n}(m) < r_n - P_s|h_{s(n)}(n)|^2](|\mathcal{J}_n| \cdot \tau_{RTS} + \tau_{CTS}). \end{aligned} \tag{3}$$

Moreover, at mth observation at sub layer, if relays stop by $\psi(m) = 2$, a reward $Y_{n,m,2} = 0$ is received, and a cost $T_{n,m,2} = T_{n,2} + T_m$ is spent.

TSOSP Formulation: Based on the theoretic framework above, if winner source stops at Nth observation in main layer and relays stop at Mth observation in sub layer, an instantaneous reward $Y_{N,M}$ is obtained and a time cost $T_{N,M}$ is spent. And the corresponding instantaneous system throughput is $Y_{N,M}/T_{N,M}$. In the sequel, capital N is called stopping time in main layer, and capital M is called stopping time in sub layer. To achieve the optimal OCA strategy, our goal is to find an optimal stopping strategy $\{N^*, R^*_{N^*}, M^*\}$ maximizing the average system throughput $\sup_{N,R_N,M} \mathbb{E}[Y_{N,M}]/\mathbb{E}[T_{N,M}]$. Here $\mathbb{E}[\cdot]$ means expectation.

3 Optimal OCA Strategy and Its Implementation

Enlightened by Lemma 1 in [14, Chapter 6], the maximal-expected-rate-of-return problem is transformed into a standard problem for maximal expected return in accordance with the following theorem. For a given price $\lambda > 0$, we define the reward function $V(\lambda) = \sup_{N,R_N,M} \mathbb{E}[Y_{N,M} - \lambda T_{N,M}]$, representing the maximal expected reward under price λ for time cost.

Theorem 1. *The optimal OCA strategy $\{N^*(\lambda^*), R^*_{N^*}(\lambda^*), M^*(\lambda^*)\}$ achieving $\sup_{N,R_N,M} \mathbb{E}[Y_{N,M}]/\mathbb{E}[T_{N,M}]$ is an optimal strategy achieving the priced expected return $V^*(\lambda^*) = \sup_{N,R_N,M} \mathbb{E}[Y_{N,M} - \lambda^* T_{N,M}]$, where λ^* uniquely exits such that $V^*(\lambda^*) = 0$. The price λ^* is the maximal expected system throughput such that $\lambda^* = \sup_{N,R_N,M} \mathbb{E}[Y_{N,M}]/\mathbb{E}[T_{N,M}]$.*

In accordance with Theorem 1, thought train towards finding an optimal strategy is that, for a given price $\lambda > 0$, an optimal strategy $\{N^*(\lambda), R^*_N(\lambda), M^*(\lambda)\}$ is found achieving $V^*(\lambda)$. Then, by replacing λ with λ^*, $\{N^*, R^*_N, M^*\}$ is acquired as an optimal OCA strategy maximizing the average system throughput.

Based on the relation between main layer and sub layer, reward function obtained from sub layer decision process conditioned on observation information $\mathscr{F}_n$ and broadcast rate R_n in main layer, is defined as that

$$W_n(\lambda) := \mathbb{E}[Y_{n,M^*} - \lambda T_{M^*} | \mathscr{F}_n, R_n].$$

Theorem 2. *For a given price $\lambda > 0$, finding optimal strategy $\{N^*(\lambda), R^*_{N^*}(\lambda), M^*(\lambda)\}$ is equivalent to a two-stage optimal stopping problem. In the sub layer, when main layer stops at nth observation, and $\phi(n) = 2$, an optimal stopping strategy $M^*(\lambda)$ is to find achieving W_n; in the main layer, an optimal strategy $\{N^*(\lambda), R^*_{N^*}(\lambda)\}$ is to find achieving $\sup_{N,R_N} \mathbb{E}\big[(Y_N - \lambda T_N)\mathbb{I}[\phi(N) = 1] + W_N(\lambda)\mathbb{I}[\phi(N) = 2]\big]$.*

Proof. We prove by deriving the two-stage optimal strategy dominates other strategy in the expected reward value.

For any strategy $\{N^\dagger, R^\dagger_{N^\dagger}, M^\dagger\}$, we analyze that[1]

$$\begin{aligned}
&\mathbb{E}\big[Y_{N^\dagger,M^\dagger} - \lambda T_{N^\dagger,M^\dagger}\big] = \\
&\mathbb{E}\big[\sum_{n=1}^{\overline{\infty}} \mathbb{I}[N^\dagger = n]\mathbb{I}[\phi(N^\dagger) = 2]\mathbb{E}[Y_{n,M^\dagger} - \lambda T_{M^\dagger}|\mathscr{F}_n] \\
&+ (Y_{N^\dagger} - \lambda T_{N^\dagger})\mathbb{I}[\phi(N^\dagger) = 1]\big] \\
&\le \mathbb{E}\big[\sum_{n=1}^{\overline{\infty}} \mathbb{I}[N^\dagger = n]\mathbb{I}[\phi(N^\dagger) = 2]\mathbb{E}[Y_{n,M^*} - \lambda T_{M^*}|\mathscr{F}_n] \\
&+ (Y_{N^\dagger} - \lambda T_{N^\dagger})\mathbb{I}[\phi(N^\dagger) = 1]\big] \\
&\le \mathbb{E}\big[(Y_{N^*} - \lambda T_{N^*})\mathbb{I}[\phi(N^*) = 1] + W_{N^*}(\lambda)\mathbb{I}[\phi(N^*) = 2]\big] \qquad (4)
\end{aligned}$$

Therefore, the optimal strategy $\{N^*(\lambda), R^*_{N^*}(\lambda), M^*(\lambda)\}$ in a two-stage form can achieve $V^*(\lambda)$.

In the light of Theorem 2, optimal OCA strategy is derived as follow. A sub-layer optimal strategy $M^*(\lambda)$ followed by relays is first presented in following theorem. We define upper and lower thresholds Th_l and Th_u, where $Th_l = \lambda\frac{\tau_d - \tau_{|\mathcal{J}_n|}}{\tau_d}$ and $Th_u = \lambda + \frac{\lambda(\tau_d + \tau_{|\mathcal{J}_n|})}{P_n \tau_d}$.

Theorem 3. *If main layer stops at nth observation with $\phi(n) = 2$ and broadcast rate R_n, an sub-layer optimal strategy has a one-of-three choices form:*

- *when $R_n \le Th_l$, a myopic strategy is optimal, i.e. $M^*(\lambda) = 1$ and $\psi(M^*) = 2$;*
- *when $R_n \in \big(Th_l, Th_u\big)$, a myopic strategy is optimal, i.e. $M^*(\lambda) = 1$ and $\psi(M^*) = 1$;*
- *when $R_n \ge Th_u$, $M^* = \inf\{m > 0 : P_r \max_{j\in\mathcal{J}_n} |g_{js(n)}(m)|^2 \ge r_n - P_s|h_{s(n)}(n)|^2\}$, where $g_{\mathcal{J}_n} := P_r \max_{j\in\mathcal{J}_n} |g_{js(n)}(m)|^2$.*

Proof. When the first-hop observation stops at observation n, an optimal stopping strategy in the second hop exists[2]. By optimal stopping theory it is of form that

$$\begin{aligned}
M^* = \inf\{m \ge 0 : Y_{n,m,\psi(m)} - \lambda(T_{n,2} + T_m) \ge \\
\mathbb{E}[S_{n,m+1}|\mathscr{F}_n \vee \mathcal{G}_{nm}]\} \qquad (5)
\end{aligned}$$

The threshold S_{nm} is conditional reward defined as

$$\sup_{M\ge m} \mathbb{E}\big[Y_{n,M,\psi(M)} - \lambda(T_{n,2} + T_{M,\psi(M)})\big|\mathscr{F}_n \vee \mathcal{G}_{nm}\big]. \qquad (6)$$

[1] Note the superscript $\overline{\infty}$ means the summation includes term at $n = \infty$.

[2] For finite n, the existence proof is similar to Theorem 6, while $n = \infty$ means the main layer does not stop.

Based on optimal stopping theory, we have bellman equation that

$$\begin{aligned} S_{nm} = &\max\{Y_{n,M,\psi(M)} - \lambda(T_{n,2} + T_{m,\psi(m)}), \\ &\mathbb{E}[S_{n,m+1}|\mathscr{F}_n \vee \mathcal{G}_{nm}]\}. \end{aligned} \tag{7}$$

Then, we calculate the threshold $\mathbb{E}[S_{n,m+1}|\mathscr{F}_n \vee \mathcal{G}_{nm}]$ in the right side of Eq. (5), and is shown in Eq. (8). Duration $\tau_{|\mathcal{J}_n|}$ denotes the time cost for relay channel sensing, with $\tau_{|\mathcal{J}_n|} = |\mathcal{J}_n|\tau_{RTS}+\tau_{CTS}$. The last line is from the definition of W_n, which is the maximal expected reward in sub layer.

$$\begin{aligned} &\mathbb{E}[S_{n,m+1}|\mathscr{F}_n \vee \mathcal{G}_{nm}] \\ &= \mathbb{E}\Big[\sup_{M \geq m+1} \mathbb{E}\big[Y_{n,M,\psi(M)} - \lambda(T_{n,2} + T_{M,\psi(M)})\big|\mathscr{F}_n \vee \mathcal{G}_{n,m+1}\big]\Big|\mathscr{F}_n \vee \mathcal{G}_{nm}\Big] \\ &= \mathbb{E}\Bigg[\sup_{M \geq m+1} \mathbb{E}\big[(R_n - \lambda)\tau_d \mathbb{I}\big[\psi(M) = 1\big] - \lambda\tau_{|\mathcal{J}_n|} \\ &\quad -\lambda(M-m)\tau_d\big|\mathscr{F}_n \vee \mathcal{G}_{n,m+1}\big]\Big|\mathscr{F}_n \vee \mathcal{G}_{nm}\Bigg] - \lambda\big(T_{n,2} + m\tau_d\big) \\ &\overset{(a)}{=} W_n(\lambda) - \lambda\big(T_{n,2} + m\tau_d\big) \end{aligned}$$

By substituting expression in Eq. (8) into Bellman Equation (7), we have that

$$\begin{aligned} &W_n - \lambda(T_{n,2} + (m-1)\tau_d) = \mathbb{E}\big[\max\big\{Y_{n,m,\psi(m)} - \\ &\lambda T_{m,\psi(m)}, W_n - \lambda\big(\tau_{|\mathcal{J}_n|} + m\tau_d\big)\big\}|\mathscr{F}_n\big] - \lambda T_{n,2}. \end{aligned} \tag{8}$$

And it is further simplified into the form that

$$\begin{aligned} &\mathbb{E}\big[\max\big\{Y_{n,m,\psi(m)} - \lambda\big(T_{m,\psi(m)} - (m-1)\tau_d\big) - W_n, \\ &-\lambda\big(\tau_{|\mathcal{J}_n|} + \tau_d\big)\big\}|\mathscr{F}_n\big] = 0. \end{aligned} \tag{9}$$

The reward $-W_n$ can be derived by solving the next equation. By the sub-layer optimal stopping strategy, at each observation m, whether stop or not is decided by relation between $Y_{n,m,\psi(m)} - \lambda\big(T_{m,\psi(m)} - (m-1)\tau_d\big)$ and $W_n - \lambda\big(\tau_{|\mathcal{J}_n|} + \tau_d\big)$. And by observing the above equation, the decision on $\psi(m)$ for stop is first determined, which maximizes the reward by stop.

$$\mathbb{E}\big[\max\big\{\mathbb{I}[\psi(m) = 1](Y_{n,m,1} - \lambda\tau_d) + \mathbb{I}[\psi(m) = 2]Y_{n,m,2} - W_n, -\lambda\tau_d\big\}|\mathscr{F}_n\big] = \lambda\tau_{|\mathcal{J}_n|}.$$

Recall that reward $Y_{n,m,1} - \lambda\tau_d$ and $Y_{n,m,2} - \lambda\tau_{|\mathcal{J}_n|}$ for $\psi(m) = 1$ and $\psi(m) = 2$ respectively, it is optimal to stop by taking action as follows.

- when $(R_n - \lambda)\tau_d > -\lambda\tau_{|\mathcal{J}_n|}$ and $g_{\mathcal{J}_n}(m) \geq r_n - P_s|h(n)|^2$, we have $\psi(m) = 1$;
- otherwise, we have $\psi(m) = 2$.

Following a similar line, we analyse the Bellman Equation. For simplicity, we define $P_n := \mathbb{P}[g_{\mathcal{J}_n} \geq r_n - P_s|h(n)|^2]$.

When $(R_n - \lambda)\tau_d > -\lambda\tau_{|\mathcal{J}_n|}$, there are two cases.

Case 1: if $R_n\tau_d \geq W_n$ and $W_n \leq \lambda\tau_d$, we have

$$W_n = P_n \cdot (R_n - \lambda)\tau_d - \lambda\tau_{|\mathcal{J}_n|}. \tag{10}$$

Case 2: if $R_n\tau_d \geq W_n$ and $W_n > \lambda\tau_d$, we have

$$W_n = R_n\tau_d - \frac{\lambda(\tau_d + \tau_{|\mathcal{J}_n|})}{P_n}. \tag{11}$$

Case 3: otherwise, when $(R_n - \lambda)\tau_d \leq -\lambda\tau_{|\mathcal{J}_n|}$, we have that

$$W_n = -\lambda\tau_{|\mathcal{J}_n|}. \tag{12}$$

Based on three cases above, we further analyze the form of optimal strategy in sub layer.

1. When $R_n < \lambda\frac{\tau_d - \tau_{|\mathcal{J}_n|}}{\tau_d}$, Case 3 applies and thus it is optimal to stop at the beginning by $\phi(m) = 2$;
2. when $R_n \in \big(\lambda\frac{\tau_d - \tau_{|\mathcal{J}_n|}}{\tau_d}, \lambda + \frac{\lambda(\tau_d + \tau_{|\mathcal{J}_n|})}{P_n\tau_d}\big)$, Case 1 applies and it is optimal to stop at the beginning by $\phi(m) = 1$;
3. when $R_n \geq \lambda + \frac{\lambda(\tau_d + \tau_{|\mathcal{J}_n|})}{P_n\tau_d}$, Case 2 applies and it is optimal to stop when $g_{\mathcal{J}_n}(m) \geq r_n - P_s|h(n)|^2$ satisfies.

Theorem 3 presents an optimal strategy $M^*(\lambda)$ in the sub layer. It is conditioned on the observed information in the main layer and the broadcast rate R_n. On this basis, an optimal strategy in the main layer, denoted as $\{N^*, R^*_{N^*}\}$ is focused. In the following, the problem is decomposed into two levels: the optimal stopping time N^* at higher level and its associated optimal transmission manner $\{\phi(N^*), R^*_{N^*}\}$ at lower level. For finding optimal stopping time in the main layer, we define optimal expected reward $\{G_n\}_{n=1,2,...}$ such that $G_n := \sup_{R_n \geq 0}\{(Y_n - \lambda T_n)\mathbb{I}[\phi(n) = 1] + W_n(\lambda)\mathbb{I}[\phi(n) = 2]\}$, which represents the maximal expected reward achieved by taking optimal actions for stop in the main layer and optimal strategy in the sub layer.

Theorem 4. *In the main layer, for a price $\lambda > 0$, the problem of finding an optimal strategy $\{N^*, R^*_{N^*}\}$ achieving $\sup_{N,R_N}\mathbb{E}\big[(Y_N - \lambda T_N)\mathbb{I}[\phi(N) = 1] + W_N(\lambda)\mathbb{I}[\phi(N) = 2]\big]$ is equivalently decomposed as that: in the lower level, at each observation $n > 0$, find optimal stop manner $\phi(n)$ and transmitting rate $R^*_n := arg \sup_{R_n \geq 0}\{(Y_n - \lambda T_n)\mathbb{I}[\phi(n) = 1] + W_n(\lambda)\mathbb{I}[\phi(n) = 2]\}$; in the higher level, find optimal stopping rule $N^* := arg \sup_{N>0}\mathbb{E}[G_N]$.*

$$
\begin{aligned}
&\mathbb{E}[Y_{N^\dagger}(R^\dagger_{N^\dagger}) - \lambda T_{N^\dagger}] \\
&= \mathbb{E}\Big[\sum_{n=1}^{\overline{\infty}} \mathbb{I}[N^\dagger = n]\big(Y_n(R_n^\dagger) - \lambda T_n\big)\Big] \\
&\quad \leq \mathbb{E}\Big[\sum_{n=1}^{\overline{\infty}} \mathbb{I}[N^\dagger = n] \sup_{R_n \geq 0} \big\{(Y_n - \lambda T_n)\mathbb{I}[\phi(n) = 1] + W_n(\lambda)\mathbb{I}[\phi(n) = 2]\big\}\Big] \\
&= \mathbb{E}\big[\sum_{n=1}^{\overline{\infty}} \mathbb{I}[N^\dagger = n] \cdot G_n\big] \leq \sup_{N \geq 0} \mathbb{E}\left[G_N\right] = \mathbb{E}\left[G_{N^*}\right]
\end{aligned} \tag{13}
$$

Proof. For any transmission strategy $\{N^\dagger, R^\dagger_{N^\dagger}\}$ in main layer, we derive an upper bound in Eq. (13). It is proved that $\mathbb{E}\left[G_{N^*}\right]$ is an upper bound for any feasible rules, and $\mathbb{E}\left[G_{N^*}\right]$ is achieved by following a stopping strategy N^* with its associated transmission manner such that reward G_n is attained for stop at observation n.

Based on the optimal rule's structure in Theorem 4, the optimal stopping strategy N^* in the top level relies on the reward sequence $\{G_n\}_{n\in\mathbb{N}}$, where $\mathbb{N}$ is the positive integer set. This sequence is acquired by solving problems in lower level. In this regards, for each observation n, we focus on derivation of the reward function G_n. Recognizing that the component λT_n in function G_n is independent with rate R_n, it suffices to maximize the following function, which is derived by using analytic form of $W_n(\lambda)$. Replacing W_n in Eqs. (10), (11) and (12), we have the objective function to maximize in Eq. (14).

$$
\begin{aligned}
&\mathbb{I}[R_n < Th_l] \cdot (-\lambda\tau_{|\mathcal{J}_n|}) + \mathbb{I}[Th_l \leq R_n \leq Th_u] \cdot \big(R_n\tau_d - \frac{\lambda(\tau_d + \tau_{|\mathcal{J}_n|})}{P_n}\big) \\
&+ \mathbb{I}[R_n > Th_u] \cdot \big(P_n \cdot (R_n - \lambda)\tau_d - \lambda\tau_{|\mathcal{J}_n|}\big)
\end{aligned} \tag{14}
$$

For stop by $\phi(n) = 2$, i.e. two-stage transmission.

To maximize the objective in Eq. (14), we solve the problem in piece-wise region. In the following, the maximization problem is analyzed.

First, we consider the problem when $R_n \leq Th_l$.

The function $P_n \cdot (R_n - \lambda)\tau_d - \lambda\tau_{|\mathcal{J}_n|}$ is to maximized based on channels gains $\mathcal{F}_n$. Observing that the variable $|\mathcal{J}_n|$ depends on the $\{R_n, \mathcal{F}_n\}$, it requires to analyze the function when $|\mathcal{J}_n| = 1, 2, ..., L$.

Therefore, we define functions Z_k, $k = 1, 2, ..., L$ below:

$$
Z_k = P_n(k) \cdot (R_n - \lambda)\tau_d - \lambda\tau_k
$$

where $P_n(k) = 1 - \Big(1 - e^{-\frac{r_n - P_s|h(n)|^2}{\sigma_g^2}}\Big)^k$.

Properties of these functions are taken into accounts in the following lemma.

Lemma 1. *Function Z_k has unique maximal solution for $k = 1, 2, ..., L$, denoted as ζ_k respectively. In particular, when $r_n \leq \zeta_k$, Z_k increases, and when $r_n > \zeta_k$, Z_k decreases. The maximal solutions satisfy that $P_s|h(n)|^2 \leq \zeta_1 < \zeta_2 < ... < \zeta_L$.*

Proof. See Appendix A.

Second, we consider the problem when $Th_l \leq R_n \leq Th_u$.

The function $R_n\tau_d - \frac{\lambda(\tau_d - \tau_{|\mathcal{J}_n|})}{P_n}$ is to maximized based on channels gains $\mathcal{F}_n$. We define functions U_k, $k = 1, 2, ..., L$ below:

$$U_k = \tau_d \log_2(1 + r_n) - \frac{\lambda(\tau_d - \tau_k)}{1 - \left(1 - e^{-\frac{r_n - P_s|h(n)|^2}{\sigma_g^2}}\right)^k}.$$

Properties of functions U_k, $k = 1, 2, ..., L$ are taken into accounts in the following lemma.

Lemma 2. *For $k = 1, 2, ..., L$, function U_k is strictly concave in $r_n \geq 0$; if $P_s|h(n)|^2 > \frac{\tau_d\sigma_g^2 \log_2 e}{\lambda(\tau_{CTS} + \tau_d)} - 1$, the optimal point of U_1, denoted by η_1, is equal to $P_s|h(n)|^2$; otherwise, for $k \geq 1$, the optimal point of U_k, denoted by η_k, is equal to $x_k + P_s|h(n)|^2$ such that $\frac{\partial U_k(x)}{\partial x}\big|_{x_k} = 0$, respectively. Also, $P_s|h(n)|^2 \leq \eta_1 < \eta_2 < ... < \eta_L$ is satisfied.*

Denote $Z_k^* := Z_k(\zeta_k)$ as the maximum of Z_k in $r_n \geq P_s|h(n)|^2$. As function Z_k increases with the number k of available relays in the second hop, it is obvious that $Z_1^* < Z_2^* < ... < Z_L^*$.

Moreover, by definition of functions $U_k, k = 1, 2, ..., L$, it is shown that $U_1 < U_2 < ... < U_L$ on $r_n \geq 0$. By denoting $U_k^* = U_k(\eta_k)$, $U_1^* < U_2^* < ... < U_L^*$ is satisfied.

Then, combing the function analysis above, we focus on finding the optimal broadcast rate R_n.

We first investigate the region of r_n such that $R_n \geq Th_u$.

Using properties of functions $\{Z_k\}_k = 1, ..., L$ and relations, we define an integer κ that

$$\kappa(\lambda) = \min\{k = 1, 2, ..., L : Z_k^* \geq \lambda\tau_d\}.$$

Correspondingly, regions $\{\zeta_k^-, \zeta_k^+\}$ for $k \geq \kappa'$ are further defined, and $r_n \in \{\zeta_k^-, \zeta_k^+\}$ satisfies $R_n \geq Th_u$. Also, for $\forall i \geq k$, it is proved that $\zeta_i^- \geq \zeta_{i+1}^-$ and $\zeta_{i+1}^+ \geq \zeta_i^+$.

Based on the region above, for channel information $\mathcal{F}_n$, we need to design an algorithm to determine the point maximizing the function. By sorting the first-hop channel gains in descending order as $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_L$, the region such that $R_n \geq Th_u$ is $\cup_{i=1}^q \big\{[\zeta_i^-, \zeta_i^+] \cap (\max\{\gamma_{i+1}(n), P_s|h_{s(n)}(n)|^2\}, \gamma_i(n)]\big\}$, where integer q satisfying $\gamma_{q+1}(n) \leq P_s|h(n)|^2 < \gamma_q(n)$.

As the process acquiring the optimal rate R_n^* within the region demands high complexity, further investigation is made to design an algorithm which significantly reduces the complexity. Algorithm 1 is presented in the following. In it, we define $\kappa^\dagger$ as the least integer κ such that $[\zeta_\kappa^-, \zeta_\kappa^+] \cap (\gamma_{\kappa+1}(n), \gamma_\kappa(n)] \neq \varnothing$. Moreover, for $k = 1, 2, ..., L$, truncated functions $U_k'(\gamma_k(n)) := U_k(\gamma_k(n))\mathbb{I}[\gamma_k(n) \geq \zeta_k^-]$

are defined. Also, we use $H_n := Y_{n,1} - \lambda\tau_d$ to denote the reward by instantaneous transmission.

Algorithm 1. Algorithm to determine optimal broadcast rate

Calculate integer q and κ' based on channel gain $\mathcal{F}_n$.
if $\gamma_q(n) \geq \max\{\eta_q, \zeta_q^+\}$ **then**
    $r_n^* = \arg\max\{U_q(\min(\eta_q, \zeta_q^+)), H_n\}$.
else
    Set $k = q - 1$.
    while $k > \kappa'$ **do**
        if $\gamma_k(n) \geq \max\{\eta_k, \zeta_k^+\}$ **then**
            if $\gamma_{k+1}(n) < \eta_k$ **then**
                $r_n^* = \arg\max_{q \geq i \geq k+1}\{U_k(\min(\eta_k, \zeta_k^+)), U_i'(\gamma_i(n)), H_n\}$.
            else if $\gamma_{k+1}(n) \geq \eta_k$ **then**
                $r_n^* = \arg\max_{q \geq i \geq k+2}\{U_{k+1}(\gamma_{k+1}(n)), U_i'(\gamma_i(n)), H_n\}$.
            end if
            $k = k - 1$.
        end if
    end while
    if $\gamma_{\kappa^\dagger}(n) < \max\{\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+\}$ **then**
        if $\gamma_{\kappa^\dagger}(n) < \min(\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+)$ **then**
            $r_n^* = \arg\max_{q \geq i \geq \kappa^\dagger}\{U_i'(\gamma_i(n)), H_n\}$
        else
            $r_n^* = \arg\max_{q \geq i \geq \kappa^\dagger+1}\{U_{\kappa^\dagger}(\min(\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+)), U_i'(\gamma_i(n)), H_n\}$.
        end if
    end if
end if

The effective of our proposed algorithm is guaranteed by the following theorem.

Theorem 5. *Based on observed information $\mathscr{F}_n$ until observation n in main layer, Algorithm 1 solves the optimal rate R_n^* achieving G_n.*

Proof. Due to page limit, the proof is omitted.

Based on Theorem 5, the optimal transmission rate $R_n^* := \arg\sup_{R_n \geq 0}\{(Y_n - \lambda T_n)\mathbb{I}[\phi(n) = 1] + W_n(\lambda)\mathbb{I}[\phi(n) = 2]\}$is derived, which is followed by multiple sources. According to Theorem 4, it remains an optimal stopping strategy N^*. In the sequel, we target in finding an optimal stopping strategy in main layer achieving $\sup_{N>0}\mathbb{E}[G_N(\lambda)]$.

Using the statistical characteristics of observed channel gains in the network, which are independent and identically distributed, the reward function G_n conditioned on the first-hop channel gains $\{h_{s(n)}(n), f_{s(n)1}(n), ..., f_{s(n)L}(n)\}$ remains invariant along observation n, And a following conclusion is derived.

Theorem 6. *The optimal stopping rule* N^* *achieving* $\sup_{N>0}\mathbb{E}[G_N(\lambda)]$ *exists and is given by* $N^* = \inf\{n > 0 : G_n(\lambda^*) \geq 0\}$*, where the optimal throughput* λ^* *uniquely satisfies* $\mathbb{E}[\max\{G_n(\lambda^*), 0\}] = \lambda^*\tau_o$*.*

Notably, as the maximal average throughput λ^* is achieved by using our joint optimal strategy, according to Theorem 6 it is inferred that a unique-form second-hop optimal rule exits, which is $M^* = \inf\{m > 0 : P_r \max_{j\in\mathcal{J}_n} |g_{js(n)}(m)|^2 \geq r_n\}$.

According to Theorems 2 and 4, our optimal OCA strategy is decomposed into three-level form: at the top level, an optimal stopping strategy is derived to decide when sources to stop (i.e., broadcast or directly transmit), at the medium level, an optimal transmitting rate is demanded to decide how a winner source to stop, and at the bottom level, another optimal strategy at second hop is to be decided on when relay(s) to stop (i.e., forward data to destinations).

Combing the results in Theorems 3, 5 and 6, our proposed optimal OCA strategy can be implemented as follows.

For channel access of multiple sources, upon a successful contention in the observation n, the winner source $s(n)$ obtains information $\mathcal{F}_n$, sorts the first-hop channel gains by $\{\gamma_1(n) \geq \gamma_2(n) \geq ... \geq \gamma_q(n) \geq P_s|h_{s(n)}(n)|^2\}$. If an integer q does not exist, the broadcast rate R_n is 0; otherwise, calculate $\{\zeta_k^-, \zeta_k^+\}$ for $k \in \{q, q-1, ..., 1\}$, pick up every $k' \in \{q, q-1, ..., 1\}$ such that $\gamma_{k'+1}(n) < \zeta_{k'}^+$ and $\gamma_{k'}(n) \geq \zeta_{k'}^-$; then records the minimal k' as $\kappa^\dagger$; if such k' does not exist, the broadcast rateR_n is 0; otherwise, for each step k falling from q to $\kappa^\dagger$, Source $s(n)$ acts as follows until the broadcast rate r_n^* is acquired.

– If $\gamma_k(n) \geq \max\{\eta_k, \zeta_k^+\}$
  - If $\gamma_{k+1}(n) < \eta_k$, $(r_n^*, Y_n^*) = \mathbf{v}_k^1$.
  - If $\gamma_{k+1}(n) \geq \eta_k$, $(r_n^*, Y_n^*) = \mathbf{v}_k^2$.
– If $\gamma_k(n) < \max\{\eta_k, \zeta_k^+\}$, go into next comparison until $\kappa^\dagger$.
– If $k = \kappa^\dagger$ and $\gamma_{\kappa^\dagger}(n) < \max\{\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+\}$
  - If $\gamma_{\kappa^\dagger}(n) < \min(\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+)$, $(r_n^*, Y_n^*) = \mathbf{v}_{\kappa^\dagger}^3$.
  - If $\gamma_{\kappa^\dagger}(n) \geq \min(\eta_{\kappa^\dagger}, \zeta_{\kappa^\dagger}^+)$, $(r_n^*, Y_n^*)) = \mathbf{v}_{\kappa^\dagger}^1$.

Then, Source $s(n)$ decides if or not to stop.

– If $\max\{Y_n^*, H_n\} < \lambda^*\tau_d$, Source $s(n)$ gives up its transmission opportunity and re-contends with other sources.
– If $\max\{Y_n^*, H_n\} \geq \lambda^*\tau_d$,
  - If $Y_n^* \leq H_n$, Source $s(n)$ transmits its data to Destination $s(n)$ in direct manner.
  - If $Y_n^* > H_n$, Source $s(n)$ broadcasts its data in rate R_n^*, and the channel probing of relays starts. The relays in $\mathcal{J}_n$ decode the signals, and based on observed channel conditions they have to decide if or not to forward data. In the mth observation, each relay, says Relay $j \in \mathcal{J}_n$, has information of channel gain $g_{js(n)}$. If $P_r|g_{js(n)}|^2 < r_n^* - P_s|h(n)|^2$, it keeps silent; otherwise, it forwards the data to Destination $s(n)$ in best single-relay transmission. After relays' transmission, a two-hop transmission is finished and a new contention is started among all sources.

It is notable that, the threshold λ^* deciding if or not to stop at first hop is calculated off-line. In the process to decide the broadcast rate, the thresholds $\{\zeta_l^+, \zeta_l^-, \eta_l\}_{l=1,2,...,q}$ are calculated; also, only one of vectors $\{\mathbf{v}_k^1, \mathbf{v}_k^2, \mathbf{v}_k^3\}$ demand to obtain for the step where the condition is satisfied (i.e., it is calculated once for each-round decision).

$$\mathbf{v}_k^1 := (\arg \max_{q \geq i \geq k+1} \{U_k(\min(\eta_k, \zeta_k^+)), U_i'(\gamma_i(n))\}, \max_{q \geq i \geq k+1} \{U_k(\min(\eta_k, \zeta_k^+)), U_i'(\gamma_i(n))\}),$$

$$\mathbf{v}_k^2 := (\arg \max_{q \geq i \geq k+2} \{U_{k+1}(\gamma_{k+1}(n)), U_i'(\gamma_i(n))\}, \max_{q \geq i \geq k+2} \{U_{k+1}(\gamma_{k+1}(n)), U_i'(\gamma_i(n))\}),$$

$$\mathbf{v}_k^3 := (\arg \max_{q \geq i \geq k} \{U_i'(\gamma_i(n))\}, \max_{q \geq i \geq k} \{U_i'(\gamma_i(n))\})$$

4 Performance Evaluation

In this section, system performance for our proposed strategy is investigated through numerical simulations. We consider a wireless cooperative network with 15 source-destination pairs under the help of 6 relays. The probability that a source sends a RTS in a mini-slot is $p = 0.1$, and channel coherence time is $\tau_d = 8$ ms. RTS transmission duration is $\tau_{RTS} = 40$ µs, CTS transmission duration is $\tau_{CTS} = 40$ µs, and mini-slot duration $\delta = 20$ µs.

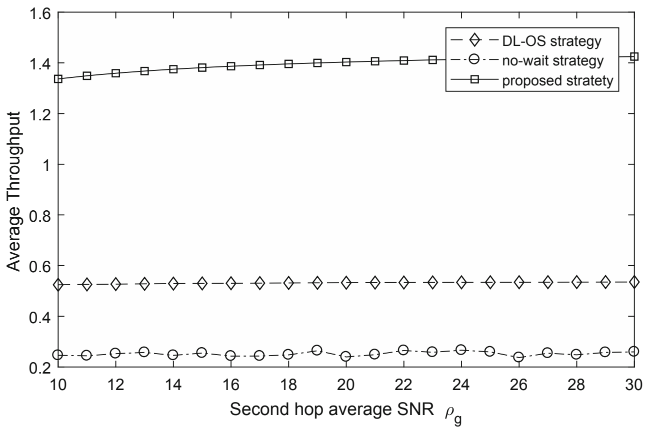

Fig. 2. Average system throughput with $\rho_g = 2 \cdot \rho_f$

Various average SNR configurations are considered. The second-hop average SNR ρ_g is from 10 dB to 30 dB, and the first-hop average SNR $\rho_f = \rho_g/2$ and $\rho_f = \rho_g/4$, respectively. To evaluate performance, the average system throughput is investigated. To verify the performance enhancement, we also compare the average system throughput with other two strategies, which are no-wait strategy and direct-link optimal stopping (DL-OS) strategy. In particular, under no-wait strategy, each source accesses the channel through direct transmission without wait, while under DL-OS strategy each source senses and accesses the direct link channel using optimal stopping strategy.

Considering different SNR configuration, two figures are shown as follows.

Figures 2 and 3 show the system performance with second-hop average SNR $\rho_g = 2 \cdot \rho_f$ and $\rho_g = 4 \cdot \rho_f$. In both figures, three curses are presented, representing the average system throughput by following our proposed strategy, no-wait strategy and DL-OS strategy, respectively. It can be seen that the average throughput with the proposed strategy performs much better than other strategies, and significant performance enhancement is harvested.

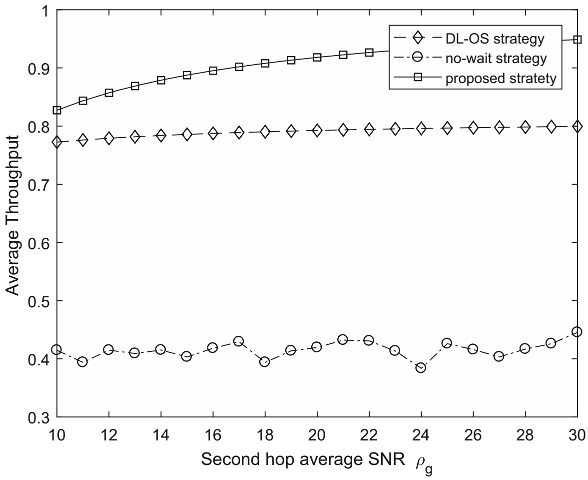

Fig. 3. Average system throughput with $\rho_g = 4 \cdot \rho_f$

5 Conclusion

In a wireless network aided by multiple relays, independent channel fading is experienced in both sources and relays. To enhance the spectrum efficiency, a joint exploitation of the multi-source diversity, multi-relay diversity, and time diversity is desired, and efficient distributed scheduling is motivated. To make a full exploitation of these diversities, the distributed DOS problem is investigated in our research. Formulating the problem as a three-level optimal stopping problem, an optimal strategy is proposed guiding distributed channel access for multiple source-to-destination communications under the help of multiple relays. The optimality of the strategy is rigorously proved, and easy implementation is presented of low complexity. The close-form expression of the maximal expected system throughput is also derived. This research should provide insights to the design of channel-aware MAC protocols in distributed cooperative network. Further research may involve the cases with quantized CSI and with QoS provision.

A Proof of Lemma 1

For relay number $k \in \{1, 2, ..., L\}$, we define $r'_n := r_n - P_s|h(n)|^2$. For function Z_k, the first-order derivative is calculated as that

$$\frac{\partial Z_k}{\partial r'_n} = \tau_d \Big(\frac{\log_2 e}{1+r'_n+P_s|h(n)|^2}(1-(1-e^{-\frac{r'_n}{\sigma_g^2}})^k) - \frac{1}{\sigma_g^2} e^{-\frac{r'_n}{\sigma_g^2}}$$
$$(1-e^{-\frac{r'_n}{\sigma_g^2}})^{k-1} k\big(\log_2(1+r'_n+P_s|h(n)|^2) - \lambda\big)\Big).$$

By replacing $1-e^{-\frac{r'_n}{\sigma_g^2}}$ with $y(r'_n)$, the derivative is rewritten in Eq. (15).

$$\begin{aligned} \frac{\partial Z_k}{\partial r'_n} &= (1-y(r'_n))ky^{1-k}(r'_n)\Big(-\frac{\tau_d}{\sigma_g^2}\big(\log_2(1+r'_n+P_s|h(n)|^2) - \lambda\big) \\ &+ \tau_d \log_2 e \frac{1}{k} \sum_{i=0}^{k-1} y^{-i}(\bar{R}_n) \frac{1}{1+r'_n+P_s|h(n)|^2} \Big). \end{aligned} \quad (15)$$

In the region $r'_n \geq 0$, the factor in Eq. (15) satisfies $(1 - y(r'_n))y^{1-k}(r'_n) > 0$. Therefore, it suffices to compare $\frac{\tau_d \log_2 e}{1+r'_n+P_s|h(n)|^2} \sum_{i=0}^{k-1} y^{-i}(r'_n)$ and $\frac{1}{\sigma_g^2} k\big(\log_2(1+r'_n+ P_s|h(n)|^2)\tau_d - \lambda\tau_d\big)$ to determine the derivative.

It can be proved that $\sum_{i=0}^{k-1} y^{-i}(r'_n) \frac{1}{1+r'_n+P_s|h(n)|^2}$ is decreasing and $\log_2(1 + r'_n + P_s|h(n)|^2) - \lambda$ is increasing in $r'_n \geq 0$, respectively. Meanwhile, as r'_n is large, $\sum_{i=0}^{k-1} y^{-i}(r'_n)$ will approach to k and $\frac{1}{1+r'_n+P_s|h(n)|^2}$ is small, while $\{\log_2(1 + r'_n + P_s|h(n)|^2)\tau_d - \lambda\tau'_d\}$ is large. Thus, the existence of stationary point, denoted by $\zeta_k - P_s|h(n)|^2$ is guaranteed, which are unique such that $\frac{\partial Z_k}{\partial r'_n} = 0$.

As a result, function Z_k increases in $r_n < \zeta_k$, and decreases in $r_n \geq \zeta_k$.

Then, relation of these points $\{\zeta_k\}, k = 1, 2, ..., L$ is further investigated.

For $k > 1$, by valuing $r'_n = 0$, we have the derivative satisfy

$$\frac{\partial Z_k}{\partial r'_n}\Big|_{r'_n=0} = \frac{\log_2 e \cdot \tau_d}{1+P_s|h(n)|^2} > 0. \quad (16)$$

which means $\zeta_k > P_s|h(n)|^2$.

For $k = 1$, we have the derivative satisfy that

$$\frac{\partial Z_k}{\partial r'_n}\Big|_{r'_n=0} = \frac{\tau_d \log_2 e}{1+P_s|h(n)|^2} - \frac{\log_2(1+P_s|h(n)|^2)\tau_d - \lambda\tau_d}{\sigma_g^2}. \quad (17)$$

Suppose ζ_1 satisfies $\frac{\partial Z_k}{\partial r'_n} = 0$, we compare the points $\{\zeta_1, \zeta_2, ..., \zeta_L\}$. The situation where $\zeta_1 = P_s|h_{s(n)}(n)|^2$ is similar as $\zeta_k > P_s|h_{s(n)}(n)|^2$ for $\forall k \geq 2$.

Since ζ_k such that $\frac{\partial Z_k}{\partial R_n} = 0$, by induction it suffices from (15) to compare $\frac{1}{k} \sum_{i=0}^{k-1} y^{-i}(r'_n)$.

Based the relation shown in the following that

$$\frac{1}{k}\sum_{i=0}^{k-1} y^{-i}(r'_n) - \frac{1}{k+1}\sum_{i=0}^{k} y^{-i}(r'_n)$$
$$= \frac{1}{k(k+1)}\Big(\sum_{i=0}^{k-1} y^{-i}(r'_n) - ky^{-k}(r'_n)\Big) < 0$$

it proves that $\zeta_k < \zeta_{k+1}$ for $\forall k = 1, 2, ..., L$–1. In other words, $\zeta_1 < \zeta_2 < ... < \zeta_L$.

References

1. Andrews, M., Kumaran, K., Ramanan, K., Stolyar, A., Whiting, P., Vijayakumar, R.: Providing quality of service over a shared wireless link. IEEE Commun. Mag. **39**(2), 150–154 (2001)
2. Viswanath, P., Tse, D.N.C., Laroia, R.: Opportunistic beamforming using dumb antennas. IEEE Trans. Inf. Theory **48**(6), 1277–1294 (2002)
3. Zheng, D., Ge, W., Zhang, J.: Distributed opportunistic scheduling for ad hoc networks with random access: an optimal stopping approach. IEEE Trans. Inf. Theory **55**(1), 205–222 (2009)
4. Ge, W., Zhang, J., Wieselthier, J., Shen, X.: PHY-aware distributed scheduling for ad hoc communications with physical interference model. IEEE Trans. Wirel. Commun. **8**(5), 2682–2693 (2009)
5. Tan, S., Zheng, D., Zhang, J., Zeidler, J.: Distributed opportunistic scheduling for ad-hoc communications under delay constraints. In: Proceedings of IEEE INFOCOM 2010 (2010)
6. Xie, L.L., Kumar, P.R.: An achievable rate for the multiple-level relay channel. IEEE Trans. Inf. Theory **51**(4), 1348–1358 (2005)
7. Laneman, J.N., Tse, D.N.C., Wornell, G.W.: Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans. Inf. Theory **50**, 3062–3080 (2004)
8. Zhang, J., Jia, J., Zhang, Q., Lo, E.M.K.: Implementation and evaluation of cooperative communication schemes in software-defined radio testbed. In: Proceedings of IEEE INFOCOM 2010 (2010)
9. Zhang, Q., Jia, J., Zhang, J.: Cooperative relay to improve diversity in cognitive radio networks. IEEE Commun. Mag. **47**(2), 111–117 (2009)
10. Gunduz, D., Erkip, E.: Opportunistic cooperation by dynamic resource allocation. IEEE Trans. Wirel. Commun. **6**(4), 1446–1454 (2007)
11. Jing, Y., Jafarkhani, H.: Network beamforming using relays with perfect channel information. IEEE Trans. Inf. Theory **55**(6), 2499–2517 (2009)
12. Zhang, Z., Jiang, H.: Distributed opportunistic channel access in wireless relay networks. IEEE J. Sel. Areas Commun. **30**(9), 1675–1683 (2012)
13. Zhang, Z., Zhou, S., Jiang, H., Dong, L.: Opportunistic cooperative channel access in distributed wireless networks with decode-and-forward relays. IEEE Commun. Lett. **19**(10), 1778–1781 (2015)
14. Ferguson, T.: Optimal stopping and applications. Mathematics Department, UCLA. http://www.math.ucla.edu/~tom/Stopping/Contents.html