Convex Optimization Algorithm for Wireless Localization by Using Hybrid RSS and AOA Measurements

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Abstract. With the development of new array technology and smart antenna, it is easier to obtain the angle of arrival (AOA) measurements. The hybrid received signal strength (RSS) and AOA measurement techniques are proposed for the wireless localization in the paper. By converting the measurement equations and relaxing the optimization function, a second order cone programming and semidefinite programming (SOCPSDP) algorithm is put forward to obtain the position estimate by considering the known or unknown transmit power. The proposed SOCPSDP algorithm provides a solution to the source position estimate and avoids the initialization process. The simulations show that the SOCPSDP algorithm performs better than the semidefinite programming (SDP) algorithm. The accuracy performance of the proposed SOCPSDP algorithm degrades as the measurement noises increase.

Keywords: Wireless localization · Received signal strength · Angle of arrival · Convex optimization

1 Introduction

Wireless localization has been playing a key role in many applications, for example, emergency services, friend finding and tracking of the elderly [7,24]. In addition, wireless localization is an indispensable component of wireless sensor networks since the readings from a large number of sensor nodes are meaningful only when the locations of these readings are known. To obtain the position information, sensor nodes are categorized into anchor node with known position and source node which is required to be localized. A localization scheme tries to localize the source node using the ranging information extracted from the signaling between anchor node and source node [10,19].

Most of the accurate localization techniques are based on the ranging information by using the techniques such as, time of arrival (TOA) [9,18], time difference of arrival (TDOA) [4,20], received signal strength (RSS) [6,17,23] and angle of arrival (AOA) [3,8]. Among the difference ranging methods, RSS-based localization scheme is the most prevalent one due to easier implementation and less complexity [25]. However, the noises of the RSS measurements are large,

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so the positioning performance is not very well. Electronic compass or vision sensor provides the possibility of AOA measurements, but it requires additional hardware configuration and the hardware cost of the node. Recently, the cost of AOA measurement is decreased with the development of new array technology and smart antenna which provide a broad space for the AOA measurements [5].

To locate the source node by using these different measurement methods, some algorithms including maximum likelihood (ML) [2,11], second order cone programming (SOCP) [12] and semidefinite programming (SDP) method [13,26] are proposed for the wireless localization. The ML estimator is always solved by the numerical method which requires initial solution to ensure the convergence. When the selected initial solution is far from the actual, it will be trapped in the local optimum. To overcome the shortcoming of the ML estimator, the convex SDP algorithm are proposed to obtain the position estimate of the source node. By relaxing the nonconvex optimization into convex problem, the SDP method provides robust solution. However, the computational complexity of SDP is high. The accuracy performance of SDP can not achieve the optimal Cramér-Rao Lower Bound (CRLB) due to the convex optimization relaxation [22].

Recently, some researches focus on the wireless localization by using the hybrid RSS and AOA measurements [1,14]. Due to the increasing of the unknown parameters, the source node is more difficult to be localized in the three-dimensional plane compared with the two-dimensional plane. To locate the source node, the required number of the anchor nodes in the three-dimensional plane is much larger than that of the two-dimensional plane. Compared with the single ranging method, the source node is easier to be estimated by using the hybrid RSS and AOA measurements, which provide more ranging information for the position estimation [15]. On the other hand, the required number of the anchor nodes would be less for locating the source nodes. In [14], semidefinite programming (SDP) relaxation techniques are proposed for the cooperative wireless localization by using the RSS and AOA measurements. However, the proposed SDP algorithm performs not very well due to the convex relaxation.

The RSS value of receiving node is relevant with the transmit power of transmitting node. However, the transmit power will be subject to a large fluctuation because its value is dependent on the height and orientation of the node antenna, as well as antenna gain and its battery which will decrease with time. So the RSS-based position estimation problem always assumes the transmit power to be known or unknown. When the transmit powers are unavailable and assumed to be unknown, the RSS-based localization scheme is designed to estimate the positions of the source nodes in [16]. The convex optimization algorithms are proposed to estimate the position parameters and compared with their performance by considering the transmit powers to be known or unknown [25]. In [21], the linear least square approach is designed to determine the locations of the source nodes, when path loss model parameters are unknown only by exploiting the RSS measurements.

In this paper a mixed SOCPSDP algorithm is proposed for the hybrid RSS and AOA wireless localization by assuming the known or unknown transmit power. By converting the nonconvex optimization problem into the convex optimization, the proposed SOCPSDP algorithm provides a solution for the source position estimate and avoids the initialization of the ML estimator. The rest of this paper is structured as follows. Section 2 presents the problem specification of the joint RSS and AOA wireless localization. Section 3 in detail describes the proposed SOCSDP algorithm by assuming known transmit power. In Sect. 4, the SOCSDP algorithm is extended to the situation of unknown transmit power. Section 5 analyzes the simulation results. The conclusion is represented in Sect. 6. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If the matrix is denoted by $(*)$, $(*)^{-1}$ and $(*)^T$ represent the matrix inverse and transpose operator, respectively. $\|*\|$ denotes ℓ_2 norm. For arbitrary symmetric matrix $\mathbf{A}$, $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is positive semidefinite.

2 Problem Specification

In a three-dimensional plane N anchor nodes are deployed with known positions which are denoted as $\mathbf{a}_i = [a_{i,x} \quad a_{i,y} \quad a_{i,z}]^T$, $i = 1, 2, \ldots, N$. In the same region, the source node is required to be located. The position of the source node is denoted as $\mathbf{x} = [x_x \quad x_y \quad x_z]^T$. To derive the position of the source node, the RSS between anchor node i and the source node is measured and denoted by p_i. Assuming that the RSS obeys the logarithmic decay model,

$$p_i = p_0 - 10\beta \log_{10} d_i + \varepsilon_i \tag{1}$$

where $i = 1, 2, \ldots, N$, β is called as path loss exponent (PLE) which is determined by the environment media and generally varied from 2 to 5. p_0 is called as the transmit power and related with the antenna gain and energy supply of the source node. d_i is the measurement distance between the anchor node i and the source node. ε_i represents the noise which conforms to the Gaussian distribution with zero mean and variance $\delta_{i,\varepsilon}^2$.

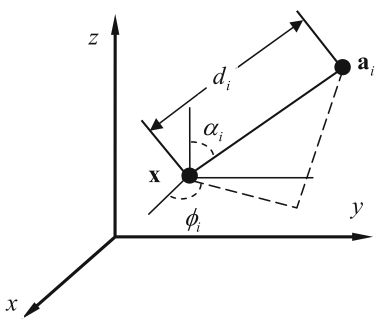


Fig. 1. AOA measurements between anchor node and source node

In the three-dimensional plane, the unknown position parameter of the source node includes the three direction of x, y and z. It is possible to be unreliable for the wireless localization only by using the RSS measurements between the anchor node and the source node. To reduce the positioning error and ensure the reliability of the position estimation, the direction angle and the elevation angle are also measured and shown in Fig. 1. The direction angle and the elevation angle are denoted as ϕ_i and α_i, respectively. By using the geographical position relationship of the nodes, the direction angle ϕ_i and the elevation angle α_i can be written as

$$\phi_i = \arctan(\frac{a_{i,y} - x_y}{a_{i,x} - x_x}) + m_i \tag{2}$$

$$\alpha_i = \arccos(\frac{a_{i,z} - x_z}{d_i}) + n_i \tag{3}$$

where m_i and n_i are the noises of the direction and the elevation measurements, respectively. Without loss of generality, it is assumed that the noises m_i and n_i are gaussian with zero mean and variance $\delta_{i,m}^2$ and $\delta_{i,n}^2$, respectively.

To derive the unknown position of the source node, the well known maximum likelihood (ML) estimator of least square cost function is written as

$$\min_{\mathbf{x}} \sum_{i}^{N} (\frac{1}{\delta_{i,\varepsilon}^2} r_{i,p}^2 + \frac{1}{\delta_{i,m}^2} r_{i,\phi}^2 + \frac{1}{\delta_{i,n}^2} r_{i,\alpha}^2) \tag{4}$$

where $r_{i,p}$, $r_{i,\phi}$ and $r_{i,\alpha}$ represent the error of the RSS, the direction and the elevation measurements. $r_{i,p}$, $r_{i,\phi}$ and $r_{i,\alpha}$ are written as

$$\begin{cases} r_{i,p} = p_i - p_0 + 10\beta \log_{10} d_i \\ r_{i,\phi} = \phi_i - \arctan(\frac{a_{i,y} - x_y}{a_{i,x} - x_x}) \\ r_{i,\alpha} = \alpha_i - \arccos(\frac{a_{i,z} - x_z}{d_i}) \end{cases} \tag{5}$$

where $d_i = \|\mathbf{x} - \mathbf{a}_i\|$. The solution to ML estimator is always solved by the numerical calculation which requires an initial point. When the initial point is enough close to the actual solution, the positioning results will be trapped in the local optimum. To overcome the shortcoming of the ML estimator and fasten the iterative calculation, the nonconvex optimization equation of (4) is converted into the convex optimization when the transmit power $\mathbf{p}_0$ is assumed to be known in Sect. 3 and unknown in Sect. 4.

3 Know Transmit Power

In the section the source location $\mathbf{x}$ is estimated by using the observed RSS measurements when the transmit power $\mathbf{p}_0$ is assumed be available. It is possible to relax the ML estimator formulation to a convex optimization problem, to provide an approximate solution that can be obtained in a globally optimum fashion with reduced computational efforts. Both SDP and SOCP relaxations are convex optimization techniques for wireless localization. To obtain the convex optimization form, the RSS, direction and elevation angle measurement equations are approximately linearized by considering the small noise conditions. In the following, we in detail describe the proposed convex optimization algorithm for the RSS and AOA wireless localization.

Firstly (1) is rewritten as

$$d_i^2 = 10^{\frac{p_0 - p_i + \varepsilon_i}{5\beta}} \tag{6}$$

where $i = 1, 2, \ldots, N$, ε_i is the noise which conforms to the gaussian distribution with zero mean and variance $\delta_{i,\varepsilon}^2$. Expanding the right side of (6) with the Taylor series and neglecting the high order terms, (6) is also equivalent to

$$d_i^2 = \lambda_i + \frac{\lambda_i \ln 10}{5\beta} \varepsilon_i \tag{7}$$

where $\lambda_i = 10^{\frac{p_0 - p_i}{5\beta}}$, $i = 1, 2, \ldots, N$. (7) represents the equivalent RSS measurement equation.

To convert into the convex form, we further introduce a new matrix

$$\mathbf{Z} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{x} \\ \mathbf{x}^T & \mathbf{y} \end{bmatrix} \tag{8}$$

where $\mathbf{y} = \mathbf{x}^T\mathbf{x}$. So d_i^2 can be given by

$$d_i^2 = \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}^T \mathbf{Z} \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix} \tag{9}$$

By transforming the direction angle measurement equation, (2) is also rewritten as

$$\tan(\phi_i - m_i) = \frac{a_{i,y} - x_y}{a_{i,x} - x_x} \tag{10}$$

Expanding both sides of (10) and neglecting the high order terms, we obtain that

$$-\sin\phi_i x_x + \cos\phi_i x_y = b_{i,\phi} + \sqrt{\lambda_i} \sin\alpha_i m_i \tag{11}$$

where $b_{i,\phi} = -\sin\phi_i a_{i,x} + \cos\phi_i a_{i,y}$, $i = 1, 2, \ldots, N$. (11) represents the equivalent direction angle measurement equation.

Similarly by transforming the elevation angle measurement equation, (3) is also rewritten as

$$d_i \cos(\alpha_i - n_i) = a_{i,z} - x_z \tag{12}$$

Since the distance d_i can be approximately obtained by

$$d_i = \sqrt{\lambda_i} + \frac{\sqrt{\lambda_i} \ln 10}{10\beta} \varepsilon_i \tag{13}$$

Substituting (13) in (11) and expanding both sides of (12), we obtain that

$$-x_z = b_{i,\alpha} + \sqrt{\lambda_i}\sin\alpha_i n_i + \frac{\sqrt{\lambda_i}\cos\alpha_i \ln 10}{10\beta}\varepsilon_i \tag{14}$$

where $b_{i,\alpha} = \sqrt{\lambda_i}\cos\alpha_i - a_{iz}$, $i = 1, 2, \ldots, N$. (14) represents the equivalent elevation angle measurement equation.

Based on the equivalent measurement equations of (7), (11) and (14), the optimization problem by using the squared target function can be written as

$$\begin{aligned}
&\min_{\mathbf{Z}, t_{i,p}, t_{i,\phi}, t_{i,\alpha}} \sum_i^N \left(\frac{1}{\delta_{i,p}^2} t_{i,p}^2 + \frac{1}{\delta_{i,\phi}^2} t_{i,\phi}^2 + \frac{1}{\delta_{i,\alpha}^2} t_{i,\alpha}^2\right) \\
&\text{s.t.} \quad t_{i,p} = d_i^2 - \lambda_i \\
&\qquad t_{i,\phi} = \mathbf{e}_{i,\phi}\mathbf{x} - b_{i,\phi} \\
&\qquad t_{i,\alpha} = \mathbf{e}_{i,\alpha}\mathbf{x} - b_{i,\alpha} \\
&\qquad d_i^2 = \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}^T \mathbf{Z} \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}
\end{aligned} \tag{15}$$

where $\delta_{i,p}^2 = \frac{\lambda_i^2 \ln^2 10}{25\beta^2}\delta_{i,\varepsilon}^2$, $\delta_{i,\phi}^2 = \lambda_i \sin^2\alpha_i \delta_{i,m}^2$, $\delta_{i,\alpha}^2 = \lambda_i \sin^2\alpha_i \delta_{i,n}^2 + \frac{\lambda_i \cos^2\alpha_i \ln^2 10}{100\beta^2}\delta_{i,\varepsilon}^2$, $\mathbf{e}_{i,\phi} = [-\sin\phi_i \quad \cos\phi_i \quad 0]$, $\mathbf{e}_{i,\alpha} = [0 \quad 0 \quad -1]$, $i = 1, 2, \ldots, N$. The optimization function of (15) can be equivalently written as its epigraph form

$$\begin{aligned}
&\min_{\mathbf{Z}, \mathbf{t}_p, \mathbf{t}_\phi, \mathbf{t}_\alpha} (\tau_p + \tau_\phi + \tau_\alpha) \\
&\text{s.t.} \quad \|\mathbf{t}_p\| \le \tau_p, \|\mathbf{t}_\phi\| \le \tau_\phi, \|\mathbf{t}_\alpha\| \le \tau_\alpha \\
&\qquad t_{i,p} = d_i^2 - \lambda_i \\
&\qquad t_{i,\phi} = \mathbf{e}_{i,\phi}\mathbf{x} - b_{i,\phi} \\
&\qquad t_{i,\alpha} = \mathbf{e}_{i,\alpha}\mathbf{x} - b_{i,\alpha} \\
&\qquad d_i^2 = \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}^T \mathbf{Z} \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}
\end{aligned} \tag{16}$$

where $\mathbf{t}_p \triangleq [\frac{t_{i,p}}{\delta_{i,p}}]$, $\mathbf{t}_\phi \triangleq [\frac{t_{i,\phi}}{\delta_{i,\phi}}]$ and $\mathbf{t}_\alpha \triangleq [\frac{t_{i,\alpha}}{\delta_{i,\alpha}}]$. The cost function of (16) is linear with the variables of $\mathbf{Z}$, so it is easy be expressed to the convex optimization form. However, the constraints in (16) make the problem nonconvex. To obtain the convex optimization form, we relax $\mathbf{y} = \mathbf{x}^T\mathbf{x}$ as $\mathbf{y} \succeq \mathbf{x}^T\mathbf{x}$. So (16) is reformulated as

$$\begin{aligned}
&\min_{\mathbf{Z}, \mathbf{t}_p, \mathbf{t}_\phi, \mathbf{t}_\alpha} (\tau_p + \tau_\phi + \tau_\alpha) \\
&\text{s.t.} \quad \|\mathbf{t}_p\| \le \tau_p, \|\mathbf{t}_\phi\| \le \tau_\phi, \|\mathbf{t}_\alpha\| \le \tau_\alpha \\
&\qquad t_{i,p} = d_i^2 - \lambda_i \\
&\qquad t_{i,\phi} = \mathbf{e}_{i,\phi}\mathbf{x} - b_{i,\phi} \\
&\qquad t_{i,\alpha} = \mathbf{e}_{i,\alpha}\mathbf{x} - b_{i,\alpha}
\end{aligned}$$

$$d_i^2 = \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}^T \mathbf{Z} \begin{bmatrix} \mathbf{a}_i \\ -1 \end{bmatrix}$$
$$\mathbf{Z} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{x} \\ \mathbf{x}^T & \mathbf{y} \end{bmatrix} \succeq \mathbf{0}_4 \tag{17}$$

The convex optimization of (17) includes three SOCP and one SDP constraints, so it is called mixed SOCPSDP algorithm. The mixed SOCPSDP algorithm trades off the positioning accuracy and computational complexity since the less variables are produced in the convex relaxation. The SOCPSDP optimization problem of (17) is convex and can be solved with well known algorithms such as interior point methods which are self initialized and requires no initialization from the user. Extracting from defined $\mathbf{Z}$, we can obtain the position estimate $\mathbf{x}$ of the source node.

4 Unknown Transmit Power

Sometimes each source node has a specific transmit power depending on, e.g., its battery and antenna gain. In addition, the transmit power might change with time, e.g., when batteries begin to exhaust. Consequently, each source node has to report its transmit power to anchor nodes constantly during RSS measurements which requires additional hardware and software in both anchor nodes and source nodes making the network more convoluted. In this section the transmit powers are considered as nuisance parameters and assumed to be unknown, so the source transmit powers are estimated jointly with the source locations.

When the source transmit powers are unknown, the convex optimization relaxation follows the same procedure as described previously for the known transmit power case but with a slightly different relaxation. When the transmit power is considered as unknown parameter, we define a new measurement related parameter μ_i and a new variable ρ_0, which are given by

$$\begin{cases} \mu_i = 10^{\frac{-p_i}{5\beta}} \\ \rho_0 = 10^{\frac{p_0}{5\beta}} \end{cases} \tag{18}$$

So (7) can be rewritten as

$$d_i^2 = \mu_i \rho_0 + \frac{\lambda_i \ln 10}{5\beta} \varepsilon_i \tag{19}$$

where $\lambda_i = \mu_i \rho_0$. So when the transmit power p_0 is unknown, the optimization problem of (15) is given by

$$
\begin{aligned}
\min_{\mathbf{Z},t_{i,p},t_{i,\phi},t_{i,\alpha},\rho_0} & \sum_{i}^{N}(\frac{1}{\delta_{i,p}^2}t_{i,p}^2+\frac{1}{\delta_{i,\phi}^2}t_{i,\phi}^2+\frac{1}{\delta_{i,\alpha}^2}t_{i,\alpha}^2) \\
\text{s.t.}\quad & t_{i,p}=d_i^2-\mu_i\rho_0 \\
& t_{i,\phi}=\mathbf{e}_{i,\phi}\mathbf{x}-b_{i,\phi} \\
& t_{i,\alpha}=\mathbf{e}_{i,\alpha}\mathbf{x}-b_{i,\alpha} \\
& d_i^2=\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix}^T\mathbf{Z}\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix}
\end{aligned} \tag{20}
$$

where $\delta_{i,p}$, $\delta_{i,\phi}$, $\delta_{i,\alpha}$, $\mathbf{e}_{i,\phi}$ and $\mathbf{e}_{i,\alpha}$ are same with the definitions in (15). Similarly the epigraph form of (20) is written as

$$
\begin{aligned}
\min_{\mathbf{Z},\mathbf{t}_p,\mathbf{t}_\phi,\mathbf{t}_\alpha,\rho_0} & (\tau_p+\tau_\phi+\tau_\alpha) \\
\text{s.t.}\quad & \|\mathbf{t}_p\|\le\tau_p,\|\mathbf{t}_\phi\|\le\tau_\phi,\|\mathbf{t}_\alpha\|\le\tau_\alpha \\
& t_{i,p}=d_i^2-\mu_i\rho_0 \\
& t_{i,\phi}=\mathbf{e}_{i,\phi}\mathbf{x}-b_{i,\phi} \\
& t_{i,\alpha}=\mathbf{e}_{i,\alpha}\mathbf{x}-b_{i,\alpha} \\
& d_i^2=\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix}^T\mathbf{Z}\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix}
\end{aligned} \tag{21}
$$

where τ_p, τ_ϕ and τ_α are same with the definitions in (16). Then by relaxing the matrix $\mathbf{Z}$, the convex optimization form is obtained with

$$
\begin{aligned}
\min_{\mathbf{Z},\mathbf{t}_p,\mathbf{t}_\phi,\mathbf{t}_\alpha,\rho_0} & (\tau_p+\tau_\phi+\tau_\alpha) \\
\text{s.t.}\quad & \|\mathbf{t}_p\|\le\tau_p,\|\mathbf{t}_\phi\|\le\tau_\phi,\|\mathbf{t}_\alpha\|\le\tau_\alpha \\
& t_{i,p}=d_i^2-\mu_i\rho_0 \\
& t_{i,\phi}=\mathbf{e}_{i,\phi}\mathbf{x}-b_{i,\phi} \\
& t_{i,\alpha}=\mathbf{e}_{i,\alpha}\mathbf{x}-b_{i,\alpha} \\
& d_i^2=\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix}^T\mathbf{Z}\begin{bmatrix}\mathbf{a}_i\\-1\end{bmatrix} \\
& \mathbf{Z}=\begin{bmatrix}\mathbf{I}_3 & \mathbf{x}\\ \mathbf{x}^T & \mathbf{y}\end{bmatrix}\succeq\mathbf{0}_4
\end{aligned} \tag{22}
$$

The weight coefficient $\delta_{i,p}$, $\delta_{i,\phi}$, $\delta_{i,\alpha}$ rely on the estimated λ_i which is determined by the transmit power and not available in the beginning. Preliminarily considering λ_i as identical we obtain the initial estimate λ_i. Then putting the initial estimate into these optimization expressions would produce better solutions for the position estimate along with the transmit power.

5 Evaluation

To test the performance of the proposed convex optimization algorithm, the simulations are implemented by the CVX toolbox using SeDuMi as the solver in the MATLAB software. Three anchor nodes are set at the points (80, 15, 5), (30, 60, 80) and (90, 95, 5) in a 3-dimensional plane region. The position of the source node is set at (50, 50, 50) in advance. The noises of RSS, direction and elevation measurements are set to δ_p^2, δ_m^2 and δ_n^2, respectively. Unless specifically mentioned, the transmit power p_0 and the true PLE β are set to –45.0 dB and 4, respectively. The accuracy performance is evaluated with root mean square error (RMSE) which is defined as

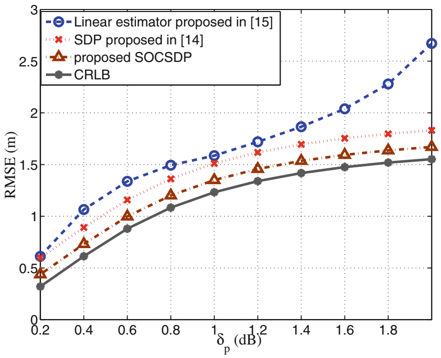
$$\text{RMSE} = \sqrt{\frac{1}{M_c}\sum_{i=1}^{M_c} \| \mathbf{x}_i - \mathbf{x}^o \|^2} \tag{23}$$

where M_c is called as the Monte Carlo times, $\mathbf{x}_i$ and $\mathbf{x}^o$ denotes the estimate and the true position of the source node in ith Monte Carlo run, respectively. In our simulation, we use the average of 1000 Monte Carlo runs to evaluate the accuracy performance of the proposed algorithm.

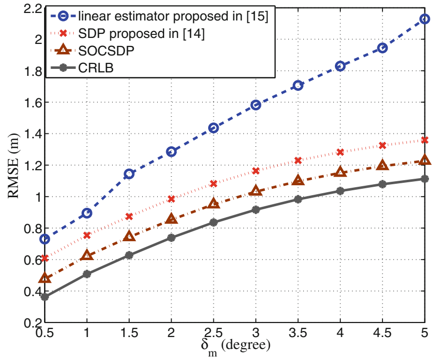
5.1 Known Transmit Power

Firstly, when the transmit power is assumed to be known, the RMSE performance of different algorithms are compared by considering the impacts of the RSS measurement noises when the noise variance δ_p is varied from 0.2 to 2 dB. Figure 2(a) plots the RMSE performance with the linear estimator proposed in [15], the SDP algorithm proposed in [14], our proposed SOCSDP algorithm and the CRLB under known transmit power. It can be seen that the RMSE performance of all algorithms degrades as the RSS noise increases. When the RSS noise δ_p is increased to 2 dB, the RMSE of the SOCPSDP algorithm achieves to 1.67 m. However, the proposed linear estimator proposed in [15] and the SDP algorithm proposed in [14] achieve 2.67 m and 1.83 m, respectively, when δ_p is set to 2 dB. The RMSE of the SOCPSDP algorithm is always less than that of the linear estimator or the SDP algorithm when the RSS noise is varied from 0.2 dB to 2 dB.

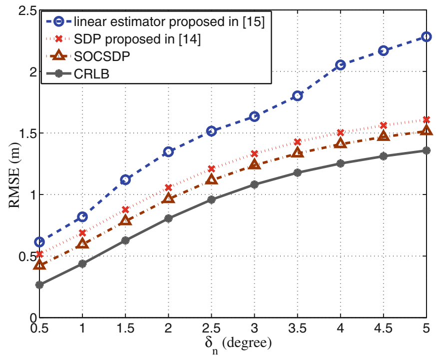
Similarly, the direction angle noise δ_m and elevation angle noise δ_n are varied from 0.5° to 5°, Fig. 2(b) and (c) plot the RMSE performance with three different algorithms. The performance order of three different algorithms is same with Fig. 2(a). When the noises are increased from 0.5° to 5°, the RMSE is greatly increased. For instance, when the direction angle noise is varied from 0.5° to 5°, the RMSE of SOCSDP algorithm shown in Fig. 2(b) is increased from 0.47 m to 1.23 m. When the elevation angle noise is varied from 0.5° to 5°, it can be shown from Fig. 2(c) that the RMSE of SOCSDP algorithm is increased from 0.42 m to 1.51 m. So the bigger noises of direction angle and elevation angle lead to the degrade of the RMSE performance.



(a) RMSE Performance with different RSS noises.



(b) RMSE Performance with different direction angle noises.



(c) RMSE Performance with different elevation angle noises.

Fig. 2. Performance comparison under known transmit power.

5.2 Unknown Transmit Power

When the transmit power is assumed to be unknown, the transmit power is estimated along with the position of the source node. When the standard deviation of the RSS noise is also varied from 0.2 dB to 2 dB, Fig. 3 plots the RMSE of the estimated source position with the linear estimator, SDP and SOCSDP algorithm. As can be seen, the RMSE performance of three proposed algorithms also becomes worse as the RSS noise increases. For instance, the RMSE of the SOCSDP is 0.64 m when the RSS noise is set to 0.2 dB. However, when the RSS noise is increased to 2 dB, the RMSE of the SOCSDP is also increased to 2.01 m. Compared with the linear estimator and SDP algorithm, the SOCSDP provides better accuracy performance for the estimate of source position.

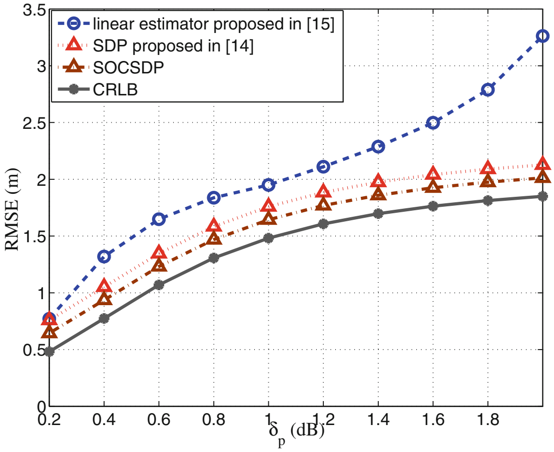


Fig. 3. Performance comparison under unknown transmit power

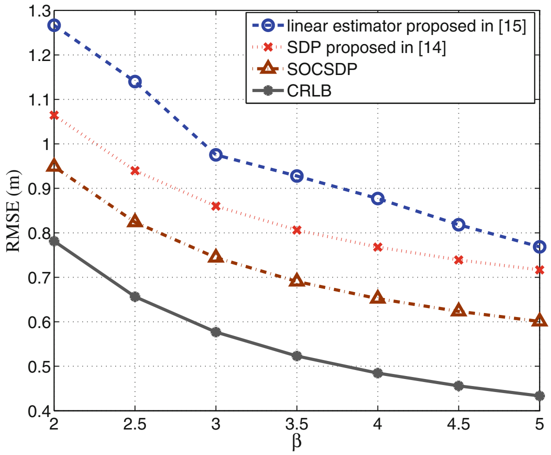


Fig. 4. Impacts of PLE

5.3 Path Loss Exponent

In this subsection, we investigate the effect of path loss exponent (PLE) on the performance of the proposed algorithms. The RSS noise δ_p, the direction angle noise δ_m and the elevation angle noise δ_m are set to 0.2 dB, 0.5° and 0.5°, respectively. When the PLE is varied from 2 to 5, Fig. 4 plots the RMSE performance versus different PLE. As can be seen, the RMSE performance of the algorithms degrades, especially when the PLE is small. Compared with the linear estimator or SDP algorithm, the SOCPSDP algorithm performs better. For instance, when the PLE is set to 2, the RMSEs are 1.27 m with the linear estimator, 1.07 m with the SDP and 0.96 m with the SOCSDP, respectively.

6 Conclusion

SOCP has a simpler structure and the potential to be solved faster than SDP, so its relaxation is weaker. Using the hybrid RSS and AOA measurements and considering the known or unknown transmit power, we introduce the convex optimization SOCPSDP algorithm for the wireless localization. The proposed SOCPSDP algorithm also provides accurate position estimate of the source node and performs better than the SDP algorithm or the linear estimator. The RMSE performance of the proposed SOCPSDP degrades as the noises increase. When the PLE is bigger, the RMSE of the estimated positions would be reduced for a given noise condition. Since the computational complexity of the proposed convex algorithm is high due to a large number of variables and equality constraints produced in the relaxation process. The next work is how to reduce the computational complexity of the convex algorithm.

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