An Efficient Approach for Rigid Body Localization via a Single Base Station Using Direction of Arrive Measurement

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Abstract. Rigid bodies are objects whose profile will not change after moving or being forced. A framework of rigid body localization (RBL) is to estimate the position and the orientation of a rigid object. In a wireless node network (WSN) based RBL approach, a few wireless nodes are mounted on the surface of the rigid target. Even though the position of the rigid body is unknown, we know how the nodes are distributed, which means that the topology of the nodes is known. Recently, a novel RBL scheme is studied, in which the rigid target is localized with just one single base station (BS) by measuring the angles between the BS and the positions of wireless nodes in the current frame, i.e., direction of arrival (DOA). However, the DOA-based RBL model is highly nonlinear and existing heuristic algorithms are generally time-consuming. In this paper, we intend to find the optimal solution of the 3-D positions of wireless nodes by fusing the topology information and DOA measurements with Newton's Iteration algorithm (NIA). Then, the rotation matrix and the translation vector can be obtained by the unit quaternion (UQ) method with the 3-D positions of wireless nodes, which completes the RBL task. Finally, we evaluate the proposed NIA-based RBL performance in terms of the root mean squared error (RMSE), as well as the computation costs.

Keywords: Rigid body localization · Single base station · Direction of arrival · Newton's iteration algorithm · The unit quaternion method

1 Introduction

1.1 Background

Generally, a rigid target is the object whose deformation of the moving object can be ignored when we consider the object as a rigid body. Compared with general positioning models, rigid body localization (RBL) considers not only the position of the object but also the attitude [1].

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The RBL plays an important role in many fields. With its maturity, RBL can be used to the small-scale instruments such as virtual reality (VR) helmets, video games and smart robots. For example, the three-dimensional (3-D) position and posture of VR helmets are essential to provide more realistic virtual images. RBL is also applied to large-scale appliance such as vehicles, ships, aircraft and spacecraft. In unmanned systems, the position and orientation information are absolutely necessary, and unmanned systems require high precision to ensure safety [2]. In addition, RBL can also be applied to monitor and estimate the oblique of buildings in real time.

Global Positioning System (GPS) is a system for positioning and navigating globally by satellites which can provide navigation information including the absolute position and the speed of users. GPS can also be used for RBL and has been applied widely in many fields such as transportation, marine exploration, aerospace, and so on. However, it has to consider the problems of period ambiguity in phase measurement and the costs of the antenna mounted on the object. Besides, GPS cannot be applied in many sheltered scenarios such as indoor positioning. The source localization in wireless node networks (WSNs) compensates well for the vacancy of GPS in accurate scenarios [3], since the position of rigid object can be estimated by the physical characteristics of the signal from the distributed wireless nodes in a WSN with relatively low cost.

1.2 Related Work

In order to estimate the position of an object in WSNs, there are many processing methods for the information which is obtained from wireless nodes, such as time of arrival (TOA), time difference of arrival (TDOA), direction of arrival (DOA) and so on. In the framework of RBL based on TOA and TDOA, several wireless nodes have to be mounted on the rigid object and then TOA and TDOA between nodes and each base station (BS) can be measured [4, 5].

The TOA method realizes the localization by converting the time of propagating signals between the wireless nodes and the BSs to a distance. However, it is obvious that most of the existing TOA-based positioning methods have limitations. It is generally difficult to measure absolute arrive time of signals and then the TDOA method is proposed. The TDOA method is an improvement to the TOA method. Instead of directly utilizing the time of signal arrival, the TDOA method determines the location of the moving object using the time difference between the signals received by multiple BSs, which reduced time synchronization requirements of TOA between wireless nodes and BSs. Even though the TDOA method relatively improves the accuracy of position, both of the TOA and TDOA method need three BSs in WSNs at least and have to make sure the positions of BS are fixed and known.

In the framework of RBL based on DOA, there likewise are wireless nodes mounted on the rigid object and DOA between these nodes and the single BS can be measured. The DOA method estimates the position of a rigid object with the angles of arrival. It means that the method does not need to know the position of the BS, and it can determine the position of the object by using just a single BS because of the 2-dimensional DOA information, including the azimuth and pitch angles. The locating measurement of object is using multi-station positioning system mostly which makes sure the position of an object by three to five BSs. Multi-station positioning system requires several BSs to synchronize data transmission, while single-station positioning system requires only one observation BS. And it does not need to transmit information actively, the only requirement is to receive information of the wireless nodes to achieve the localization of the target. Compared with multi-station positioning system, the single BS positioning system greatly reduces the consumption of resources.

Recently, several heuristic search positioning algorithms based on the evolutionary algorithm paradigm are studied for the DOA-based RBL scheme, such as particle swarm optimization (PSO) [6] and participatory searching algorithm (PSA) [7]. They all realized the RBL purpose of the DOA-based single BS method, starting from the random solution and achieving the optimal solution through iterations. PSO is to initialize a bunch of random particles (random solution) and to follow the current searched optimal value to find the global optimal. The method uses PSO to optimize the target cost function to obtain the true distance of the object and achieves accurate solution. PSA is a population-based search procedure derived from the participatory learning paradigm. The algorithm is forming search as a pool with individuals, reserving the one nearest to the current best individuals and adding new random individuals in each step.

The PSO algorithm is easy to be accomplished and does not need to adjust too many parameters. It has great development value and can be applied to multiple fields such as pattern recognition, image processing and neural network training. Even though the methods of PSO and PSA are superior for their easy to implement in practical problems, they also have shortcomings of slow running speed and low success rate. The two methods run slowly because they are both heuristic algorithms. And they are easy to fall into a local optimum when the model to be optimized is highly nonlinear, so that the global optimum solution cannot be obtained.

In this paper, the rigid object is located by DOA with single BS, and the position of wireless nodes in the current frame is estimated by Newton's iteration algorithm (NIA). The reference frame is a preset system of information about the position and attitude of wireless nodes in the 3-D space, and the current frame is the position and attitude of wireless nodes after transforming relative to the reference frame. The algorithm is used to linearize the nonlinear equation to find the approximate root. When the positions of wireless nodes in the reference frame and current frame are known, we can obtain the rotation matrix and the translation vector by the unit quaternion (UQ) method [9].

The remainder of the paper is organized as follows. The model of positioning system and the considered problems are introduced in Sect. 2. Section 3 discusses the relative process about determining location information of rigid object by NIA and solving transformation information of object by the UQ method. In Sect. 4, the performance of proposed method is analyzed by contrast results based on simulations. Finally, a conclusion of the paper is drawn in Sect. 5.

For clarity, the notations used in this paper are shown as below. Uppercase or lowercase bold letters are used to represent matrix and vectors. $\mathbf{X}^T$ is the transpose of $\mathbf{X}$, $\otimes$ is the notation of the Kronecker product, and $\mathbf{I}_n$ is the identity matrix of $n \times n$. $\|*\|$ means the Euclidian norm and $\mathrm{vec}(\mathbf{X})$ denotes a column vector by stacking the columns $\mathbf{X}$.

2 DOA-Based RBL Model

As mentioned above, a framework of network that K wireless nodes are mounted is built to determine the position and orientation of a rigid object in the 3-D space. These wireless nodes are mounted on the rigid object with a certain accuracy and their topology is known. As illustrated in Fig. 1, the coordinate system is set to make the single BS as the origin O of and a reference frame is assumed at the origin O to be the starting point of the rigid object. After a series of transformations, the object arrives at another position which defined as the current frame.

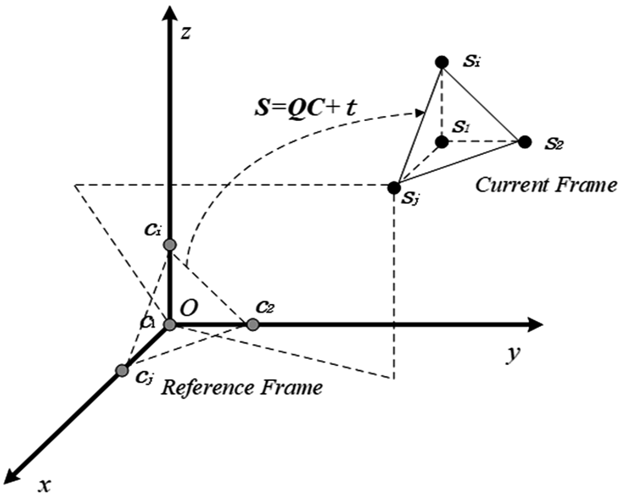

Fig. 1. DOA-based RBL framework

In the reference frame, the coordinate of the kth wireless node is denoted as 3-D vector $\mathbf{c}_k = \left[c_{k,x}, c_{k,y}, c_{k,z}\right]^T$ and the wireless nodes topology is determined by the matrix $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K] \in \mathbb{R}^{3\times K}$. The position of wireless node $\mathbf{c}_k$ is determined because of the known topology. Let the absolute coordinate of the kth wireless node in the current frame be denoted as 3-D vector $\mathbf{s}_k = \left[s_{k,x}, s_{k,y}, s_{k,z}\right]^T$ and the absolute coordinates of these wireless nodes are collected in $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \in \mathbb{R}^{3\times K}$. The current frame is transformed from the reference frame by rotation and translation, mathematically $\mathbf{S}$ can be expressed by $\mathbf{C}$ that

$$\mathbf{S} = \mathbf{RC} + \mathbf{t} \otimes \mathbf{1}_{1\times K}, \tag{1}$$

where $\mathbf{t} = [x, y, z]^T \in \mathbb{R}^{3\times 1}$ is the unknown translation vector, and $\mathbf{R} \in \mathbb{R}^{3\times 3}$ is the unknown rotation matrix which is orthogonal matrix with determinant is 1, i.e. $\mathbf{R}^T\mathbf{R} = \mathbf{R}\mathbf{R}^T = \mathbf{I}_3$.

Specifically, the 3-D vector $\mathbf{c}_k$ is the known coordinate of the kth wireless node in the reference frame while the $\mathbf{s}_k$ is the unknown coordinate of the kth wireless node in the current frame that we need to solve. The unknown orthogonal matrix $\mathbf{R}$ actually represents how the rigid object rotates from the reference frame. It means that the rotation matrix $\mathbf{R}$ refers to the orientation of the transformation of the rigid target. And the unknown vector $\mathbf{t}$ represents the moving position of the rigid target. When the rigid object does not experience rotation, the rotation matrix $\mathbf{R} = \mathbf{I}_3$.

In order to obtain the positions of the wireless nodes in the current frame, two conditions are needed in the RBL estimation. The first one is the Euclidean distances between the node pairs on the rigid target. We denote the distance between the ith and jth wireless nodes in the reference frame as $d_{i,j}$. The distance $d_{i,j}$ can be obtained since the topology of wireless nodes is known. The known distance $d_{i,j}$ can be calculated by the position of wireless nodes in the reference frame that $d_{i,j} = \left\| c_i - c_j \right\| = \left\| s_i - s_j \right\|$. Collecting the distance $d_{i,j}$, and we obtain a column vector

$$\mathbf{d} = \left[d_{1,2}, \ldots, d_{i,j}, \ldots, d_{K-1,K} \right]^T, i,j = 1, \ldots, K, i > j. \tag{2}$$

The second condition for RBL is not known information, it need the single BS to measure. Assuming that the BS is equipped with phased array radar (PAR), which is used to measures the DOA of wireless nodes signals in 3-D space for RBL. In the phased array, signals can be used for both reception and transmission. The angles between nodes signals and the x- and z-axes can be obtained and we denote the kth node's DOA as α_k and β_k corresponding to the x- and z-axes.

The distance of wireless nodes pairs $d_{i,j}$ can be rewritten by the coordinates of nodes in reference frame. Combining with the DOAs, α_k and β_k, the equation of the nonlinear system can be obtained according to the spatial geometric:

$$\begin{cases} \alpha_k = \arccos \frac{x_k}{\sqrt{x_k^2 + y_k^2 + z_k^2}}, \\ \beta_k = \arccos \frac{z_k}{\sqrt{x_k^2 + y_k^2 + z_k^2}}, \\ d_{i,j} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \left(z_i - z_j\right)^2}. \end{cases} \tag{3}$$

Usually, there is no root formula to solve the nonlinear system exactly in most cases. So, it is difficult or even impossible to find the exact solution. In order to obtain the position of wireless nodes in current frame, it is particularly important to find the approximate solution of the nonlinear equation. The method proposed to solve the problem is given in the next section.

3 Proposed RBL Algorithm

3.1 Newton's Iteration Method

Newton's iteration is one of the important methods to find the root of the nonlinear model and it is a classical method of optimization. The advantage of the method is that there is performance of square convergence around the single root of the equation. Newton's method is the process of using the iteration to continuously recur the new value with the old value of the variable. Therefore, it is necessary to set an appropriate initial value to guarantee the algorithm converges and ensure its convergence speed. In this paper, the initial value is determined in the reference frame.

Solving nonlinear equations by Newton's method is an approximation algorithm for linearizing the nonlinear equations. The equation is expanded into Taylor series (TS) at the initial value, and we can get an iterative relation by letting linear part of TS be zero if the derivative of the equation is not equal to zero. The method utilizes the derivative, and the direction of each iteration is the direction in which the value of the current point of the function decreases.

In order to iteratively estimate the node position, we define a measurement vector

$$\mathbf{F} = \left[\boldsymbol{\omega}^T \mathbf{d}^T\right]^T, \tag{4}$$

where $\boldsymbol{\omega} = \left[\alpha_{1,x}, \ldots, \alpha_{K,x}, \beta_{1,z}, \ldots, \beta_{K,z}\right]^T + \left[\upsilon_{1,x}, \ldots, \upsilon_{K,x}, \upsilon_{1,z}, \ldots, \upsilon_{K,z}\right]^T$ containing DOA observations with Gaussian noise and $\mathbf{d}$ is the true value measured from the known position of wireless nodes without noise. The noise distribution of DOA observations is $\mathcal{N}(0, \sigma^2)$. These measurements of wireless nodes in RBL are expressed in (2). Expanding nonlinear equations (2) at the initial value by TS until the expansions are nonlinear, the initial value in this paper is the positions of wireless nodes in the reference frame $\mathbf{C}$, and we obtain

$$\mathbf{F} \approx \mathbf{F}(\mathrm{vec}(\mathbf{C})) + \mathbf{G} \cdot (\mathrm{vec}(\mathbf{S}) - \mathrm{vec}(\mathbf{C})), \tag{5}$$

where $\mathrm{vec}(\mathbf{C}) = \left[\mathbf{c}_1^T, \ldots, \mathbf{c}_K^T\right]^T$, $\mathrm{vec}(\mathbf{S}) = \left[\mathbf{s}_1^T, \ldots, \mathbf{s}_K^T\right]^T$ and $\mathbf{G}$ is the Jacobian matrix of $\mathbf{F}$ which partially derived with respect to the position of wireless nodes in the current frames $\mathbf{S}$ and defined as

$$\mathbf{G} = \begin{bmatrix} \frac{\partial \boldsymbol{\omega}^T}{\partial \mathrm{vec}(\mathbf{S})}\Big|_{\mathbf{S}=\mathbf{C}} \\ \frac{\partial \mathbf{d}^T}{\partial \mathrm{vec}(\mathbf{S})}\Big|_{\mathbf{S}=\mathbf{C}} \end{bmatrix}, \tag{6}$$

where $\frac{\partial \boldsymbol{\omega}^T}{\partial \mathrm{vec}(\mathbf{S})}$ is the partial derivate of $\boldsymbol{\omega}$ with respect to $\mathbf{S}$ and is a matrix of $2K \times 3K$, $\frac{\partial \mathbf{d}^T}{\partial \mathrm{vec}(\mathbf{S})}$ is the partial derivate of $\mathbf{d}$ with respect to $\mathbf{S}$ and is a matrix of $\frac{(K-1)K}{2} \times 3K$. The expressions of $\frac{\partial \boldsymbol{\omega}^T}{\partial \mathrm{vec}(\mathbf{S})}$ and $\frac{\partial \mathbf{d}^T}{\partial \mathrm{vec}(\mathbf{S})}$ are as follows respectively:

$$\frac{\partial \boldsymbol{\omega}^T}{\partial \mathrm{vec}(\mathbf{S})} = \begin{bmatrix} \frac{\partial \alpha_1}{\partial x_1} & \frac{\partial \alpha_1}{\partial y_1} & \frac{\partial \alpha_1}{\partial z_1} & \cdots & \frac{\partial \alpha_1}{\partial x_K} & \frac{\partial \alpha_1}{\partial y_K} & \frac{\partial \alpha_1}{\partial z_K} \\ & \vdots & & \ddots & & \vdots & \\ \frac{\partial \alpha_K}{\partial x_1} & \frac{\partial \alpha_K}{\partial y_1} & \frac{\partial \alpha_K}{\partial z_1} & \cdots & \frac{\partial \alpha_K}{\partial x_K} & \frac{\partial \alpha_K}{\partial y_K} & \frac{\partial \alpha_K}{\partial z_K} \\ \frac{\partial \beta_1}{\partial x_1} & \frac{\partial \beta_1}{\partial y_1} & \frac{\partial \beta_1}{\partial z_1} & \cdots & \frac{\partial \beta_1}{\partial x_K} & \frac{\partial \beta_1}{\partial y_K} & \frac{\partial \beta_1}{\partial z_K} \\ & \vdots & & \ddots & & \vdots & \\ \frac{\partial \beta_K}{\partial x_1} & \frac{\partial \beta_K}{\partial y_1} & \frac{\partial \beta_K}{\partial z_1} & \cdots & \frac{\partial \beta_K}{\partial x_K} & \frac{\partial \beta_K}{\partial y_K} & \frac{\partial \beta_K}{\partial z_K} \end{bmatrix}, \quad (7)$$

$$\frac{\partial \mathbf{d}^T}{\partial \mathrm{vec}(\mathbf{S})} = \begin{bmatrix} \frac{\partial d_{1,2}}{\partial x_1} & \frac{\partial d_{1,2}}{\partial y_1} & \frac{\partial d_{1,2}}{\partial z_1} & \cdots & \frac{\partial d_{1,2}}{\partial x_K} & \frac{\partial d_{1,2}}{\partial y_K} & \frac{\partial d_{1,2}}{\partial z_K} \\ & \vdots & & & & \vdots & \\ \frac{\partial d_{(K-1),K}}{\partial x_1} & \frac{\partial d_{(K-1),K}}{\partial y_1} & \frac{\partial d_{(K-1),K}}{\partial z_1} & \cdots & \frac{\partial d_{(K-1),K}}{\partial x_K} & \frac{\partial d_{(K-1),K}}{\partial y_K} & \frac{\partial d_{(K-1),K}}{\partial z_K} \end{bmatrix}. \quad (8)$$

According to the conditions of Newton's method, we are still need the formula that arrives at its next value from the previous value of the variable, i.e., the iterative relation, which can be shown to be given by

$$\boldsymbol{\delta} = \mathrm{vec}(\mathbf{S}) - \mathrm{vec}(\mathbf{C}) = \mathbf{F}/\mathbf{G}, \quad (9)$$

We can update the iterated value by replacing $\mathbf{C}$ with $\mathbf{C} + \boldsymbol{\delta}$ and obtain the final estimated value by repeating the above update progress until $\boldsymbol{\delta}$ is sufficiently small which means the solutions convergence or a maximum number of iterations has been reached during execution.

The solution of the nonlinear equations can be found by constantly iterating, and it is the position coordinates of wireless nodes in the current frame. Now the positions of the wireless nodes in the reference frame and current frame are known, it is equivalent that in the transformation formula (1), $\mathbf{S}$ and $\mathbf{C}$ both have been known. And the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$ can be obtained by all known information next.

3.2 The Unit Quaternion Method

There are many ways to express rotation, such as Euler angle, shaft angle and the UQ, etc. [9] In this paper, the UQ method is used to solve the rotation matrix and the translation vector. The UQ method uses a vector to represent the rotation axis and an angular component to represent the angle of rotation around this axis. The UQ is defined as

$$\mathbf{q} = \left[\cos\frac{\theta}{2}, l \cdot \sin\frac{\theta}{2}, m \cdot \sin\frac{\theta}{2}, n \cdot \sin\frac{\theta}{2}\right], \quad (10)$$

where θ is the rotation angle, and 3-D vector (l, m, n) which meet $l^2 + m^2 + n^2 = 1$ is the rotation axis. And the rotation matrix $\mathbf{R}$ can be derived from Rodrigues formula:

$$\mathbf{R} = \mathbf{I} + 2\mathbf{U} \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} + 2\sin^2\frac{\theta}{2} \cdot \boldsymbol{U}^2$$

$$= \begin{bmatrix} 1 - 2(m^2 + n^2) \cdot \sin^2\frac{\theta}{2} & 2lm \cdot \sin^2\frac{\theta}{2} - 2n \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} & 2ln \cdot \sin^2\frac{\theta}{2} + 2m \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} \\ 2lm \cdot \sin^2\frac{\theta}{2} + 2n \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} & 1 - 2(l^2 + n^2) \cdot \sin^2\frac{\theta}{2} & 2mn \cdot \sin^2\frac{\theta}{2} + 2l \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} \\ 2ln \cdot \sin^2\frac{\theta}{2} - 2m \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} & 2mn \cdot \sin^2\frac{\theta}{2} + 2l \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} & 1 - 2(l^2 + m^2 \cdot \sin^2\frac{\theta}{2}) \end{bmatrix}, \tag{11}$$

where $\mathbf{I}$ is the unit matrix of size 3, $\mathbf{U} = \begin{bmatrix} 0 & -n & m \\ n & 0 & -l \\ -m & l & 0 \end{bmatrix}$ is the Cross Product matrix of rotary axis (l, m, n), substituting the UQ $\mathbf{q}$ to the equation and we can get rotation matrix:

$$\mathbf{R} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_0q_3 + q_1q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_0q_1 + q_2q_3) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \tag{12}$$

The UQ $\mathbf{q}$ is the solution which minimums the least square error:

$$\varepsilon^2 = \sum\nolimits_{i=1}^{N} \|\mathbf{s}_i - \mathbf{R}\mathbf{c}_i + \mathbf{t}\|^2 = \sum\nolimits_{i=1}^{N} \|\mathbf{s}_{ri} - \mathbf{R}\mathbf{c}_{ri}\|^2 = \sum\nolimits_{i=1}^{N} \left(\mathbf{s}_{ri}^T\mathbf{s}_{ri} + \mathbf{c}_{ri}^T\mathbf{c}_{ri} - 2\mathbf{s}_{ri}^T\mathbf{R}\mathbf{c}_{ri}\right) \tag{13}$$

where $\mathbf{s}_{ri} = \mathbf{s}_i - \bar{\mathbf{s}}$, $\mathbf{c}_{ri} = \mathbf{c}_i - \bar{\mathbf{c}}$, and $\bar{\mathbf{s}}$, $\bar{\mathbf{c}}$ are mean of $\mathbf{s}_i$ and $\mathbf{c}_i$ respectively. The least square error can be rewritten as $\varepsilon = \mathbf{q}^T\mathbf{P}\mathbf{q}$ according to the attributes of quaternion, and $\mathbf{P}$ is a matrix of 4×4:

$$\mathbf{p} = \begin{bmatrix} H_{xx} + H_{yy} + H_{zz} & H_{yz} - H_{zy} & H_{zx} - H_{xz} & H_{xy} - H_{yx} \\ H_{yz} - H_{zy} & H_{xx} - H_{yy} - H_{zz} & H_{xy} + H_{yx} & H_{zx} + H_{xz} \\ H_{zx} - H_{xz} & H_{xy} + H_{yx} & H_{yy} - H_{xx} - H_{zz} & H_{yz} + H_{zy} \\ H_{xy} - H_{yx} & H_{zx} + H_{xz} & H_{yz} + H_{zy} & H_{zz} - H_{xx} - H_{yy} \end{bmatrix} \tag{14}$$

where $H_{ab} = \sum\limits_{i=1}^{N} \mathbf{s}_{ri_a}\mathbf{c}_{ri_b}$, the UQ $\mathbf{q}$ is the feature vector corresponding to the largest eigenvalue of matrix $\mathbf{P}$. Finally, the rotation matrix $\mathbf{R}$ can be obtained from (10) with the known quaternion $\mathbf{q}$, and the translation vector $\mathbf{t}$ can be obtained from transformation formula (1).

4 Performance Evaluation

In the DOA-based RBL with a single BS, we consider $K = 4$ wireless nodes mounted on a rigid object. The single BS is fixed at the origin and the wireless nodes are distributed as a four-sided pyramid with a bottom of equilateral triangle and three sides of isosceles right triangle with side length of 3 m. According to wireless nodes distribution, the reference frame is

$$\mathbf{C} = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}. \tag{15}$$

In the DOA measurements, the noise of the DOA measurement $\boldsymbol{\omega}$ from wireless nodes in the current frame to the single BS are independent, zero-meaning, additive white gaussian noise, with the standard deviation of σ. There are several different values of the Gaussian noise tried in the simulation. The rigid object transforms the angle in the direction of the rotation matrix and moves with a translation vector in the 3-D space.

In the simulation experiment, the rotation angles are set as $[0, 0, 0]^T$, implying that the rigid object does not change direction when moving, and the translation vector is set as $\mathbf{t} = [\,5 \quad 5 \quad 3\,]^T$, meaning that the rigid target moves $\sqrt{59}$ m from the reference frame. The simulations are counted average over $N = 1000$ independent Monte Carlo iterations.

The performance of the proposed method is shown in terms of the root mean squared error (RMSE) of the estimates, success rate and running speed of the simulation experiment. RMSE of the estimates of $\mathbf{R}$ and $\mathbf{t}$ are computed by

$$\mathrm{RMSE}(\mathbf{R}) = \sqrt{\frac{1}{N}\sum\nolimits_{n=1}^{N} \left\| \mathbf{R} - \widehat{\mathbf{R}}_n \right\|}, \tag{16}$$

$$\mathrm{RMSE}(\mathbf{t}) = \sqrt{\frac{1}{N}\sum\nolimits_{n=1}^{N} \left\| \mathbf{t} - \widehat{\mathbf{t}}_n \right\|}, \tag{17}$$

where $\widehat{\mathbf{R}}_n$ and $\widehat{\mathbf{t}}_n$ mean the estimates in the nth Monte Carlo iteration. As shown in Fig. 2, CCRBs [10] of the DOA-based RBL framework and the RMSEs of the estimates of $\mathbf{R}$ and $\mathbf{t}$ basing on the NIA and UQ algorithms are compared under different noise magnitudes. As can be seen from the figure, the estimation RMSE curves of both rotation matrix and the translation vector are close to their CCRB, which means that the estimates almost reach the optimal value in theory. From the trend of the curve in the figure, as we expected, the RMSE of estimates decreased as the noise level of DOA reduced.

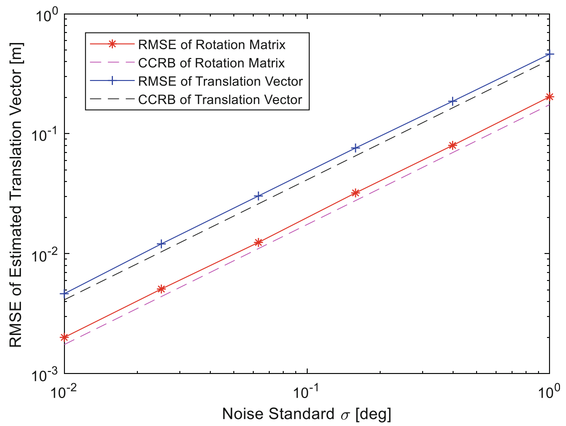

Fig. 2. RMSEs of the RBL estimation vs CCRB of rotation matrix and translation vector

The optimization performance of proposed NIA algorithm is compared with existing heuristic methods including the PSO [11] and PSA methods [10]. Simulation shows that the iteration number of the NIA method for convergence process is around 10 times, when we set the initial solution as the initial frame **C** (it is reasonable since the initial frame is known information). As shown in Table 1, the NIA method for 3-D position estimation of the wireless nodes in current frame are obviously shorten (ms-level) than the heuristic methods while guaranteeing the success rate.

Table 1. The optimization performance comparison of NIA, PSA and PSO methods

σ [deg]	10^{-2}	10^{-1}	10^{0}
PSO [s/%][a]	7.9/100	7.3/95	10.4/51
PSA [s/%]	5.3/100	5.9/100	8.9/70
NIA [s/%]	1.1×10^{-3}/100	0.9×10^{-3}/100	1.8×10^{-3}/76

[a] optimization performance is evaluated in terms of convergence time [s] and success rate [%]

5 Conclusions

In this paper, a framework was proposed to solve the DOA-based RBL problem for joint position and orientation estimation with a single BS. Several wireless nodes were mounted on the rigid object and the topology of these wireless nodes is known. Combining the topology information and the DOA measurements, we obtained the nonlinear model about the position of the wireless nodes. We optimized the model and find the 3-D position of the nodes in the current frame using Newton's method. Then the moving and rotating information of the rigid target with respect to reference frame, which is determined by a translation vector and a rotation matrix respectively, can be determined by the UQ method. The rotation and translation of wireless nodes represents the rotation and translation of the rigid target, and we can obtain the position of the rigid target from the transform information. Finally, the simulation of the proposed method is performed, and then its performance is compared with the CCRB. The simulation results shows that the accuracy of proposed method is close to the CCRB, besides, it optimization success rate and convergence speed significantly outperform the existing heuristic methods.

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