A Homology Based Coverage Optimization Algorithm for Wireless Sensor Networks

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Abstract. Simplicial complex provides a precise and tractable representation of the topology of wireless sensor networks. In this paper, a coverage optimization algorithm based on Rips complex is given for the purpose of energy conservation of wireless sensor networks. Considering an area of interest which is covered by sensor nodes completely and even superfluously, our algorithm is performed to turn off redundant sensor nodes effectively in the network while maintaining the coverage consistently. Simulation results show that this distributed algorithm can remove more than 70% internal sensor nodes, and complexity analysis for our algorithm is given.

Keywords: Coverage optimization · Homology · Wireless sensor networks

1 Introduction

Coverage of target fields provides a metric of quality of service of wireless sensor networks (WSNs). Sensor nodes in WSNs are capable of collecting, storing and processing environmental information, and communicating with neighboring nodes. So

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sensor nodes are responsible for monitoring the area of interest. However, these nodes are supported by limited energy. Power conservation is focused on to prolong the lifetime of WSNs. The aim of coverage optimization in this paper is to turn off redundant sensor nodes as many as possible to save energy while maintaining the coverage of a network.

There are a number of researches concerning the coverage problem of WSNs [1]. One type of approaches is based on location of sensor nodes, in which some computational geometry tools are used, such as Voronoi diagrams or Delaunay triangulation [2–4]. Such location-based approaches require too much knowledge of the target field and the pattern of deployment of sensor nodes. Moreover, they suffer from the weakness that they can be too expensive to compute in real-time. Distance-based approaches are another kind of solutions to the coverage issues of WSNs [5, 6]. Location of sensor nodes may not be the necessity to the algorithms of this kind. Instead, they require to measure the distance between sensor nodes and obtain the precise geometry of target field, which is gained at high expense in practice. The third type of approaches is based on connectivity information between nodes in WSNs. By means of algebraic tools, homology theory more specifically, the topology of network is prone to building and analyzing [7]. Structure of the network is modeled by Rips complex in [8] to propose a sufficient coverage criterion. However, the work in [7] and [8] does not solve the problem about how to place sensor nodes to maximize the coverage. The distributed version of the ideas proposed in [7] and [8] is firstly implemented in [9]. By means of combinatorial Laplacians, coverage holes are localized by computing the sparse generator of the first homologous class and redundancies in the sensor network are reduced by finding a sparse cover of the region. A centralized reduction algorithm is proposed in [10] to remove superfluous vertexes to make simplicial complex as planar as possible while maintaining connectivity and coverage of a network. But the computation complexity analyzed of it is explosive as the dimension of simplicial complex increases. Extending this idea further, two homology based algorithms, a simulated annealing one and a robust one, are introduced in [11] for the sake of power conservation, but they are still accomplished at high expenses when constructing the complex and performing these algorithms. Simple-connectedness graph and fundamental group persevering transformation are posed in [12] to do the skeleton extraction of network in distributed fashion. Research in [13] develops this approach to propose a homology preserving transformation to delete redundant vertexes and edges to make simplicial complex more planar. According to a strong collapse approach, a reduction algorithm for abstract simplicial complex is proposed in [14] to simplifying the topology of network.

This paper presents a coverage optimization algorithm for WSNs in a 2-dimensional plane through reduction of simplicial complex in a distributed way with low complexity. We firstly construct the simplicial complex, specifically Rips complex [7], corresponding to the connectivity graph of network; then weight of all vertexes are computed after the definition of weight information; lastly, those superfluous vertexes and edges are identified and removed recurrently based on our complex expansion algorithm to make the Rips complex as planar as possible. Meanwhile, the first two homology of the complex is maintained during this process. Simulation results imply an excellent performance on the coverage optimization of the network.

The rest of the paper is organized as follows. Section 2 gives necessary assumptions of sensor nodes and network, together with some knowledge of mathematical basis; detailed description of our coverage optimization algorithm is presented in Sect. 3; then simulation results and discussion are given in Sect. 4 and conclusion is in Sect. 5 of this paper.

2 Models and Preliminaries

2.1 Network Models and Assumptions

In this connectivity-based algorithm, sensor nodes are capable to sense their neighborhood within a circle of radius R_s and communicate with neighbors within a circle of radius R_c for the construction of Rips complexes, where all the vertexes are modeled from sensor nodes in the WSN. Different from directional sensor networks, e.g. in [15], sensors in this paper are assumed to be omni-directional, that is they sense in all directions. As for the boundary of target field, it is covered by fence nodes and its coverage is required to be consistent, which implies that fence nodes should be retained. All the other sensor nodes in the network are internal ones, whose responsibility is keeping the inner area of network covered.

2.2 Mathematical Preliminaries

Simplicial complexes give a representation of higher order relations than graphs with respect to the topology of WSNs. Given a set of points V, a k-simplex $(k \in Z)$ is an unordered k-tuples $[v_0, v_1, \ldots, v_k]$ where $v_i \in V$ and $v_i \neq v_j$ for all $0 \leq i \neq j \leq k$. For example, a 0-simplex is a vertex, a 1-simplex is an edge, a 2-simplex is a triangle with its interior included, and a 3-simplex is a solid tetrahedron. The faces of a k-simplex include all the (k-1)-simplexes in form of unordered subset $[v_0, v_{i-1}, v_{i+1}, \ldots, v_k]$ for $0 \leq i \leq k$. So there are k + 1 faces in a k-simplex. An abstract simplicial complex is a finite collection of simplexes that is closed with regard to inclusion of faces of all the simplexes in it, and its dimension is the highest dimension of all the simplexes in it. For example, a geometric realization of abstract simplicial complex is depicted in Fig. 1, which consists of one 2-simplex and four 1-simplexes.

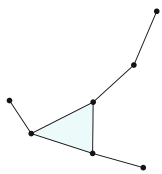


Fig. 1. An example of abstract simplicial complex

In this paper, Rips complex is adopted to represent the topology of networks. Definition of Rips complex is presented below. The construction of the Rips complex is not difficult to understand: a k-simplex $[v_0, v_1, \ldots, v_k]$ is included in $\mathcal{R}_{\in}(\mathcal{S})$ when the Euclidean distance between every two vertexes v_l and v_m is no longer than the fixed radius $\in$, that is $\| v_l - v_m \| \leq \in$ for all $0 \leq l \neq m \leq k$.

Definition 1 (Rips Complex). *Given a finite set of points $\mathcal{S}$ in $\mathbb{R}^n$ and a fixed radius $\in$, the Rips complex of $\mathcal{S}$, $\mathcal{R}_{\in}(\mathcal{S})$, is the abstract simplicial complex whose k-simplexes correspond to unordered (k + 1)-tuples of points in $\mathcal{S}$ that are pairwise within Euclidean distance $\in$ of each other.*

Now, the theoretical basis of simplicial homology is to be presented. The r-chain group $C_r(K)$ of a simplicial complex K is a free Abelian group generated by the oriented r-simplexes of K. Let $\sigma_r(p_0, \ldots, p_r)(r > 0)$ be an oriented r-simplex. The boundary $\partial_r \sigma_r$ of σ_r is an (r-1)-chain defined by $\partial_r \sigma_r \equiv \sum_{0}^{r} (-1)^r (p_0 p_1 \ldots \hat{p}_i \ldots p_r)$, where $\hat{p}_i$ means point p_l is omitted. If $c \in C_r(K)$ satisfies $\partial_r c = 0$, c is called a r-cycle. The set of r-cycle $Z_r(K)$ is a subgroup of $C_r(K)$ and is called the r-cycle group. If there exists an element $d \in C_{r+1}(K)$ such that $c = \partial_{r+1} d$ then c is called a r-boundary of d. The set of r-boundaries $B_r(K)$ is a subgroup of $C_r(K)$ and is called the r-boundaries group. The rth homology group $H_r(K), 0 \leq r \leq n$, associated with K is defined by $H_r(K) \equiv Z_r(K)/B_r(K)$. Homology is topological invariants. According to the knowledge of homology group, two r-cycles z and z' are in one homology class if and only if $z - z' \in B_r(K)$, in which case z is said to be homologous to z'. More detailed information may turn to [16].

3 Algorithm Descriptions

3.1 Model Construction and Weight Computation

The neighboring set of any vertex v_i in the Rips complex is denoted by $N(v_i)$ in this paper. The definition of the neighboring set of any k-simplex is given as follow.

Definition 2 (Neighboring Set of *k*-simplex). *For a certain k-simplex $s_k = [v_{k0}, v_{k1}, \ldots, v_{kk}]$, the neighboring set of s_k $N(s_k)$ is the intersection of neighboring sets of all vertexes in s_k, that is, $N(s_k) = \bigcap_{i=0}^{k} N(v_{ki})$.*

The concept of weight is introduced here to serve as a measure of proximity redundancy of vertexes. The weight w_v of an internal vertex v reflects the density of neighbors in the surrounding area. There are two kinds of internal vertexes: redundant and crucial vertexes. The former are referred to as ones whose all the 2-simplexes have at least one neighbor individually, which implies that they are vertexes of higher dimension simplexes than 2-simplex and thus are candidates of redundant vertexes. While the latter are referred to as ones at least one of whose 2-simplexes has no neighbor. In addition, fence vertexes are required to keep active all the time as mentioned before. All these idea is reflected in the weight information computed according to algorithm 1 given below. A simple example of redundant vertex is any one of the vertexes of a tetrahedron, because any vertex is superfluous to the other three ones in the plain.

Because the attention of this paper is focused on planar plane, the dimension of simplicial complex here is 2, which reduces the complexity of constructing Rips complex corresponding to the network. For the convenience of representation, $T(v)$ is used to denote the set of all 2-simplexes of which vertex v is a part. For any 2-simplex $t \in T(v)$, $n(t)$ denotes the neighboring set of t. Then the weight computation of vertexes is given as follow:

Algorithm 1. Weight Computation

Begin

```
for any fence vertex v
    w_v = 0
end for
for any internal vertex v
    if ∃t ∈ T(v), n(t) is empty then
        w_v = 0
    else
        w_v = 2
    end if
end for
```

End

3.2 Complex Expansion

Homological invariance is inspected in some previous papers from the metric of Betti numbers [17]. However, the cost of computing Betti numbers is expensive. Complex expansion algorithm given here reduces the simplicial complex corresponding to the WSN while maintaining the homology in much lower complexity.

There is no 1-dimensional or 2-dimensional coverage holes in the neighboring graph of an internal redundant vertex or edge. The aim of complex expansion algorithm is to identify them. Take an internal vertex v as an example. For two different vertex v_i and v_j in the neighboring set $N(v)$, if there is at least one path between them, they are connected to each other. If any two vertexes in the neighboring set are connected, the neighboring graph is connected accordingly. No 1-dimensional coverage holes means there is only one connected component in the neighboring graph, that is, the neighboring graph is connected. While no 2-dimensional coverage holes in the neighboring graph implies all the area of target field is within the union of covering range of all sensor nodes. It is the same case with internal redundant edges.

Obviously, there is no 1-dimensional and 2-dimensional coverage hole in the interior of a 2-simplex. In addition, ideally reduced Rips complex should remain as planar as possible, which is the source of our idea. Considering a 2-simplex t in $N(v)$ as the origin of complex expansion, denoted by R_{cov}. According to the knowledge given above, another 2-simplex t_1 in $N(v)$ is able to bond with R_{cov} exactly when the intersection of them is a 1-simplex, namely a common edge of them. As shown in Fig. 2, the 2-simplex t, that is R_{cov}, and t_1 form a new sub Rips complex R_{cov1} which is in the same homologous class with R_{cov} because the difference between the boundaries of them is the boundary of t_1. Therefore, the expansion from R_{cov} to R_{cov1} maintains the homology. Then update R_{cov} with R_{cov1} and continue to search for 2-simplex in $N(v)$ that is able to bond with R_{cov} exactly. Repeat the above steps until no more 2-simplex in $N(v)$ can join R_{cov}. Now, if all the vertexes in R_{cov} are the same to $N(v)$, the vertex v is proved as superfluous and can be swept off. All internal vertexes and edges are subjected to the process of complex expansion above independently. If a vertex or an edge, along with its some neighbors, is determined concurrently to be redundant, then the one with the least id among them has the priority to be removed. Thus, no two internal neighboring vertexes or edges will be removed simultaneously. All the neighbors of the removed vertex or edge will eliminate it from their neighboring sets respectively and continue to perform the next round decision.

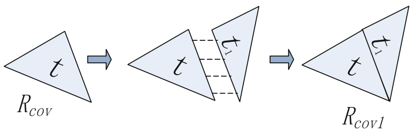


Fig. 2. Bonding of two 2-simplexes

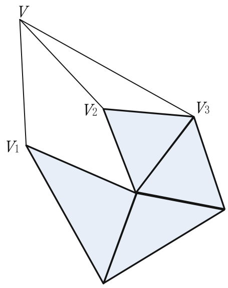


Fig. 3. Example of illegitimate vertex in complex expansion

During the process of complex expansion, the 2-simplex t_1 sharing a common edge e_{com} with any 2-simplex in R_{cov} can join it. Therefore, search of t_1 is actually the search of $N(e_{com})$ in the remaining vertexes of $N(v)$. That is, the complex expansion is proceeded by adding legitimate vertexes to R_{cov} formed by the subset of neighboring set $N(v)$. A legitimate vertex v' is required to meet the following two conditions: (1) v' has at least two neighbors in R_{cov}; (2) these neighbors of v' are one-hop connected. As shown in Fig. 3, vertex v cannot join R_{cov} because the neighbors of it in R_{cov}, v_1, v_2 and v_3, fail to satisfy pairwise one-hop connectivity, which would cause the difference between the boundaries of R'_{cov} and the original R_{cov} is not a boundary. An example of complex expansion for a vertex is presented in Fig. 4. Vertex 1 in Fig. 4(a) is the candidate to investigate, and the sub-complex incident to its neighboring graph is shown. Beginning by the 2-simplex in (b), the process of expansion is carried out with the addition of legitimate vertexes to the current sub-complex within neighboring set of vertex 1, as shown in (c) and (d). This expansion is successful since the set of vertexes in final sub-complex in (d) is exactly the neighboring set of vertex 1 and thus it is proved to be redundant. Details of complex expansion are presented in algorithm 2.

Algorithm 2. Complex Expansion

```
Begin
construct Rips sub-complex in active neighbors set Neighb
for every 2-simplex tri do
    get the vertexes set vert_set of tri and set rest_vert of rest vertexes in Neighb
    while rest_vert is not empty do
        for every vertex v in rest_vert do
            if vertex v is legitimate then
                add v to vert_set and update the vert_set
            end if
        end for
        remove common vertexes shared by rest_vert and vert_set from rest_vert
        if rest_vert is not changed after this removal then
            ex_flag = 1
            break
        end if
    end while
    if ex_flag == 1 then
        continue
    end if
    if rest_vert is empty then
        c_flag = 1;
        return;
    end if
end for
End
```

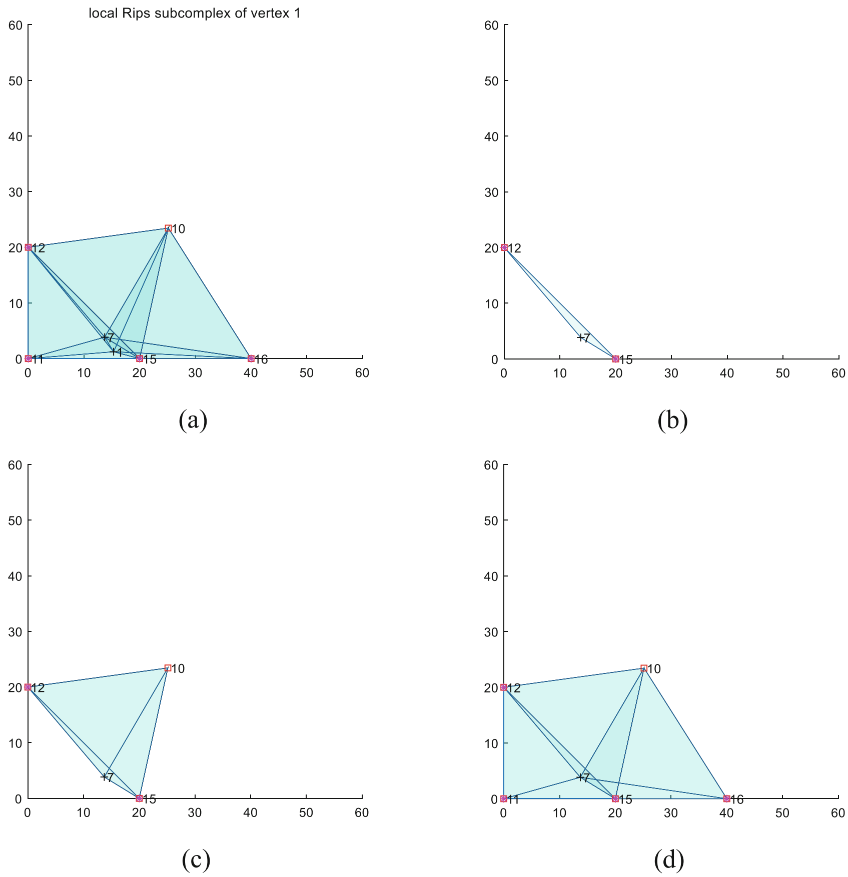


Fig. 4. Example of complex expansion for vertex 1

3.3 Complex Expansion Based Coverage Optimization

Algorithm 3. Complex Expansion Based Coverage Optimization

```
Begin
Construct initial Rips complex
Get 1-skeleton of initial Rips complex, flag every edge and get the maximum edge_flag
Compute weight of every vertex and the maximum weight_max
while ve_flag == 0 do
   while weight_max > 0 do
      find the vertex p with least ID whose wight is weight_max
      if result of complex expansion operated on Neighb_p is true then
         remove vertex p and update weight of vertexes in Neighb_p
         mark ve_flag as 1
      else
         weight of vertex p is set to 0
      end if
      update weight_max
   end while
   while edge_flag > 0 do
      find the first edge e with edge_flag
      if result of complex expansion operated on Neighb_e is true then
         remove edge e and update neighbor sets of two endpoints of e
         mark ve_flag as 1
      else
         flag edge e by 0
      end if
   end while
   if ve_flag == 1 then
      update the weights of those vertexes with weight 0
      update the flags of those edges with flag 0 to 1
      ve_flag = 0
   end if
end while
End
```

Based on the complex expansion algorithm described above, coverage optimization algorithm proposed here delete redundant vertexes and edges recurrently to make the Rips complex as planar as possible. Before every round of identification, weight information is updated. Then the determination for every candidate vertex and edge is performed in distributed way. Once a vertex or an edge is recognized as redundant, it will be removed with its simplexes. And the neighboring set of its neighbors are required to update. While if it is recognized as critical, it will not be identified in the current round and will remain critical in the following rounds till the end of the algorithm. At the end of every round, if there are some vertexes or edges deleted, the algorithm will proceed to next round; otherwise, it means there is no more redundancy in the complex and the algorithm comes to an end. The whole reduction algorithm for simplicial complex is given in algorithm 3 above.

4 Simulation and Performance Analysis

4.1 Simulation Discussion

The results of our algorithm are simulated with MATLAB R2017b. We deploy the set of vertexes randomly using Poisson point process in the given area. It can be seen in Fig. 5 that one implementation of Complex Expansion algorithm on the Rips complex with parameter $\in = 30$ is presented within a square of side length $a = 60$. Internal vertexes are distributed randomly within the area with their average number μ subject to Poisson point process, while fence vertexes are fixed along the boundary, with covering radius $r = 15$ of them all, as shown in Fig. 5(a). Corresponding initial Rips complex is shown in Fig. 5(b). Internal vertexes are starred while fence vertexes are squared and starred.

Vertex 14 in Fig. 5(b) is squared indicating that it is crucial according to weight computation, and it remains crucial during the whole process of our algorithm. Now, vertex deletion and edge deletion of first round are performed. From Fig. 5(c) and (d), it is not difficult to find vertex 4, 8 and 16 are removed and so is the case with 64 edges. The remaining vertexes are marked with red square to imply that they are crucial in this round. So it is necessary to update the weight information for the continuation of our algorithm. What follows is similar to the first round. After the second round, 6 vertexes and their simplexes are deleted shown in Fig. 5(e). Then after 5 rounds of execution, the Rips complex is reduced as much as possible and remains stable. As shown in Fig. 5(f) and (g), the snapshots after round 4 and round 6 are chosen to present the redundancy of the current Rips complex. Due to the deletion of 3 internal vertexes and their simplexes, the Rips complex after 4 rounds is more planar with 6 internal vertexes. While another 2 rounds of iteration takes more 2 redundant internal vertexes away and the Rips complex in Fig. 5(g) is the most planar one that our algorithm are able to give. It is obvious that only 4 internal vertexes exist in the final reduced Rips complex and all of them are squared indicating that they are essential for the coverage of internal area, which is proved in the corresponding covering discs presented in Fig. 5(h). The ratio of removed internal vertexes to all the internal vertexes is 77.78% this time, which is excellent.

With a series of parameters chosen to ensure coverage, the mean values of 1000 simulation results in different vertex density are collected and the corresponding ratios of removed internal vertexes to all internal vertexes after the execution of our algorithm and Reduction Algorithm in [10] respectively are shown in Fig. 6 and runtime of them is shown in Fig. 7. From Fig. 6, the ratio increases with the increase of vertex density, which is reflected by average number μ of internal vertexes. Furthermore, the performance of Complex Expansion Algorithm is closer to that of Reduction Algorithm when

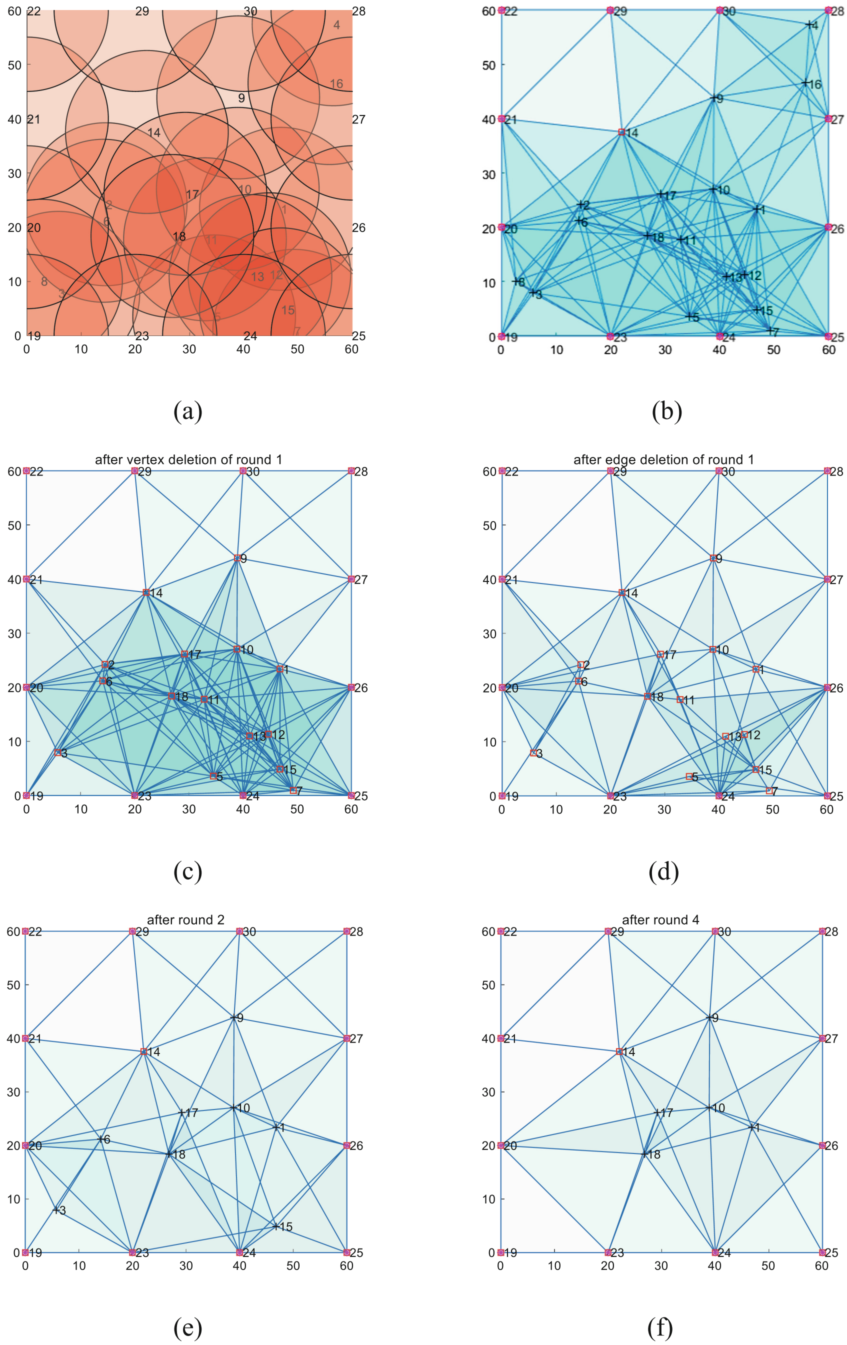
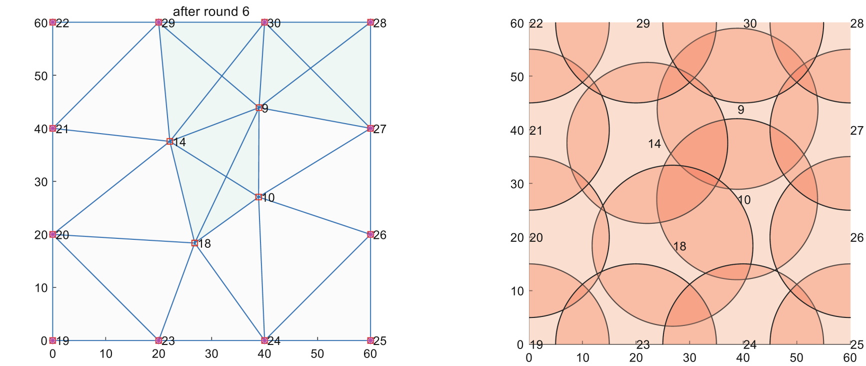


Fig. 5. Process of complex expansion based coverage optimization algorithm. (a) Initial covering discs. (b) Initial Rips complex. (c) After vertex deletion of round 1. (d) After edge deletion of round 1. (e)–(g) Reduced Rips complex after round 2, 4, 6. (h) Final covering discs



(g) (h)

Fig. 5. (*continued*)

there are more and more internal vertexes in the initial Rips complex. For example, when μ is chosen as 16, ratio of removed internal vertexes to all internal vertexes for RA and CE algorithms is 73.80% and 70.55% respectively; while that of them is 80.60% and 80.00% respectively when μ is 26.

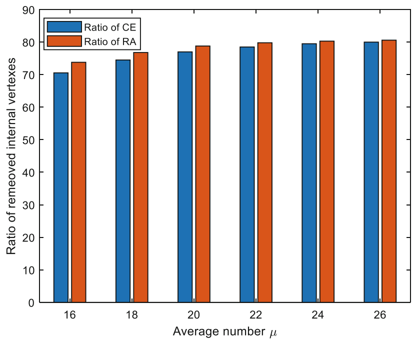


Fig. 6. Ratio of removed internal vertexes to all internal vertexes of algorithm CE and RA

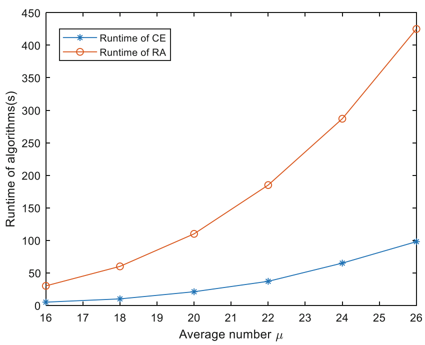


Fig. 7. Runtime of algorithm CE and RA

4.2 Complexity Analysis

As shown in Fig. 7, the computational complexity between them is remarkable. Complexity of Reduction Algorithm in [10] increases exponentially with the number of vertexes of the Rips complex, which has been analyzed in detail. Let n denote the average number of neighbors of individual vertexes in the Rips complex in CE algorithm. As for construction of 1-simplexes and 2-simplexes, each vertex needs to obtain its neighboring set and its neighbors' sets, which is achieved by two rounds of broadcast. By checking n times, the set of its 1-simplexes is obtained; while it is necessary to check C_n^2 times to obtain its set of 2-simplexes. So the complexity of constructing 1-simplexes and 2-simplexes is $O(n)$ and $O(n^2)$ respectively. Since weight computation of a vertex is equivalent to the inspection of neighbors of its 2-simplexes, the complexity of weight computation is $O(n^2)$. As for the execution of complex expansion, the worst case is to check the neighboring set of every 2-simplex of a vertex or an edge, which is done for up to nC_n^2 times, so the complexity of vertex or edge deletion is $O(n^3)$. Then the weight of neighbors of a redundant vertex is updated and the sub-complex is reconstructed in the neighboring graph of it, which is $O(n^2)$. So the complexity of our algorithm is $O(n^3)$. In the scenario where the number of sensor nodes deployed to monitor target area is tremendous, the runtime of RA is unbearable and our CE algorithm is more practical.

5 Conclusion and Future

In this paper, a homology based coverage optimization algorithm is proposed to simplify the wireless sensor network when it gives superfluous coverage to the target field. Different from most existing homological approaches, our algorithm performs redundant sensor nodes deletion from the perspective of connectivity of the neighboring graph of internal sensor nodes. Moreover, the complexity of this algorithm is much lower. Yet, our algorithm is constrained in full coverage situation and suitable for two-dimensional plane. Applying this algorithm to k-coverage scenarios is a research direction in the future.

References

1. Elhabyan, R., Shi, W., St-Hilaire, M.: Coverage protocols for wireless sensor networks: review and future directions. J. Commun. Netw. **21**(1), 45–60 (2019)
2. Ma, H.C., Sahoo, P.K., Chen, Y.W.: Computational geometry based distributed coverage hole detection protocol for the wireless sensor networks. J. Netw. Comput. Appl. **34**(5), 1743–1756 (2011)
3. Soundarya, A., Santhi, V.: An efficient algorithm for coverage hole detection and healing in wireless sensor networks. In: Proceedings of the 1st International Conference on Electronics, Materials Engineering and Nano-Technology, pp. 1–5. IEEE, Kolkata (2017)
4. Rachid, B., Amar, L.: Boundary and holes recognition in wireless sensor networks. J. Innovation Digital Ecosyst. **3**(1), 1–14 (2016)
5. Novella, B., Tiziana, C., Tom, F.L.P., Simone, S.: Autonomous deployment of heterogeneous mobile sensors. IEEE Trans. Mobile Comput. **10**(6), 753–766 (2011)
6. Zhang, Y., Zhang, X., Fu, W., Wang, Z., Liu, H.: HDRE: coverage hole detection with residual energy in wireless sensor networks. J. Commun. Netw. **16**(5), 493–501 (2016)
7. Ghrist, R., Muhammad, A.: Coverage and hole-detection in sensor networks via homology. In: Proceedings of the 4th International Conference on Information Processing in Sensor Networks, pp. 254–260. IEEE, Boise (2005)
8. Silva, V.D., Ghrist, R.: Coordinate-free coverage in sensor networks with controlled boundaries via homology. Int. J. Robot. Res. **25**(12), 1205–1222 (2006)

9. Tahbaz-Salehi, A., Jadbabaie, A.: Distributed coverage verification in sensor networks without location information. IEEE Trans. Autom. Control **55**(8), 1837–1849 (2008)
10. Vergne, A., Decreusefond, L., Martins, P.: Reduction algorithm for simplicial complexes. In: 2013 Proceedings of IEEE INFOCOM, pp. 475–479. IEEE, Turin (2013)
11. Le, N.-K., Martins, P., Decreusefond, L., Vergne, A.: Simplicial homology based energy saving algorithms for wireless networks. In: 2015 IEEE International Conference on Communication Workshop (ICCW), pp. 166–172. IEEE, London (2015)
12. Dong, D., Liu, Y., Liao, X.,: Fine-grained boundary recognition in wireless ad hoc and sensor networks by topological methods. In: Proceedings of the 10th ACM International Symposium on Mobile Ad Hoc Networking and Computing, pp. 135–144. MobiHoc, New Orleans (2009)
13. Yan, F., Vergne, A., Martins, P., Decreusefond, L.: Homology-based distributed coverage hole detection in wireless sensor networks. IEEE/ACM Trans. Netw. **23**(6), 1705–1718 (2015)
14. Ma, W., Yan, F., Zuo, X., Hu, J., Xia, W., Shen, L.: Simplicial complex reduction algorithm for simplifying WSN's topology. In: Zheng, J., Xiang, W., Lorenz, P., Mao, S., Yan, F. (eds.) ADHOCNETS 2018. LNICST, vol. 258, pp. 25–35. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-05888-3_3
15. Guvensan, M.A., Yavuz, A.G.: On coverage issues in directional sensor networks: a survey. Ad Hoc Netw. **9**(7), 1238–1255 (2011)
16. Munkres, J.R.: Elements of Algebraic Topology, 2nd edn. Addison-Wesley, New Jersey (1993)
17. Edelsbrunner, H., Parsa, S.: On the computational complexity of betti numbers: reductions from matrix rank. In: Proceedings of the Twenty-Fifth Annual ACM-SIAM Symposium on Discrete Algorithms, pp. 152–160. SIAM, Portland (2014)