Joint Evaluation of Spectral Efficiency, Energy Efficiency and Transmission Reliability in Massive MIMO Systems

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Abstract. The main goals planned to achieve in fifth generation (5G) networks are to increase capacity, improve data rate, decrease latency, improve energy efficiency and provide a better quality of service. To achieve these goals, massive multiple input multiple output (MIMO) is considered as one of the competing technologies that provide high spectral efficiency (SE) and energy efficiency (EE). Hence, energy efficiency, spectral efficiency and transmission reliability are the main performance metrics for massive MIMO systems. Although these performance metrics are thoroughly studied independently, their joint effects are not considered and evaluated for massive MIMO systems. Hence, in this work, we investigate a mathematical model that jointly evaluates the spectral efficiency, energy efficiency and transmission reliability in downlink massive MIMO systems with linear precoding techniques. Closed-form analytical formulation is derived that jointly evaluates the impacts of spectral efficiency and transmission reliability on energy efficiency. Finally, numerical results are provided to validate the theoretical analysis.

Keywords: Massive MIMO · Spectral efficiency · Energy efficiency · Precoding techniques · Transmission reliability

1 Introduction

By using large numbers of small physical size and inexpensive low-power antennas at the base station (BS), massive MIMO improves spectral efficiency and energy efficiency [1]. Deploying a large number of antennas at the BS helps to focus the transmission energy into a smaller region of space. With a very large number of antenna arrays, things that were random before start to look deterministic so that the effect of small-scale fading is averaged out [2]. Besides, when

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the number of BS antennas grows large, the random channel vectors between the users become pair-wisely orthogonal. In the limit of an infinite number of BS antennas, with simple matched filter processing at the BS, uncorrelated noise and intra-cell interference disappear completely [2].

The authors in [3] evaluate the achievable spectral efficiency and energy efficiency in downlink multiuser MIMO systems. To reap the benefits provided by large number of BS antennas, accurate channel state information (CSI) is necessary which makes possible to have a reliable communication. With perfect CSI, downlink precoding is employed to maximize link the performance. Equipping large number of antennas at the BS makes linear precoding techniques to perform nearly the same as nonlinear precoding techniques [4]. A comprehensive review of precoding techniques in massive MIMO systems is provided in [5]. The authors in [6] compares the performance of maximum ratio transmission (MRT) and zero forcing (ZF) precoding in downlink massive MIMO systems under perfect CSI. The results show that when the number of BS antennas increases, linear precoding techniques give near optimal performance with low complexity. To study the trade-off on energy efficiency and spectral efficiency, a new metric called resource efficiency is proposed in [7,8]. The resource efficiency helps to balance the spectral efficiency and energy efficiency through optimization algorithms.

To the best of our literature, there are no recent works that jointly evaluate the spectral efficiency, energy efficiency and transmission reliability in massive MIMO systems. Thus, in this work, we develop a more general analytical expression for energy efficiency in terms of spectral efficiency and transmission reliability. In this regard, the contributions of this work are summarized as follows:

- Develop a mathematical model that relates the spectral efficiency, energy efficiency and transmission reliability in single cell downlink massive MIMO system.
- Formulate analytical expression to the proposed model with linear precoding techniques.
- Perform numerical simulation to validate the theoretical analysis.

The rest of this paper is organized as follows. In Sect. 2, the system model for single cell downlink massive MIMO system is provided. Spectral efficiency and system power consumption is formulated in Sects. 3 and 4, respectively. Energy efficiency is formulated in Sect. 5. Transmission reliability and its impact on energy efficiency is provided in Sects. 6 and 7, respectively. Simulation results and discussions are presented in Sect. 8 and conclusions are drawn in Sect. 9.

2 The Massive MIMO System Model

We consider a single-cell downlink massive MIMO system shown in Fig. 1, where the BS is equipped with M antennas to support K single-antenna users in the same time-frequency resource. Let $\mathbf{x}$ denotes the complex valued $M \times 1$ transmitted signal vector from the M antennas, the $K \times 1$ received signal vector $\mathbf{y}$ at the users is given by [9]

$$\mathbf{y} = \sqrt{p_d}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{K \times M}$ is a channel matrix between the BS antennas and K users [2, 21]. The channel is assumed to be a Rayleigh fading and ergodic with perfect CSI at the BS. Thus, the elements of $\mathbf{H}$ are assumed to be independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance. $\mathbf{x} = \mathbf{W}\mathbf{s}$ is the precoded signal at the BS, $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the information bearing signal with $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$, $\mathbf{W} \in \mathbf{C}^{M \times K}$ is the precoding matrix at the BS and p_d is the downlink transmit power for user k. $\mathbf{n}$ is additive white Gaussian noise vector at the users with zero mean and unit variance elements.

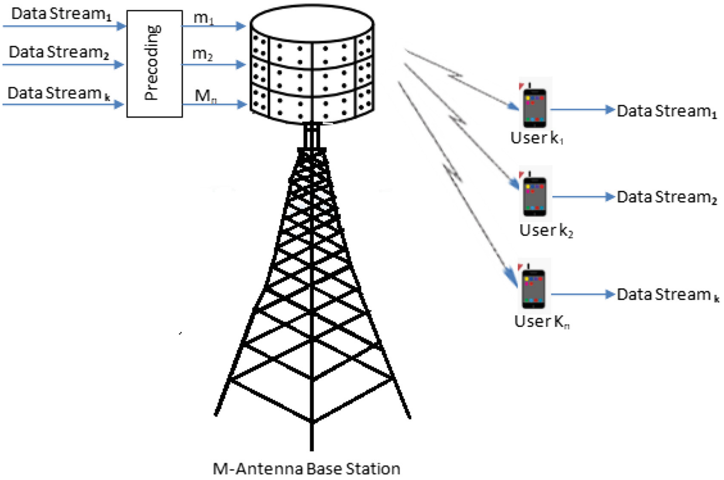

Fig. 1. Downlink massive MIMO system model [9].

Then, the received signal at the kth user is given by [8]

$$y_k = \sqrt{p_d}\mathbf{h}_k\mathbf{w}_k s_k + \sum_{i=1, i\neq k}^{K} \sqrt{p_d}\mathbf{h}_k\mathbf{w}_i s_i + n_k \tag{2}$$

where $\sqrt{p_d}\mathbf{h}_k\mathbf{w}_k s_k$ is the desired signal for user k and $\sum_{i\neq k}^{K} \sqrt{p_d}\mathbf{h}_k\mathbf{w}_i s_i$ is the multi-user interference signal. The signal to interference plus noise ratio (SINR) at user k is given by [12]

$$\text{SINR}_k = \frac{p_d|\mathbf{h}_k\mathbf{w}_k|^2}{p_d\sum_{i=1,i\neq k}^{K} |\mathbf{h}_k\mathbf{w}_i|^2 + 1} \tag{3}$$

which is a function of the channel $\mathbf{h}_k$ and transmit precoding vector $\mathbf{w}_k$.

3 Spectral Efficiency in Massive MIMO Systems

One of the metrics to quantify the performance of a massive MIMO system is the achievable rate. It gives the lower bound on the spectral efficiency that the massive MIMO systems can transmit over the fading channel [10]. Based on the Shannon's channel capacity theory, the achievable rate per user in a single cell downlink massive MIMO system is given by [10]

$$R_k = B \log_2(1 + \mathrm{SINR}_k) \tag{4}$$

and the achievable sum rate of the system with K users is given by [6]

$$R_s = B \sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k) \tag{5}$$

where B is the bandwidth of the system.

3.1 Spectral Efficiency with Linear Precoding Techniques

Spectral Efficiency with ZF Precoding: Zero forcing or null-steering precoding is a method of spatial signal processing by which multiple antenna transmitter can null multiuser interference signals. That is, with ZF precoding the inter-user interference can be average out at each user. The ZF precoding matrix is generated by implementing the pseudo-inverse of the channel matrix. Hence, the ZF precoding matrix employed by the base station is given by [9]

$$\mathbf{W}_{\mathrm{ZF}} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1}. \tag{6}$$

For large values of M and K, the SINR of a user in (3) with ZF precoding is given by [9]

$$\mathrm{SINR}_k^{\mathrm{ZF}} \approx p_d \Big(\frac{M-K}{K}\Big). \tag{7}$$

Thus, the achievable sum rate in (5) with ZF precoding is

$$R_s^{\mathrm{ZF}} \approx K \log_2 \big(1 + p_d (\frac{M-K}{K})\big). \tag{8}$$

Spectral Efficiency with MRT Precoding: By implementing the conjugate transpose of the channel matrix, MRT precoding aims to maximize the signal to interference plus noise ratio at the intended user. Thus, the MRT precoding matrix employed by the BS is given by [9]

$$\mathbf{W}_{\mathrm{MRT}} = \mathbf{H}^H. \tag{9}$$

For large values of M and K, the SINR of a user in (3) with MRT precoding is given by [6]

$$\mathrm{SINR}_k^{\mathrm{MRT}} \approx \frac{p_d M}{K(p_d+1)}. \tag{10}$$

Thus, the achievable sum rate of the system in (5) with MRT precoding is become

$$R_s^{\mathrm{MRT}} \approx K \log_2 \big(1 + \frac{p_d M}{K(p_d+1)}\big). \tag{11}$$

4 Power Consumption in Massive MIMO Systems

Accurate modeling of the system power consumption is required to formulate the energy efficiency and to obtain reliable guidelines on energy efficiency optimization with respect to the system parameters. The total power consumption of the proposed downlink massive MIMO system is given by the sum of transmitted power and circuit power consumption as [3]

$$P_t = \mu \sum_{k=1}^{K} p_k + MP_c + P_{\text{fix}} \tag{12}$$

where μ is the inverse of the power amplifier efficiency at the BS, p_k is the downlink transmitter power allocated to each user, P_c is the constant circuit power consumption per antenna that includes power dissipation in the transmit filter, mixer, frequency synthesizer and digital-to-analog converter, P_{fix} is the fixed power consumption at the BS which is independent of the number of transmit antennas.

5 Energy Efficiency in Massive MIMO Systems

The energy efficiency (in bits/Joule) of the massive MIMO system is commonly defined as a benefit-cost ratio where the achievable rate is compared with the associated energy consumption of the system [7,8]. One of the well known established metrics to measure this benefit-cost ratio is the global energy efficiency (GEE) which is given by [7,8,19]

$$\text{EE} = \frac{\text{Achievable sum rate}}{\text{Total power consumption}}. \tag{13}$$

Plugging (5) and (12) into (13), the energy efficiency of a downlink massive MIMO system is given by

$$\text{EE} = \frac{B \sum_{k=1}^{K} \log_2(1 + \text{SINR}_k)}{\mu \sum_{k=1}^{K} p_k + MP_c + P_{\text{fix}}}. \tag{14}$$

Generally, increasing the transmit power increases the achievable sum-rate. But, the energy efficiency is a unimodal function with the transmit power and thus, it increases until some transmit power and then decrease after that power.

6 Transmission Reliability in Massive MIMO Systems

The transmission reliability or symbol transmission success rate of a communication system describes the probability of receiving a symbol correctly over the communication link. Since any symbol error results in loss of the symbol, the symbol transmission success rate is defined in terms of the bit error probability as [15]

$$f(\gamma) = (1 - P_e(\gamma))^{\frac{L}{b}} \tag{15}$$

where γ is the received SINR of the communication link, P_e is the bit error probability, L is the information bits in the transmitted symbol and b is the number of bits per symbol which is determined based on the modulation type.

6.1 BER in Massive MIMO Systems

The bit error rate (BER) which is also termed as the bit error probability measures the errors in received bits over a communication channel that is altered due to noise, distortion, interference, or synchronization errors. It is the ratio of the numbers of bits in error to the total number of transmitted bits during a predefined time interval [16]. For the proposed massive MIMO system, the BER is determined based on the modulation scheme employed for the transmission. When the massive MIMO system employs ZF precoding and Gray-coded square NQAM modulation, the average BER for the kth user can be expressed as [17,18,20]

$$P_e(\gamma_k) \approx \frac{c_N}{2d_N} \frac{\Gamma(\tau + \frac{1}{2})/\Gamma(\tau + 1)}{(\gamma_k d_N^2 + 1)^{(\tau + \frac{1}{2})} \sqrt{\pi \gamma_k}} \tag{16}$$

where $\tau = M - K$ is the degree of freedom, $\gamma_k = \frac{P_T}{K\sigma^2}$ is the transmission SNR of the user, P_T is the total transmission power at the BS which is divided equally for each user and $\Gamma(.)$ is the Gamma function. The constant c_N and d_N is derived from the modulation level as

$$(c_N, d_N) = \begin{cases} (1, 1) & N = 2 \\ (2\frac{1 - 1/\sqrt{N}}{\log_2(\sqrt{N})}, \sqrt{\frac{3/2}{N-1}}) & N \geq 4. \end{cases} \tag{17}$$

The effect of $\Gamma(\tau + \frac{1}{2})/\Gamma(\tau + 1)$ is negligible and if we omit it, the bit error rate is approximated as

$$P_e(\gamma_k) \approx (\gamma_k d_N^2 + 1)^{-(\tau + \frac{1}{2})} \gamma_k^{-\frac{1}{2}}. \tag{18}$$

The result in (18) shows that the BER is improved by deploying large number of antennas at the BS; where as the BER is become worse when the scheduled users become very large.

7 Impact of Transmission Reliability on Energy Efficiency

In this section, different from the previous works that only focus on the relationship between spectral efficiency and energy efficiency, we develop a framework that relates the spectral efficiency, energy efficiency and transmission reliability.

The proposed formulation defines an energy efficiency expression that incorporates all necessary wireless performance metrics. Mathematically, the problem of interest is formulated as [22,23]

$$\text{EE} = \frac{[\text{Achievable sum rate}] \times [\text{Transmission reliability}]}{\text{Total power consumption}}. \tag{19}$$

By using the results from (5), (12) and (15), the proposed formulation for the energy efficiency interms of the spectral efficiency and transmission reliability is given by

$$\text{EE} = \frac{B\sum_{k=1}^{K}\log_2(1+\text{SINR}_k)(1-P_e(\gamma))^{\frac{L}{b}}}{\mu\sum_{k=1}^{K}p_k + MP_c + P_{\text{fix}}}. \tag{20}$$

With ZF precoding, the energy efficiency of the proposed system is expressed as

$$\text{EE}^{\text{ZF}} \approx \frac{KB\big(\log_2(1+P_d(\frac{M-K}{K}))\big)(1-P_e(\gamma))^{\frac{L}{b}}}{\mu\sum_{k=1}^{K}p_k + MP_c + P_{\text{fix}}}. \tag{21}$$

Similarly, with MRT precoding, the energy efficiency of the proposed system is expressed as

$$\text{EE}^{\text{MRT}} \approx \frac{KB\big(\log_2[1+\frac{P_dM}{K(P_d+1)}\big)(1-P_e(\gamma))^{\frac{L}{b}}}{\mu\sum_{k=1}^{K}p_k + MP_c + P_{\text{fix}}}. \tag{22}$$

8 Simulation Results and Discussions

In this section, we validate the theoretical analysis via numerical simulation. For the simulation, we consider single cell downlink massive MIMO system with perfect CSI. To evaluate the spectral efficiency, energy efficiency and transmission reliability for the proposed system, we use the mathematical expressions that are formulated in the previous sections. Performance analysis is done for both ZF and MRT precoding techniques under different systems and propagation parameters. The spectral efficiency with Monte-Carlo realizations from (5) is added to validate the tightness of the asymptotic lower bound spectral efficiency expressions in (8) and (11). Some of the standard simulation parameters are summarized in Table 1 [2].

Figure 2 shows the achievable sum rate versus the number of BS antennas for both precoding techniques. The result shows that as the number of BS antennas increases, the achievable sum rate increases for both precoding techniques. As expected when the BS antennas grows large, ZF achieves better sum rate than MRT. Because, at large number of BS antennas, ZF precoding can completely null-out the inter-user interference. The result also shows that the gaps between analytical approximation and the simulated values are very small. Thus, it is quite reasonable to formulate the system energy efficiency by using the closed-form lower bound achievable sum rate approximations in (8) and (11).

Table 1. Some of the simulation parameters

Parameter	Value
Circuit Power Consumption per BS Antenna (P_c)	2 W
Fixed Power Consumption P_{fix}	10 W
Downlink Transmit Power (P_d)	1 W
Power Amplifier Efficiency (η_{UE})	0.38
System Bandwidth (B)	20 MHz
Information Bits (L)	64
Number of bits per symbol (b)	2, 4, 6
Modulation level (NQAM)	4, 16, 64

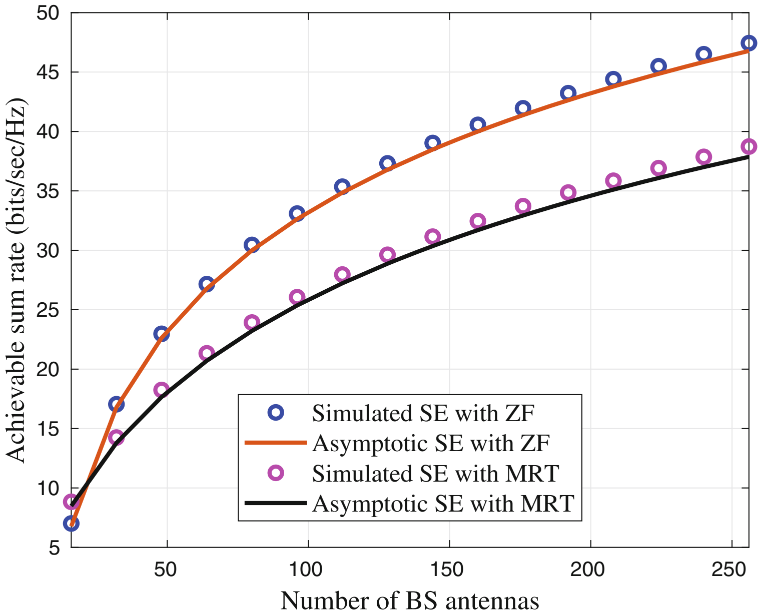

Fig. 2. Spectral efficiency versus the number of BS antennas.

Figure 3 shows the energy efficiency with the number of BS antennas. The result shows that the energy efficiency increases until some number of BS antennas and then decreases with the number of BS antennas. This is because as shown in (12) when the number of BS antennas grows large, the total circuit power consumption of the BS increases and this results in a lower energy efficiency. Thus, although increasing the number of BS antennas can help to reduce the transmission power for the system, it decreases the energy efficiency due to the increment on internal power consumption. Hence, a design trade-off is required to obtain the optimal number of BS antennas that give maximum spectral efficiency and energy efficiency simultaneously.

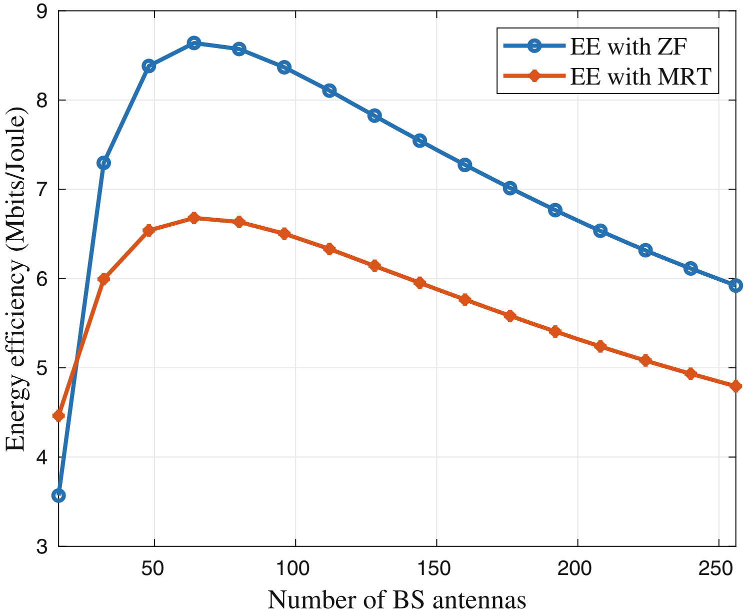

Fig. 3. Energy efficiency versus the number of BS antennas. We assume $\gamma = 1\,\text{dB}$ and 16 QAM.

The energy efficiency versus the spectral efficiency performance of the proposed massive MIMO system is shown in Fig. 4. The results shows that in lower spectral efficiency regime, when the spectral efficiency increases, the energy efficiency is also increased. Whereas at high spectral efficiency regime, the energy efficiency decreases. This is because, the spectral efficiency grows in logarithm function whereas the power consumption grows linearly and thus after a certain time the increment on the power consumption overtakes the spectral efficiency increment and the energy efficiency starts to decrease.

Finally, Fig. 5 shows the impact of the modulation level on the energy efficiency of the system. As shown in (15), when the modulation level increases, the symbol transmission success rate is decreased. This shows that under fixed bandwidth when the modulation level increases, the reliable data transmission function is deteriorated. Thus, the energy efficiency decreases with modulation level.

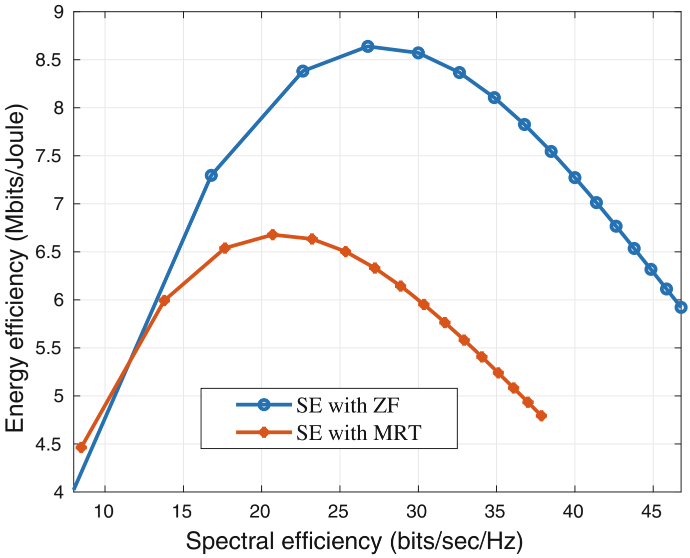

Fig. 4. Spectral efficiency versus energy efficiency. We assume $\gamma = 1\,\mathrm{dB}$ and 16 QAM.

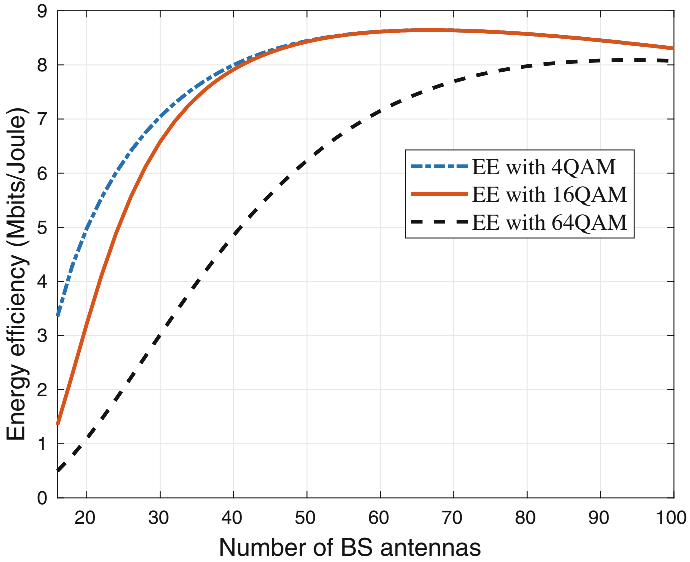

Fig. 5. Impact of the modulation level on energy efficiency. We consider ZF precoding.

9 Conclusion

In this work, we have investigated a mathematical model that jointly evaluates the spectral efficiency, energy efficiency and transmission reliability in downlink massive MIMO systems with linear precoding techniques. Closed-form analytical formulation is derived that jointly evaluates the impacts of spectral efficiency and transmission reliability on energy efficiency. Numerical results have been done to validate the effectiveness of the proposed analytical expression for energy efficiency. The impacts of system and propagation parameters on the spectral and energy efficiency are evaluated. The result shows that energy efficiency is a unimodal function that increases until some value and decreases above that value. Hence, a design trade-off is required to obtain the optimal system and propagation parameters that maximize the spectral efficiency and energy efficiency simultaneously.

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