



A New Method for Deriving Upper Bound of OCR-TDMA Performance

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Abstract. With the rapid development of flash memory technology, the buffer capacity of device becomes higher. In this case, infinite buffer could be introduced to derived the upper bound of system performance. In this paper, a new method which considers Enqueue rate and Dequeue rate of buffer is proposed to derive the performance of OCR-TDMA when buffer length of relay $L = \infty$. In simulation part, buffer lengths from 1 to ∞ are considered as parameters to compare system performances, and the theoretical results and simulation results match well. Therefore, the proposed method can be applied in other similar model to simplify the derivation of system performance.

Keywords: Cooperative MAC · TDMA · Throughput · Buffer

1 Introduction

In modern 5G era, the applications of edge computing and content enhance the experiences of end users in cellular network, which has less redundant transmissions, more efficient bandwidth resources and faster service response.

Actually the concept of edge computing has been proposed for several year, and one of the key points to achieve this scheme is to design reliable tactics. In [1, 2], many technologies and tactics for mobile edge computing (MEC) are studied. The idea of collaboration is a significant concept in edge computing, which utilizes the distributed end users to save and run their own demands [3, 4]. However in future cellular network, end users are not just clustered around macro base station. Micro cells and femto cells are deployed everywhere and the topology of cellular network could be multi-hop. In this case, the tactics of edge computing should be considered in a scene which contains source node, destination node and relay nodes.

The study of relay channel have over forty years history. But what made it be focused by the worldwide is some cooperative communication schemes which proposed in the end of last century. Cooperative communication utilize the available resource near the transmission link and make use of the diversity gain of Multiple-Input Multiple-Output (MIMO). Cooperative technique is able to harvest spatial diversity, increase the reliability of transmission and the throughput of whole system [5, 6].

Cooperative communication need a cooperative group work together not like the point-to-point transmission scheme. Thus a good cooperative Media Access Control (MAC) scheme can collect great performance gain. Based on 802.11 MAC system, several cooperative MAC, such as CoopMAC [7, 8], Distributed Cooperative MAC [9, 10], Cooperative Aloha [11]. However, these MAC scheme are designed for packet-based (or contention-based) networks like Ad Hoc networks or WLANs, but not for cellular networks or some other networks with high dense nodes.

Channel-based MAC schemes are widely applied in high dense network. C-TDMA [12] and CR-TDMA [13] are two TDMA-based cooperative MAC scheme proposed for this kind of network. The former use the MISO technique to improve the uplink transmission, in which many mobile devices forward their data to base station. The latter works for a mesh network, in which few helper nodes exist near the default relay works and they can help transmitting packets if they have empty buffer. In previous work [14], we proposed a new TDMA-based MAC scheme Opportunistic Cooperative Relaying TDMA (OCR-TDMA), which provides superior throughput performance compared with CR-TDMA in theoretical analysis and simulation results. But there is a improper special case in our previous analysis. In this paper, we set the buffer capacity $L = \infty$, which means R would not have full buffer anymore. This case is more practical and can be applied to some situations of which relay node has low traffic or large buffer capacity. And we also treat the performance of this model as the upper bound of OCR-TDMA.

2 System Model

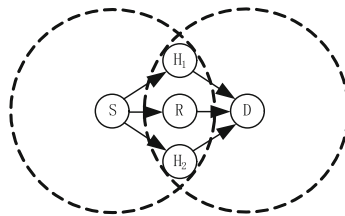


Fig. 1. System model of a two-hop relay network containing a default relay and several helpers

In this two-hop network model, there are four types of nodes, a source S, a destination D, a single relay R surrounded by a few available helpers $H_i, i \in \{1, \dots, n_h\}$, as shown in Fig. 1. Each node has only one antenna and works in half-duplex mode. R is default relay which means R will save the new relay packet from S if the buffer of R is not full while H_i have intention to receive relay packet for cooperation only if H_i is idle (empty buffer).

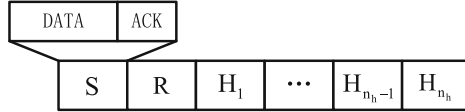


Fig. 2. Structure of a time frame containing $n_h + 2$ identical time slots

Let E_s , E_r and E_h be the transmission powers of S, R, H_i respectively. The channel from node i to node j has an channel coefficient h_{ji} and it suffers the Rayleigh fading. So the signal-to-noise ratio in receiving node is given by $SNR_{ji} = |h_{ji}|^2 E_i / N_0$, where N_0 means the power density of AWGN.

Each node has its own buffer, of which capacity for packets is L . Without considering about channel coding, each packet has N BPSK-Modulated bits. So the average BER of transmission from node i to node j can be calculated as $p_{ji} = 0.5(1 - \sqrt{\gamma_{ji}/(1 + \gamma_{ji})})$, where the average received SNR $\gamma_{ji} = E[|h_{ji}|^2] E_i / N_0$. Therefore, the probability of successful transmission is $P_{succ,ji} = (1 - p_{ji})^N$, while the probability of occurring error is $P_{err,ji} = 1 - P_{succ,ji}$.

Considering this is a TDMA-based system, as shown in Fig. 2, each node of S, R, H_i is allocated to a time slot in every frame. These nodes could only send packet in their own time slot. After transmitting, each time slot reserves a very short period for returning ACK. Thus each frame contains $n_h + 2$ time slots.

In the first time slot of each frame, we assume that the buffer of S is not empty and all packets in buffer are addressed to D. Actually, besides receiving the relay packets from S, R and H_i will generate non-relay packets by themselves and these packets are not forwarded to D. σ_{nr} is the generating rate of non-relay packet in each time slot at R while it is simplified in H_i that the probability of empty buffer is P_{idle} .

3 Performance Analysis

Previously [14], the performance analysis of OCR-TDMA with a special case that $P_{succ,dh} = 1$. has completed. But this case is not practical in a realistic environment. In this paper we proposed a more practical case that the buffer of R has infinite capacity which means $L = \infty$ (“the buffer of R” would be replaced by “buffer” in the following passage). Comparing with limited buffer, R can have more opportunities to transmit relay packet. In this section we divide the analysis into two situations, Infinite packets in buffer and Almost no packet in buffer.

Before discussing these two circumstances, we propose two variables, Enqueue rate and Dequeue rate. Enqueue rate means the rate when packet enters the buffer and Dequeue rate refers to the rate when packet goes out of the buffer. In this paper, the packets, which will access the buffer, are composed of relay packets and non-relay packets. Meanwhile, there are two approaches that packets leave the buffer, one is transmitted by R itself and another is cooperated by helper nodes (Fig. 3).

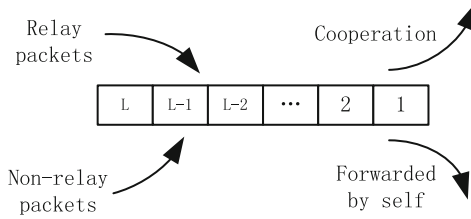


Fig. 3. The situations of packets entering and leaving R’s buffer

When the buffer capacity is limitless, Enqueue rate and Dequeue rate will not be influenced by L . So if Enqueue rate is larger than Dequeue rate, buffer would be filled frame by frame and the length will be infinite finally, otherwise the buffer length would always fluctuate above zero.

3.1 Infinite Packets in Buffer

Analyze Features. Although the buffer is empty in the beginning, which is only a very short period during the whole working time, it has little impact on the sum throughput if Enqueue rate is faster than Dequeue rate. So we assume this system start working with infinite buffer. In this case, the system possesses two features:

- (a) The packets which at the tail of Buffer cannot reach the head of buffer if this packet was held by H_i . Because the probability, of which H_i didn’t help transmit the relay packet to D successfully before this packet reach the head of buffer, is nearly 0 (maybe it could happen after a very very long time about 10000 years, this situation can be neglected now).
- (b) The packets which at the head of buffer will not be held by H_i . On the one hand, H_i will absolutely not get any non-relay packets. On the other hand, the relay packet, as mentioned in (a), can be held by H_i with nearly 0 probability.

When ensuring these two features, we can separate the throughput into two parts. Here are the packet at the head of buffer which could be forwarded by R only, as well as a new relay packet which can only be transmitted by H_i as long as H_i get it, or it will get into the buffer as normal if R get it while H_i not.

Analyze the Situation at Buffer’s Tail. If R received a new relay packet but H_i didn’t, this packet would be a member of buffer and it would distribute steadily all over the buffer in accompanied with other relay packets and non-relay packets. After a very long time it will reach the head of buffer and be forwarded by R. But if H_i received this relay packet, we consider this packet will certainly be transmitted by H_i and it will not exist in Buffer whether R has received it or not.

We propose a vector $[y_1, y_2, \dots, y_{n_h}]$ to characterize the states of helper nodes. Helper nodes can hold n_h different relay packets at most so the range of y_i can be $[0, n_h]$, where 0 means there is no relay packet in this helper node and $[1, n_h]$ represents disparate relay packets.

After deriving all state vectors, we put all the state vectors in one collection \mathbf{y} with a fixed order. The next task is to calculate all the transition probability from state i to state j , where i and j are the sequence numbers of the i_{th} and the j_{th} state vectors in \mathbf{y} respectively. In order to simplify the calculation, we construct two transition sub-matrix \mathbf{B}_{hs} and \mathbf{B}_{dh} to characterize the procedures of $S \rightarrow H$ and $H \rightarrow D$. The final matrix \mathbf{B} is given by:

$$\mathbf{B} = [\mathbf{B}_{j,i}]_{N \times N} = \mathbf{B}_{hs} \mathbf{B}_{dh} = [P_{hs(j,i)}]_{N \times N} [P_{dh(j,i)}]_{N \times N} \quad (1)$$

where N is the number of state vectors.

To calculate P_{hs} and P_{dh} , we propose another two vectors $[t_{h_1s}, \dots, t_{h_{n_h}s}]$ and $[t_{dh_1}, \dots, t_{dh_{n_h}}]$ which represent whether $S \rightarrow H$ or $H \rightarrow D$ is successful or not respectively. For instance, if $[t_{h_1s}, t_{h_2s}, t_{h_3s}] = [1, 0, 1]$, that means $S \rightarrow H_1$, $S \rightarrow H_3$ are successful but $S \rightarrow H_2$ is failed. Meanwhile the probability of procedure $S \rightarrow H_i$ can be calculated as:

$$P_{s \rightarrow h}(t_{h_1s}, \dots, t_{h_{n_h}s}) = \prod_{i=1}^{n_h} [P_{succ,hs}(t_{h_i s} = 1) + P_{err,hs}(t_{h_i s} = 0)] \quad (2)$$

Therefore the algorithm of generating matrix \mathbf{B}_{hs} is given by Algorithm 1. where $\mathbf{y}[i]$ is the i_{th} vector element in \mathbf{y} and $Index(\mathbf{y}, \mathbf{x})$ is a function which find the sequence number of \mathbf{x} in \mathbf{y} , and then return this number as result.

As same as the above, the probability of procedure $H_i \rightarrow D$ can be calculated as

$$P_{h \rightarrow d}(t_{dh_1}, \dots, t_{dh_{n_h}}) = \prod_{i=1}^{n_h} [P_{succ,dh}(t_{dh_i} = 1) + P_{err,dh}(t_{dh_i} = 0)] \quad (3)$$

and the algorithm of generating matrix \mathbf{B}_{dh} is given by Algorithm 2.

After finishing the construction of \mathbf{B}_{hs} and \mathbf{B}_{dh} , we derive the final matrix \mathbf{B} . Let $\Pi_h = [\pi_1, \pi_2, \dots, \pi_N]$ be the vector of the steady probability of state vectors. Then we can derive the results Π_h from solving the equation $\Pi_h \mathbf{B} = \Pi_h$.

In this system model, if H_i has received relay packets, we consider these packets will certainly reach D. Then the throughput is given by

$$Th_{A,tail} = P_{recv,H} \quad (4)$$

where $P_{recv,H}$ represent the probability that helper nodes could receive a new relay packet, which is given by

$$P_{recv,H} = \sum_{i=1}^N \pi_i \left[1 - (1 - P_{succ,hs} P_{idle})^{N_0(\mathbf{y}[i])} \right] \quad (5)$$

in this formula, $N_0(\mathbf{y}[i])$ represent the number of 0 in the i_{th} state vectors of \mathbf{y} .

Algorithm 1. Calculation of $P_{hs(j,i)}$

```

1: Previous State: ( $i$ )
   Initialization:
2: for  $j = 1$  to  $N$  do
3:    $P_{hs(j,i)} = 0$ 
4: end for
   Calculation:
5: for  $index = 0$  to  $2^{n_h} - 1$  do
6:    $(t_{h_{1s}}, \dots, t_{h_{n_h s}}) = dec2bin(index)$ 
7:    $\mathbf{x} = \mathbf{y}[i]$ 
8:    $x_{max} = \max\{\mathbf{x}\}$ 
9:   for  $k = 1$  to  $n_h$  do
10:    if  $t_{h_{ks}} = 1$  then
11:       $\mathbf{x}[k] \leftarrow (x_{max} + 1)(\mathbf{x}[k] = 0) + \dots$ 
         $\mathbf{x}[k](\mathbf{x}[k] > 0)$ 
12:    end if
13:  end for
14:   $j = Index(\mathbf{y}, \mathbf{x})$ 
15:   $P_{hs(j,i)} \leftarrow P_{hs(j,i)} + P_{s \rightarrow h}(t_{h_{1s}}, \dots, t_{h_{n_h s}})$ 
16: end for

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Algorithm 2. Calculation of $P_{dh(j,i)}$

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1: Previous State: ( $i$ )
   Initialization:
2: for  $j = 1$  to  $N$  do
3:    $P_{dh(j,i)} = 0$ 
4: end for
   Calculation:
5: for  $index = 0$  to  $2^{n_h} - 1$  do
6:    $(t_{h_{1s}}, \dots, t_{h_{n_h s}}) = dec2bin(index)$ 
7:    $\mathbf{x} = \mathbf{y}[i]$ 
8:    $x_{max} = \max\{\mathbf{x}\}$ 
9:   for  $k = 1$  to  $n_h$  do
10:    if  $t_{h_{ks}} = 1$  then
11:       $x_k = \mathbf{x}[k]$ 
12:      for  $m = 1$  to  $n_h$  do
13:         $\mathbf{x}[m] \leftarrow (\mathbf{x}[m] - 1)(x_k > 0)(\mathbf{x}[m] > x_k) \dots$ 
           $+ \mathbf{x}[m](\mathbf{x}[m] < x_k)$ 
14:      end for
15:    end if
16:  end for
17:   $j = Index(\mathbf{y}, \mathbf{x})$ 
18:   $P_{dh(j,i)} \leftarrow P_{dh(j,i)} + P_{h \rightarrow d}(t_{dh_1}, \dots, t_{dh_{n_h}})$ 
19: end for

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Analyze Situation at Buffer's Head. We consider the packets which at the head of buffer will not exist in H_i , so both of these relay packets and non-relay packets have to experience an enough long period to make themselves distribute steadily. So the ratio of both non-relay packets and relay packets is given by

$$(n_h + 2)\sigma_{nr} : (1 - P_{recv,H})P_{succ,rs} \quad (6)$$

Then the throughput can be given by

$$Th_{A,head} = \frac{(1 - P_{recv,H})P_{succ,rs}}{(1 - P_{recv,H})P_{succ,rs} + (n_h + 2)\sigma_{nr}} P_{succ,dr} \quad (7)$$

where the fraction is the probability that the first packet is a relay packet and $P_{succ,dr}$ refers to the probability of successful transmission.

Final Throughput. In this system model, since a relay packet could only be transmitted by R or H_i , the final throughput can be calculate as follow

$$Th_A = Th_{A,tail} + Th_{A,head} \quad (8)$$

All the analysis above are based on the special case that the length of buffer will always keep infinite. The next section we will discuss the situation on the contrary.

3.2 Almost No Packet in Buffer

Condition of Classification. Before discussing the new case, we need to know, what is the requirement of numerical relationship that could generate a result, which Enqueue rate would slower than Dequeue rate.

Enqueue rate is composed of two parts, which are the access of both relay packets and non relay packets to the buffer. Therefore we propose a formula of Enqueue rate which can be described as

$$R_{in} = P_{succ,rs} + (n_h + 2)\sigma_{nr} \quad (9)$$

where $P_{succ,rs}$ is the probability of receiving relay packet successfully and $(n_h + 2)\sigma_{nr}$ represent the expected rate of generating non-relay packets in each frame.

Due to our obtaining of $P_{recv,H}$, Dequeue rate consists of R's transition and H_i 's help. So it can be given by

$$R_{out} = P_{succ,dr} + P_{succ,rs}P_{recv,H} \quad (10)$$

Obviously, $P_{succ,dr}$ represents the transition ability of R itself, and $P_{succ,rs}P_{recv,H}$ refers to the probability of H_i 's successful cooperation. The reason why $P_{recv,H}$ need to multiply $P_{succ,rs}$ is that in last section, we didn't consider about R's receiving when calculating $P_{recv,H}$.

The new case is under certain condition when $R_{out} > R_{in}$.

Acquirement of New \mathbf{B}' . In this section, we use an approximate method to get a new transition matrix \mathbf{B}' . New \mathbf{B}' can be derived by updating matrix \mathbf{B}_{hs} to \mathbf{B}'_{hs} . The updating method is shown below

$$P'_{hs(j,i)} = \begin{cases} P_{hs(j,i)} + [1 - (n_h + 2)\sigma_{nr}] \times \dots & i = j \\ P_{succ,rs}P_{succ,dr} \sum_{k>j} P_{hs(j,k)} & i = j \\ P_{hs(j,i)} \{1 - [1 - (n_h + 2)\sigma_{nr}] \times \dots & i > j \\ P_{succ,rs}P_{succ,dr}\} & i > j \\ 0 & i < j \end{cases} \quad (11)$$

where $P_{hs(j,i)}$ is the transition probability which transfers from state i to state j in \mathbf{B}_{hs} . Unlike the case that buffer has lower Dequeue rate, in the procedure $\mathbf{S} \rightarrow \mathbf{H}$, \mathbf{R} may get the same relay packet in \mathbf{S} slot and transmit this packet successfully in its own slot. If that happened, we consider state vectors $[y_1, y_2, \dots, y_{n_h}]$ didn't change so that the values of diagonal line in \mathbf{B}'_{hs} should be larger than it in \mathbf{B}_{hs} .

The updating factor possessed three parts as listed below:

1. The probability \mathbf{R} has no non-relay packet in its buffer is $1 - (n_h + 2)\sigma_{nr}$;
2. The probability \mathbf{R} receive relay packet successfully is $P_{succ,rs}$;
3. The probability \mathbf{R} forward this packet successfully is $P_{succ,dr}$;

In each line of \mathbf{B}'_{hs} , case $i = j$ represent \mathbf{H}_i didn't change it states after this transition, so we use the sum of the other state vectors' probability multiplies the updating factor and add this value to the element of diagonal line.

Throughput. As same as above, we can easily use the same method to get $\Pi'_h = [\pi'_1, \pi'_2, \dots, \pi'_N]$ by $\Pi'_h \mathbf{B}' = \Pi'_h$ after deriving $\mathbf{B}' = \mathbf{B}'_{hs} \mathbf{B}_{dh}$. And $P'_{recv,H}$ can be calculated by

$$P'_{recv,H} = \sum_{i=1}^N \pi'_i \left[1 - (1 - P_{succ,hs} P_{idle})^{N_0(\mathbf{y}^{[i]})} \right] \quad (12)$$

Finally we propose the formula of calculating throughput which is given by

$$Th_B = P'_{recv,H} + (1 - P'_{recv,H})P_{succ,rs} \quad (13)$$

Throughput can be increased in the following three situations, both of \mathbf{R} and \mathbf{H}_i received a same relay packet, \mathbf{H}_i got one packets while \mathbf{R} didn't or \mathbf{R} got that but \mathbf{H}_i didn't. $P'_{recv,H}$ includes the first two situations because in this section we updating the \mathbf{B} to \mathbf{B}' by considering the influence of \mathbf{R} . And it is no difficult task to understand that $(1 - P'_{recv,H})P_{succ,rs}$ is the third situation.

4 Simulations and Results

To make simulation more practical, we assume transmission powers of S, R and H_i are the same, represented by $E_i = E$. All the channels suffer the Rayleigh fading with different distance between every two nodes, resulting different SNR for each link. In the following comparison, we set the packet length $N_{bit} = 1024$, and the number of helpers $n_h = 3$.

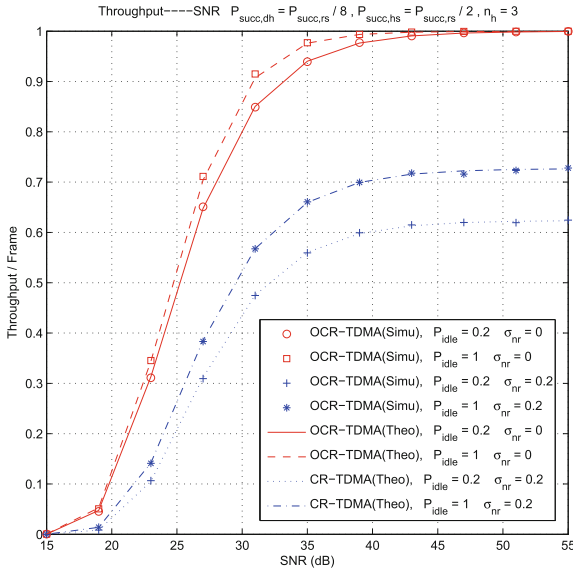


Fig. 4. The throughput curves of OCR-TDMA given different σ_{nr} and P_{idle}

Figure 4 shows the throughput of OCR-TDMA with P_{idle} chooses 0.2 or 1 and σ_{nr} chooses 0 or 0.1. The x axis is SNR, or equivalent $P_{succ,rs}$. In this simulation, buffer has infinite capacity and we set $P_{succ,dh} = P_{succ,hs}/8$, which make us control the Enqueue rate is larger than Dequeue rate or not more easily. In this figure, the red line is the case that Enqueue rate is lower than Dequeue rate and the blue line is on the contrary. The numerical results, which calculated in Part III, match the simulation results very well.

Figure 5 shows the influence of buffer capacity on the throughput and the other parameters are same as above. We set 5 simulations with L given 1, 10, 100, 1000, 100000, which represent the very small, small, medium, large and nearly infinite buffer capacity. The results present the gap between our special case and the normal case. This figure show us the numerical results with the case $L = \infty$ can be treated as proper upper bound of OCR-TDMA.

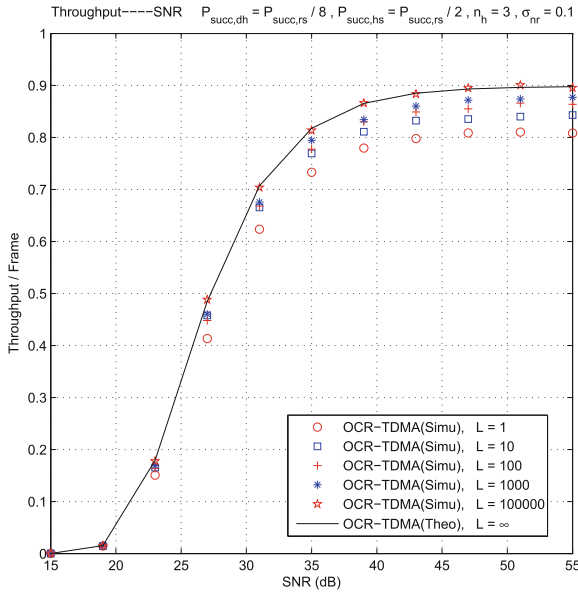


Fig. 5. The throughput curves of OCR-TDMA with different L

5 Conclusion

In this paper, we propose a more practical case $L = \infty$ to approximately calculate the throughput of OCR-TDMA and the results of this case can be treated as upper bound of OCR-TDMA. The numerical results and simulation results match very well. Meanwhile we provide a new method to solve the problem of analyzing buffer, which is very direct and useful. However, the method to evaluate the performance of OCR-TDMA with normal case still need to be explored, and this will be our future work.

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