Optimal Packet Size Analysis for Intra-flow Network Coding Enabled One Hop Wireless Multicast

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Abstract. Network coding has received great attention for its ability to greatly improve the performance of wireless networks. However, there still lacks of study to optimize the packet size for maximizing the throughput performance of network coding enabled multicast in wireless networks. In this paper, we study network coding enabled multicast from a base station to multiple receivers using random linear network coding. We build a network throughput model and derive the optimal packet size for maximal multicast throughput. Simulation results verify the high accuracy of our analysis results.

Keywords: Intra-flow network coding · Optimal packet size · Network throughput · Error-prone wireless networks

1 Introduction

It has been known that intra-flow network coding (NC) can significantly improve the multicast throughput. Existing work in this aspect has been mainly focused on designing practical intra-flow multicast protocols. How to improve the intra-flow NC assisted multicast from the perspective of packet size optimization has not been fully investigated. Packet size has big impact on the performance of wireless multicast. In the case of poor channel condition, the larger a packet is, the greater the probability of packet loss; In the case of good channel condition, the smaller a packet is, the bigger the overhead due to fixed packet header will be. Therefore, how to optimize the packet size to maximize the throughput of wireless multicast is a key problem in the study of intra-flow network coding.

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In this paper, we focus on studying one-hop networks constituent of one base station and a set of wireless clients. In such a network, we investigate how to maximize the network throughput by optimizing the packet size for single-hop wireless multicast using intra-flow network coding, which adopts random linear network coding (RLNC) for batch forwarding to multiple multicast destinations (receivers). To address this issue, we formulate the network throughput model for our scenario. Based on the network throughput model, we derive the optimal packet size for maximal network throughput for the single-hop wireless multicast scenario. Simulation results verify the high accuracy of our analytical results.

The rest of this paper is organized as follows. In Sect. 2, we briefly introduce related work. In Sect. 3, we build the network throughput model and then derive the optimal packet size for maximal network throughput. In Sect. 4, we conduct simulation results for performance validation. In Sect. 5, we conclude this paper.

2 Related Work

Network coding technology [1] has been proven to have good potential for the enhancement of network throughput. Linear codes are sufficient to achieve the maximum capacity bounds for a multicast traffic [2]. Especially the inherent broadcasting peculiarity makes the network coding more suitable for one-to-many flows in a wireless multi-hop network. Then [3] shows that random linear network coding can take advantage of redundant network capacity for improved packet delivery probability and robustness. Paper [4] studied the integration of opportunistic routing and intra-flow network coding for improved network throughput performance in multi-hop wireless networks.

Packet size optimization has been studied in various aspects. In [5], the authors studied the issue of optimal packet size in energy-constrained wireless sensor networks where optimal packet size is determined for a set of radio and channel parameters by maximizing the energy use efficiency. In [6], Wu et al. built link lifetime models for characterizing the temporal nature of wireless links and subsequently wireless paths by considering node mobility and then computed the optimal packet size as a function of mobility for improving the network throughput. However, none of them have studied the issue of optimal packet size for network coding enabled wireless networks. In [7], Cui et al. studied how to optimize the packet size of unicast packets for maximizing the network throughput of two-hop wireless networks with IEEE 802.11 for medium access. The analysis model in [7] is not suitable for our scenario in this paper due to the following reasons. First, the focus on [7] is on how to introducing various coding/broadcasting gains into a Markov throughput analysis model for establishing the relationship between network throughput and packet size, while in this paper, we focus on allocated wireless channels for transmission, which is quite different from the multi-access channel in [7]. Second, [7] studied unicast traffic while in this paper, we study multicast communications. Third, [7] studied inter-flow network coding while in this paper, we focus on studying intra-flow network coding.

Ref. [8] is a paper relevant to our work in this paper. In [8], the authors studied the throughput performance of RLNC assisted multicast under the requirement of 100%

packet delivery. They derived the normalized throughput in this case, which does not consider the impact of packet size in the throughput calculation, and further its implementation involves operations under asymptotical conditions, paper [8] also derived the upper and lower bounds of throughput of the RLNC assisted multicast with 100% packet delivery with simplified expressions. However, the simplified expressions for the bounds cannot be used to find the optimal packet size leading to maximal throughput. More importantly, achieving 100% delivery may not be necessary in many cases and applications and further it can greatly affect the multicast throughput performance. This happen when different multicast receivers have different packet loss rates since the throughput in this case will be largely determined by the receiver with the worst link loss rate. Different from [8], in this paper, we focus on studying how to optimize the packet size in RLNC assisted multicast while achieving maximal network throughput without considering 100% packet delivery.

3 Optimal Packet Size Analysis

In this section, we first describe the system model, then formulate the problem under study, and finally build our analytical model for optimal packet size analysis.

3.1 System Model

The network under study consists of one base station and multiple clients connected via wireless links. The issue under study is to deliver packets from the base station to N $(N \geq 2)$ multicast receivers $\{R_1, R_2, \ldots, R_N\}$. Assume that each node is equipped with an omnidirectional antenna. All the network nodes have the same communication range. In this paper, we assume that the channel between the source and the multicast receivers is a dedicated broadcast channel (such as 3G/4G/LTE) such that the transmissions from the base station can be received by all the N receivers when no transmission error occurs. We assume transmissions on the wireless channel may suffer from packet loss due to channel fading, multipath effects, etc. We assume a uniform random bit error model although analytical model can also be easily extended to work in other link loss models. The packet receptions at different multicast receivers are assumed to be independent. Table 1 lists the notations used in this paper.

In this paper, we focus on a scenario where the source node S is to multicast a large bulk of data to multiple receivers. The whole data bulk is divided into multiple blocks (batches). The random linear network coding (RLNC) is used to facilitate the batch forwarding such that each batch contains $K + \theta$ coded packets, each of which is a random linear combination of K original data packets to be delivered in the current batch. The coding coefficients are taken from the finite field $GF(q)$. When a receiver receives K linearly independent coded packets, it will be able to decode all the K original data packets of the batch. For the source node, after it sends out the $K + \theta$ coded packets of a batch, it will move to the next batch. This process continues until the source complete the transmissions of all the batches. Therefore, there is no guarantee that a receiver receives all the original packets. This case is of interest for delivering video contents, sensor readings, etc. The value of θ determines the successful delivery probability of the original packets and also has a big impact on the bandwidth efficiency. From the viewpoint of flow rate, if the data flow rate is r, then the actual transmission rate will be $r \times (K + \theta)/K$. Figure 1 illustrates the system model for delivering a batch.

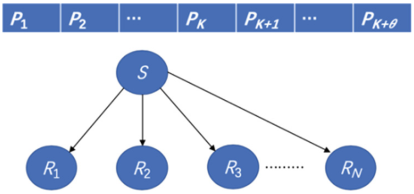

Fig. 1. The multicast scenario using random linear network coding.

Let BER denote the link bit error rate, the packet loss rate between the multicast source and a receiver, denoted by P_e, will be as follows:

$$P_e = 1 - (1 - BER)^{H+h+L} \tag{1}$$

where H is the length of the packet header, h is the length of the encoding vector, and L is the payload length of the packet.

3.2 Problem Formulation

Next, we formulate the problem under study. When a receiver receives a number i $(1 \leq i \leq K + \theta)$ of coded packets of a batch, we are concerned about at which probability the receiver can decode the K original packets of the batch. Note that the possibility of certain linear correlation between received coded packets needed to be considered. Accordingly, we define $P_{i,K}$ as "the probability that a receiver can successfully decode the K code-independent encoded packets of a batch when it successfully receives i coded packets belonging to the batch". Since the theoretical derivation of $P_{i,K}$ involves too many matrix operations and is hard to obtain a closed form, in this paper, the value of $P_{i,K}$ will be obtained empirically via simulations.

The packet decoding at a receiver is as follows. Suppose the source sends out $K + \theta$ coded packets for a batch, for a receiver to decode all the K original packets of the batch, it needs to get K linearly independent coded packets; Otherwise, it will be unable to decode any original packets[1].

[Definition] The batch based multicast transmission problem subject to given transmission redundancy constraint: Given a transmission redundancy θ, for delivering a batch generated by K original data packets, the multicast source will send out a total number $K + \theta$ of RLNC coded packets. For this case, what is the optimal packet size for maximizing the network throughput?

[1] Such decoding feature is due to the decoding characteristics of the RLNC package in the NS-3 simulator (i.e., the Kodo-RLNC module developed by Steinwurf) [9].

3.3 Analytical Model

In this subsection, we build our analytical model for deriving optimal packet size for achieving maximal throughput subject to transmission given redundancy.

Consider the lossy nature of wireless channel, when the source sends out $K + \theta$ packets, and the probability that a receiving node receives a number i $(1 \leq i \leq K + \theta)$ of packets is as follows:

$$\begin{aligned} P_{ri} &= C^{i}_{K+\theta}(1 - P_e)^{i} P_e^{K+\theta-i} \\ &= C^{i}_{K+\theta}(1 - BER)^{i(H+h+L)} \left[1 - (1 - BER)^{H+h+L}\right]^{K+\theta-i} \end{aligned} \tag{2}$$

Therefore, in the transmission of a batch, the source sends out $K + \theta$ coded packets in total, and the probability that a receiving node can decode all the original packets of the batch, denoted by P_{decode}, is as follows:

$$P_{\text{decode}} = \sum\nolimits_{i=K}^{K+\theta} P_{ri} P_{i,K} \tag{3}$$

In summary, the throughput at a particular receiver can be obtained as follows:

$$\begin{aligned} S_{RLNC} &= \frac{L}{H+h+L} \times \frac{K}{K+\theta} P_{\text{decode}} \\ &= \frac{L}{H+h+L} \times \frac{K}{K+\theta} \sum\nolimits_{K}^{K+\theta} P_{i,K} C^{i}_{K+\theta}(1 - BER)^{i(H+h+L)} \\ &\quad \left[1 - (1 - BER)^{H+h+L}\right]^{K+\theta-i} \end{aligned} \tag{4}$$

Table 1. Notations used.

Notations	Definitions
H	Data packet header length
L	Payload length
h	Coding vector length
$P_{i,K}$	The probability that a receiving node successfully decodes the K original packets in a batch when it successfully receives i coded packets
P_e	Packet error probability
θ	The number of redundant packets in a batch
K	The number of original data packets in a batch
P_{ri}	The probability that one receiving node receives i packets belonging to a batch
P_{decode}	The probability that a receiver can decode all the original packets belonging to a batch
S_{RLNC}	Normalized throughput at a receiver due to the use of random linear network coding for batch forwarding

It can be seen that S_{RLNC} is a convex function with respect to the payload L. Therefore, there exists an optimal L value within a reasonable range so that S_{RLNC} reaches the maximum. Therefore, let $\partial S_{RLNC}/\partial L = 0$, we can obtain the optimal packet size L^*.

Following (4), for multicast scenario, the mean throughput at an individual multicast receiver (i.e., normalized throughput) can be obtained as follows:

$$\begin{aligned} S_{RLNC} &= \frac{1}{N} \times \frac{L}{H+h+L} \times \frac{K}{K+\theta} \sum\nolimits_{j=0}^{N} P_{\text{decode}} \\ &= \frac{1}{N} \times \frac{L}{H+h+L} \times \frac{K}{K+\theta} \\ &\quad \sum_{j=0}^{N} \sum_{K}^{K+\theta} P_{i,K} C_{K+\theta}^{i} \left(1 - BER_j\right)^{i(H+h+L)} \left[1 - \left(1 - BER_j\right)^{H+h+L}\right]^{K+\theta-i} \end{aligned} \tag{5}$$

where BER_j is the bit error rate of the link between a source node and multicast receiver j. Again, the optimal packet size L^* for this case can be obtained by let the derivative of (5) to be 0 since the sum of convex functions is still convex.

4 Simulation Results

In this section, we conduct simulations using NS-3 to validate the accuracy of our analytical results. Specifically, we use the random linear network coding module Kodo-RLNC developed by Steinwurf to generate batches of packets. Moreover, in the generation of coded packets, we chose finite field $GF(4)$ to generate coding vectors, i.e., the coding coefficients will be random integers in $[0, 2^4-1]$. In the simulations, the source node needs to send a file with a size of 1Mbytes toward multicast destinations like shown in Fig. 1. Each batch contains 10 original data packets. The default number of redundant coded packets for each batch is 5, which will be adjusted later. The topology used in the simulation is shown in Fig. 1. UDP is used for multicast data packet transmission. All links in the network are assumed to have the same BER. In the reported results in terms of network throughput and packet size, only packet payload is counted. The parameter settings are listed in Table 2.

Table 2. Simulation settings.

Parameters	Values
Packet header length	24 bytes
Channel rate	1 Mbps
Packet type	UDP
Payload range L	(0,1500 bytes]
Number of multicast receivers	2, 5
Coding vector length h	4 bytes

In the simulations, we measure the normalized throughput due to different settings. The normalized throughput can be seen as the average number of data bytes from decoded original packets received by an individual multicast receiver when the source node sent out a whole batch of packets over the total number of bytes in all the original data packets in a batch. For example, if the source node sends out 15 coded packets for a batch, and a certain receiver receives some of the packets and decodes all the 10 original packets, then the normalized throughput will be $10 \times L/(15 \times (H + h + L))$, where h is the coding vetch length. Obviously, for $K = 10$ and $\theta = 5$, the upper bound of normalized throughput is 2/3.

4.1 $P_{i,K}$ Training

The first test is to determine (train) the value of each $P_{i,K}$ via simulations (note that $K = 10$ in our case). During the training process, there exist just one source and one receiver. Moreover, the link between the source and the receiver is error free in the training process. In this process, the source has totally 1000 batches to send. Each batch contains 15 coded packets, which are generated based on 10 original data packets. The batch decoding process at receiver works as follows: When the receiver receives i ($i \geq$ K) packets belonging to a batch, it will try to decode the K original data packets in the batch; when $i < K$, the Kodo-RLNC module will not produce any original packets (i.e., $P_{i,10} = 0$, $\forall i < 10$). The value of $P_{i,10}$ ($i \geq 10$) is the average ratio between the number of tests that decoding the K original packets when i packets were received and the total number of tests that i packets were received. Table 3 shows the values of $P_{i,10}$ ($i \geq 10$). The obtained $P_{i,K}$ values will be taken into Eqs. (4) (5) for deriving normalized throughput at different bit error rates.

Table 3. Values of $P_{i,K}$.

$P_{i,K}$ ($K = 10$)	Probability
$P_{10,10}$	0.953
$P_{11,10}$	1
$P_{12,10}$	1
$P_{13,10}$	1
$P_{14,10}$	1
$P_{15,10}$	1

Figure 2 shows the packet loss rate with varying bit error rate and payload. It can be seen that the packet loss rate increases with payload and also bit error rate.

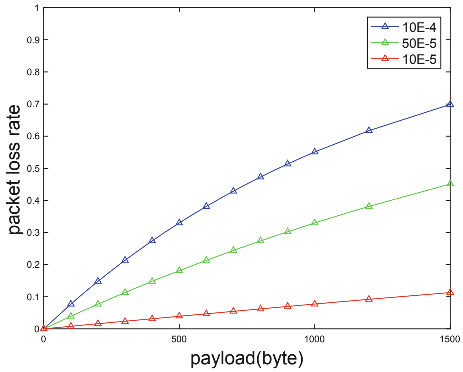

Fig. 2. Packet loss rate with varying payload and bit error rate.

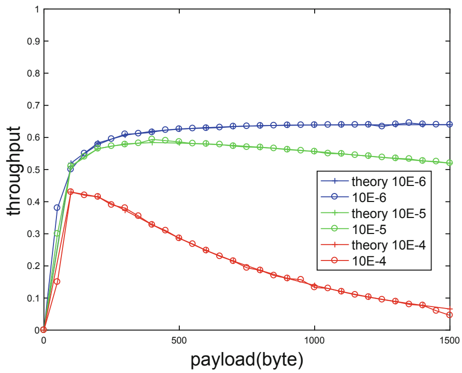

Fig. 3. Comparison of throughput with varying payload and BER. $\theta = 5$.

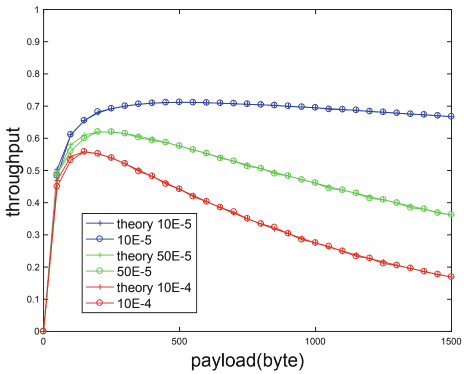

Fig. 4. Comparison of throughput with varying payload and BER. $\theta = 2$.

4.2 Homogeneous BER Case

In this test, the BERs for different multicast receivers were set to be the same. This case is actually equivalent to the unicast case since we are concerned about the normalized throughput performance. That is, the average normalized throughput performance for multicast scenario in this case is the same to the counterpart for unicast scenario under the same BER.

Figure 3 compares the throughput performance by analysis and simulations with varying bit error rate and payload when $\theta = 5$. Figure 4 compares the throughput performance when $\theta = 2$. Other parameters were the same to those used for Fig. 3.

In Fig. 3, it can be seen, in the case of a small BER, the optimal packet size is large. As the BER continues to increase, the channel keeps deteriorating, resulting in lower throughput, and also smaller optimal packet size. However, under the use of random linear network coding, the throughput can still approach the maximum value of 0.66 in the case of lossy channel due to the redundancy introduced by the source node for the delivery of each individual batch. In Fig. 4, when the number of redundant packets θ is reduced to 2. When the BER is low (i.e., 10^{-5}), the channel utilization is increased and the throughput is high, but with BER increasing to 5×10^{-5}, the packet loss rate increases, and the system throughput drops faster (compared to the case of $\theta = 5$). When the BER continues to increase (10^{-4}), the channel conditions are too bad. Even if $\theta = 5$, the receiver may still be unable to successfully receive enough coded packets in many cases. At this time, the case $\theta = 2$ performs better than the case $\theta = 5$. Furthermore, it can be seen from both figures that our analytical results are consistent with the simulation results.

Table 4 shows the optimal packet sizes for different cases. The table shows that the analytical results are very close to the simulation results. The difference between the analysis results and the simulation results is mainly due to the limited granularity of packet size setting in simulations. Table 5 lists the average delivery rates of original packets under different redundancies and bit error rates. The delivery rate is a ratio between the number of original packets received (decoded) and the number of original packets sent out. It is seen that although the bandwidth utilization ratio when $\theta = 5$ is much lower than that when $\theta = 2$, the average delivery rate for the former case is much higher than that for the latter case.

Table 4. Comparison of optimal packet sizes by analysis and simulation.

BER	$\theta = 5$		$\theta = 2$	
	Simulation	Theoretical	Simulation	Theoretical
10^{-5}	1250	1243	459	450
5×10^{-5}	400	370	217	200
10^{-4}	100	123	153	150

Table 5. Average delivery rate of original packets under different redundancies and different bit error rates.

BER	$\theta = 5$	$\theta = 2$
10^{-5}	95%	76%
5×10^{-5}	83%	64%
10^{-4}	41%	32%

Table 6. Comparison of optimal packet sizes due to analysis and simulation.

BER settings	$\theta = 5$		$\theta = 2$	
	Simulation	Theoretical	Simulation	Theoretical
10^{-4}, 5×10^{-5}, 10^{-5}	200	179.8	200	203.3
10^{-5}, 5×10^{-5}, 10^{-6}	300	303.2	350	334.0

4.3 Heterogeneous BER Case

Next, we extend the multicast scenario to heterogeneous bit error rates. The number of multicast receiver was fixed to three. In the first test in this aspect, the BERs of the links between the source node and each of the three receivers were set to 10^{-4}, 5×10^{-5}, and 10^{-5}, respectively, and the transmission redundancy $\theta = 2, 5$. Figure 5 compares the throughput performance by analysis and simulations for this case. The results show that our analytical results are consistent with the simulation results. In the second test, we fixed the redundancy $\theta = 2$, varied the BERs of different multicast receivers (one case is: 10^{-5}, 5×10^{-5}, 10^{-6}, and another case is 10^{-4}, 5×10^{-5}, 10^{-5}). Figure 6 compares the throughput performance by analysis and simulations. Again, the analytical results are consistent with the simulation results. Table 6 lists the optimal packet sizes for different cases. It is seen that the analytical results are very close to the simulation results.

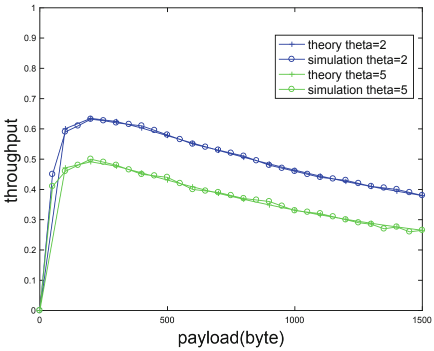

Fig. 5. Comparison of throughput under different θs in heterogeneous BER cases.

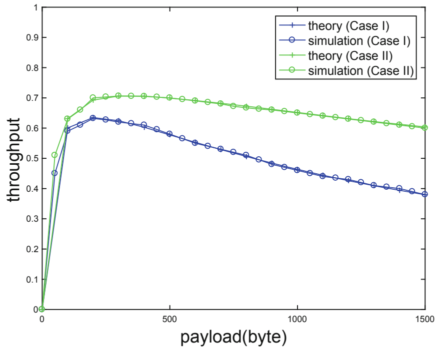

Fig. 6. Comparison of throughput in heterogeneous BER cases. Case I: 10^{-4}, 5×10^{-5}, 10^{-5}, and Case II: 10^{-5}, 5×10^{-5}, 10^{-6}.

5 Conclusion

In this paper, we studied the optimal packet size problem for single-hop wireless multicast using intra-flow network coding with lossy links. Random linear network coding is used for batch forwarding. In this study, we first model the network throughput when given transmission redundancy and then derive the optimal packet size for maximal network throughput. Simulation results verified the high accuracy of our analytical results. The RLNC based throughput-maximization multicast in this paper can also work well with the D2D recovery via localized network coding for further improved delivery performance [10–12].

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