Toward Aggregating Fuzzy Graphs a Model Theory Approach

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Abstract. In this paper, we study fuzzy graph represents by using model theory. We use hedge algebra and linguistic variables for modeling and aggregating two graphs. We prove theorem of limiting in models state space. We also figure out preserved property of aggregation operator.

Keywords: Fuzzy logic · Linguistic variable · Hedge algebra · Aggregating fuzzy graphs

1 Introduction

In everyday life, people use natural language (NL) for analysing, reasoning, and finally, make their decisions. Computing with words (CWW) [5] is a mathematical solution of computational problems stated in an NL. CWW based on fuzzy set and fuzzy logic, introduced by Zadeh is an approximate method on interval [0, 1]. In linguistic domain, linguistic hedges play an important role for generating set of linguistic variables. A well known application of fuzzy logic (FL) is fuzzy cognitive map ($\mathbb{FCM}$), introduced by Kosko [1], combined fuzzy logic with neural network. $\mathbb{FCM}$ has a lots of applications in both modeling and reasoning fuzzy knowledge [3,4] on interval [0, 1] but not in linguistic values, However, many applications cannot model in numerical domain [5], for example, linguistic summarization problems [6]. To solve this problem, in the paper, we use an abstract algebra, called hedge algebra ($\mathbb{HA}$) as a tool for computing with words.

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The remainder of paper is organized as follows. Section 2 reviews some main concepts of computing with words based on $\mathbb{HA}$ in Subsect. 2.1 and describes several primary concepts for $\mathbb{FCM}$ in Subsect. 2.2. Section 3 reviews modeling with words using $\mathbb{HA}$. Important Sect. 4 proves a new method to model fuzzy graph from model theory. Section 5 presents aggregation method to combine two fuzzy graphs. Section 6 outlines discussion and future work.

2 Preliminaries

This section presents basic concepts of $\mathbb{HA}$ and $\mathbb{FCM}$ used in the paper.

2.1 Hedge Algebra

In this section, we review some $\mathbb{HA}$ knowledges related to our research paper and give basic definitions. First definition of a $\mathbb{HA}$ is specified by 3-Tuple $\mathbb{HA} = (X, H, \leq)$ in [7]. In [8] to easily simulate fuzzy knowledge, two terms G and C are inserted to 3-Tuple so $\mathbb{HA} = (X, G, C, H, \leq)$ where $H \neq \emptyset$, $G = \{c^+,\ c^-\}$, $C = \{0, W, 1\}$. Domain of X is $\mathbb{L} = Dom(X) = \{\delta c|\ c \in G,\ \delta \in H^*(\text{hedge string over H})\}$, $\{\mathbb{L}, \leq\}$ is a POSET (partial order set) and $x = h_n h_{n-1} \ldots h_1 c$ is said to be a canonical string of linguistic variable x.

Example 1. Fuzzy subset X *is Age*, $G = \{c^+ = young;\ c^- = old\}$, $H = \{less; more; very\}$ so term-set of linguistic variable Age X is $\mathbb{L}$(X) or $\mathbb{L}$ for short:

$$\mathbb{L} = \{very\,less\,young; less\,young; young; more\,young; very\,young; very\,very\,young\ldots\}$$

Fuzziness properties of elements in $\mathbb{HA}$, specified by *fm* (fuzziness measure) [8] as follows:

Definition 2.1. A mapping $fm : \mathbb{L} \rightarrow [0, 1]$ is said to be the fuzziness measure of $\mathbb{L}$ if:

1. $\sum_{c\in\{c^+,c^-\}} fm(c) = 1,\ fm(0) = fm(w) = fm(1) = 0.$
2. $\sum_{h_i\in H} fm(h_i x) = fm(x)$, $x = h_n h_{n-1} \ldots h_1 c$, the canonical form.
3. $fm(h_n h_{n-1} \ldots h_1 c) = \prod_{i=1}^{n} fm(h_i) \times \mu(x)$.

2.2 Fuzzy Cognitive Map

Fuzzy cognitive map ($\mathbb{FCM}$) is feedback dynamical system for modeling fuzzy causal knowledge, introduced by Kosko [1]. $\mathbb{FCM}$ is a set of nodes, which present concepts and a set of directed edges to link nodes. The edges represent the causal links between these concepts. Mathematically, a $\mathbb{FCM}$ bis defined by.

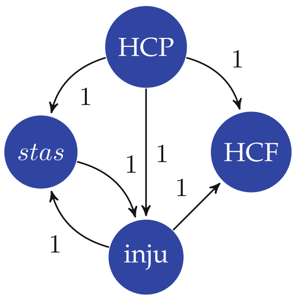
Definition 2.2. A $\mathbb{FCM}$ is a 4-Tuple:

$$\mathbb{FCM} = \{C, E, \mathcal{C}, f\} \tag{1}$$

In which:

1. $C = \{C_1, C_2, \ldots, C_n\}$ is the set of N concepts forming the nodes of a graph.
2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \{-1,\ 0,\ 1\}$ is a function associating e_{ij} with a pair of concepts (C_i, C_j), so that e_{ij} = "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N \times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i(t) \in [0, 1], t \in N$
4. With $C(0) = [C_1(0, C_2(0), \ldots, C_n(0)] \in [0, 1]^N$ is the initial vector, recurring transformation function f defined as (Fig. 2):

$$C_j(t + 1) = f(\sum_{i=1}^{N} e_{ij} C_i(t)) \tag{2}$$



Example 2. Fig.1 shows a medical problem from expert domain of strokes and blood clotting involving. Concepts C={blood stasis (stas), endothelial injury (inju), hypercoagulation factors (HCP and HCF)} [2]. The conection matrix is:

$$E = (e_{ij})_{4\times4} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Fig. 1. A simple $\mathbb{FCM}$

$\mathbb{FCM}$s have played a vital role in the applications of scientific areas, including expert system, robotics, medicine, education, information technology, prediction, etc. [3,4].

3 Modeling with Words

Fuzzy model, based on linguistic variables, is constructed from linguistic hedge of $\mathbb{HA}$ [10,11].

Definition 3.1 (Linguistic lattice). With $\mathbb{L}$ as in the Sect. 2.1, set $\{\wedge, \ \vee\}$ are logical operators, defined in [7,8], a linguistic lattice $\mathcal{L}$ is a tuple:

$$\mathcal{L} = (\mathbb{L}, \vee, \wedge, 0, 1) \tag{3}$$

Property 3.1. *The following are some properties for $\mathcal{L}$:*

1. $\mathcal{L}$ is a linguistic-bounded lattice.

2. $(\mathbb{L}, \ \vee)$ and $(\mathbb{L}, \ \wedge)$ are semigroups.

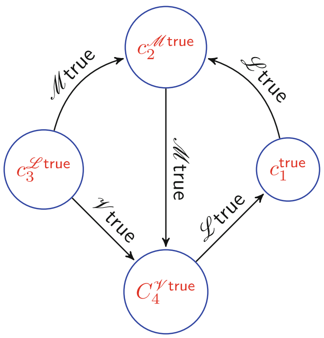
Definition 3.2. A linguistic cognitive map ($\mathbb{LCM}$) is a 4-Tuple:

$$\mathbb{LCM} = \{C, E, \mathcal{C}, f\} \tag{4}$$

In which:

1. $C = \{C_1, C_2, \dots, C_n\}$ is the set of N concepts forming the nodes of a graph.
2. $E : (C_i, C_j) \longrightarrow e_{ij} \in \mathbb{L}$; $e_{ij} =$ "weight of edge directed from C_i to C_j. The connection matrix $E(N \times N) = \{e_{ij}\}_{N\times N} \in \mathbb{L}^{N\times N}$
3. The map: $\mathcal{C} : C_i \longrightarrow C_i^t \in \mathbb{L}, t \in N$
4. With $\mathcal{C}(0) = [C_1^0, C_2^0, \dots, C_n^0] \in \mathbb{L}^N$ is the initial vector, recurring transformation function f defined as:

$$C_j^{t+1} = f(\sum_{i=1}^{N} e_{ij} C_i^t) \in \mathbb{L} \tag{5}$$



Example 3. Fig. 3 shows a simple $\mathbb{LCM}$. Let

$$\mathbb{HA} = <\mathcal{X} = \text{truth}; c^+ = \text{true}; \mathcal{H} = \{\mathscr{L}, \mathscr{M}, \mathscr{V}\}> \tag{6}$$

be a $\mathbb{HA}$ with order as $\mathscr{L} < \mathscr{M} < \mathscr{V}$ ($\mathscr{L}$ for less, $\mathscr{M}$ for more and $\mathscr{V}$ for very are hedges).
$C = \{c_1, c_2, c_3, c_4\}$ is the set of 4 concepts with corresponding values
$\mathcal{C} = \{\text{true}, \mathscr{M}\text{true}, \mathscr{L}\text{true}, \mathscr{V}\text{true}\}$

Fig. 2. A simple $\mathbb{LCM}$

Square matrix:

$$M = (m_{ij} \in \mathbb{L})_{4\times 4} = \begin{vmatrix} 0 & \mathscr{L}\mathsf{true} & 0 & 0 \\ 0 & 0 & 0 & \mathscr{M}\mathsf{true} \\ 0 & \mathscr{M}\mathsf{true} & 0 & \mathscr{V}\mathsf{true} \\ \mathscr{L}\mathsf{true} & 0 & 0 & 0 \end{vmatrix}.$$

is the adjacency matrix of $\mathbb{LCM}$. Causal relation between c_i and c_j is m_{ij}, for example if $i = 1, j = 2$ then causal relation between c_1 and c_2 is: *"if* c_1 *is* true *then* c_2 *is* $\mathscr{M}\mathsf{true}$ *is* $\mathscr{L}\mathsf{true}$*"* or let $\mathcal{P}$ = "if c_1 is true then c_2 is $\mathscr{M}\mathsf{true}$" be a fuzzy proposition $\mathcal{FP}$ [9] then $\mathsf{truth}(\mathcal{P}) = \mathscr{L}\mathsf{true}$

Definition 3.3. A $\mathbb{LCM}$ is called complete if between any two nodes alway having a connected edge (without looping edges).

4 $\mathbb{LCM}$ Modeling with Binary Structure

We use logical structures with relational symbols to represent $\mathbb{LCM}$. For specifying vetex set and edge set, we utilize relations whose arity are whole number. A relational signature $\mathscr{G}$ is a set of relational symbols.

Definition 4.1. A binary relational signature $\mathscr{G}$ as:

$$\mathscr{G} = \{lab_\alpha,\ succ_\beta\} \tag{7}$$

In which $\alpha \in \mathbb{L}$ and $\beta \in \mathbb{L}$. $lab_\alpha,\ succ_\beta$ are relational symbols.

Structures are generated from $\mathscr{G}$ called $\mathsf{struct}[\mathscr{G}]$. By using $\mathsf{struct}[\mathscr{G}]$, vetex set C and edge set E of $\mathbb{LCM}$ can be formalized as follow:

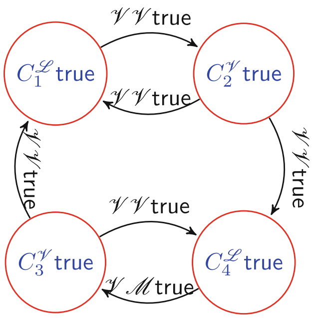
Definition 4.2. A $\mathfrak{C}$ $\mathsf{struct}[\mathscr{G}]$ is a tuple:

$$\mathfrak{C} = \langle \mathbb{C},\ lab_\alpha^{\mathfrak{C}},\ succ_\beta^{\mathfrak{C}} \rangle \tag{8}$$

Where:

- Set $\mathbb{C}$ is domain of $\mathfrak{C}$
- $lab_\alpha^{\mathfrak{C}}$ is a unary relation: $\{\exists C \in \mathbb{C} \mid lab_\alpha^{\mathfrak{C}} C\}$, $\alpha \in \mathbb{L}$
- $succ_\beta^{\mathfrak{C}}$ is a binary relation: $\{C_1,\ C_2 \in \mathbb{C} \mid succ_\beta^{\mathfrak{C}}(C_1,\ C_2)\}$, $\beta \in \mathbb{L}$. C_2 is a *sucessor* of C_1 and $(C_1,\ C_2)$ is a directed edge.

We denote graphs on $\mathbb{HA}$ by $\mathcal{GR}[\mathbb{HA}]$ and represent $\mathbb{LCM}$ by using $\mathsf{struct}[\mathscr{G}]$.
Give a $\mathfrak{C}$ $\mathsf{struct}[\mathscr{G}]$ in (8), the complexity of $\mathfrak{C}$ is proportion to $\|\mathbb{C}\|,\ \|lab_\alpha\|$ and $\|succ_\beta\|$ - The sign $\|.\|$ is short for size of.



Example 4. Left figure shows a simple $\mathbb{LCM}$ on :

$$\mathcal{GR}[X = \mathsf{truth}; c^+ = \mathsf{true}; H = \{\mathscr{L}, \mathscr{M}, \mathscr{V}\}]$$

and $\mathfrak{C}$ $\mathsf{struct}[\mathscr{G}]$ as:

$$\mathfrak{C} = \{\{C_1,\ C_2,\ C_3,\ C_4\},\ lab_\alpha C_k,\ succ_\beta(C_i, C_j)\}$$

Values $i, j, k \in \{1,\ 2,\ 3,\ 4\}$, $\alpha,\ \beta \in \mathbb{L}$.

Fig. 3. A simple $\mathbb{LCM}$ on $\mathsf{struct}[\mathscr{G}]$

Theorem 4.1. *There are:*

$$2^{2\times\binom{\|\mathbb{C}\|+1}{2}} \tag{9}$$

different $\mathfrak{C}$ $\mathsf{struct}[\mathscr{G}]$ *of size* $\|\mathbb{C}\|$.

We prove Theorem 4.1 by using combinatorial relations, symbol $\mathscr{P}(.)$ is power set.

Proof. Because a $n\!-\!ary$ relation on a set $\mathbb{C}$ is a subset of $\overbrace{\mathbb{C} \times \mathbb{C} \times \ldots \times \mathbb{C}}^{n\ times}$, therefore:

- Monadic relation lab_α has $\|lab_\alpha\| = \|\mathscr{P}(\mathbb{C})\| = 2^{\|\mathbb{C}\|}$
- Binary relation $succ_\beta$ has $\|succ_\beta\| = \|\mathscr{P}(\mathbb{C} \times \mathbb{C})\| = 2^{\|\mathbb{C}\|^2}$
- $\mathbb{QED}$: $\|\mathscr{P}(\mathbb{C})\| \times \|\mathscr{P}(\mathbb{C} \times \mathbb{C})\| = 2^{2\times\binom{\|\mathbb{C}\|+1}{2}}$

□

5 Aggregating Two $\mathbb{LCM}$s

$\mathbb{LCM}$s allow a aggregation of knowledge constructed from a few experts to form the final $\mathbb{LCM}$ which reduce potentially errors. We study a aggregation procedures for combining multiple $\mathbb{LCM}$s preserved its properties.

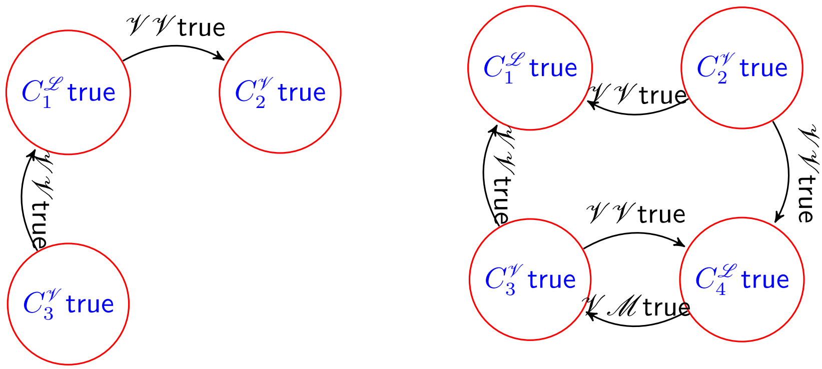
Definition 5.1. Suppose $\mathfrak{A} = \langle \mathbb{A},\ lab_\alpha^{\mathfrak{A}},\ succ_\beta^{\mathfrak{A}} \rangle$ and $\mathfrak{B} = \langle \mathbb{B},\ lab_\alpha^{\mathfrak{B}},\ succ_\beta^{\mathfrak{B}} \rangle$ are $\mathsf{struct}[\mathscr{G}]$.

$$\mathfrak{D} = \mathfrak{A} \bigcup \mathfrak{B} = \langle \mathbb{D},\ lab_\alpha^{\mathfrak{D}},\ succ_\beta^{\mathfrak{D}} \rangle \tag{10}$$

On the conditions that:

- $\mathbb{D} = \mathbb{A} \cup \mathbb{C}$
- $lab_\alpha^{\mathfrak{D}} = \begin{cases} lab_\alpha^{\mathfrak{A}} = lab_\alpha^{\mathfrak{B}} \text{ if } lab_\alpha^{\mathfrak{A}} = lab_\alpha^{\mathfrak{B}} \\ lab_\alpha^{\mathfrak{A}} \vee lab_\alpha^{\mathfrak{B}} \text{ if } lab_\alpha^{\mathfrak{A}} \neq lab_\alpha^{\mathfrak{B}} \end{cases}$
- $succ_\beta^{\mathfrak{D}} = \begin{cases} succ_\beta^{\mathfrak{A}} = succ_\beta^{\mathfrak{B}} \text{ if } succ_\beta^{\mathfrak{A}} = succ_\beta^{\mathfrak{B}} \\ succ_\beta^{\mathfrak{A}} \vee succ_\beta^{\mathfrak{B}} \text{ if } succ_\beta^{\mathfrak{A}} \neq succ_\beta^{\mathfrak{B}} \end{cases}$

Example 5. Using Eq. (10), graph in Fig. 3 is a aggregation of two graphs below:



Property 5.1. *The aggregation operator defined in (10) preserved causal relation properties, that is:*

$$succ_\beta^{\mathfrak{D}}(lab_\rho^{\mathfrak{D}},\ lab_\delta^{\mathfrak{D}}) \models succ_\beta^{\mathfrak{A}}(lab_\rho^{\mathfrak{A}},\ lab_\delta^{\mathfrak{A}}) \vee succ_\beta^{\mathfrak{B}}(lab_\rho^{\mathfrak{B}},\ lab_\delta^{\mathfrak{B}}) \quad (11)$$

6 Conclusions and Future Work

We have study a new method to present $\mathbb{LCM}$ using model theory. The impotant theorem in complexity of model space limited by expression $2^{2\times\binom{\|\mathbb{C}\|+1}{2}}$.

We also introduce a method for aggregating fuzzy graphs. This aggregation operator preserves causal relation properties. Our next study is as follow:
Suppose $\mathbb{LCM}$s are fuzzy graphs on:

$$\mathcal{GR}[\{X,\ \mathcal{H},\ \{c^+,\ c^-\},\ \{0,\ W,\ 1\},\ \leq\}] \quad (12)$$

so that:

$$\mathbb{LCM} = \langle V^{\mathbb{LCM}},\ succ^{\mathbb{LCM}},\ lab^{\mathbb{LCM}} \rangle \quad (13)$$

Let $\mathcal{H}$ be all string generated from $\mathcal{GR}[.]$. $V^{\mathbb{LCM}}$ is the finite set of *vertices*; Relation $succ^{\mathbb{LCM}} \subseteq V^{\mathbb{LCM}} \times \mathcal{H} \times V^{\mathbb{LCM}}$ saying that if two vertices are linked

by an edge with label in $\mathcal{H}$. Total map $lab^{\mathbb{LCM}}: V^{\mathbb{LCM}} \rightarrow \mathcal{H}$ assigning a label in $\mathcal{H}$ to each vertex of $\mathbb{LCM}$.
The set of all $\mathbb{LCM}$ over $\mathcal{H}$ is denote $\mathbb{LCM}_{\mathcal{H}}$, and the set of all graphs isomorphic to $\mathbb{LCM}$ is denote $[\mathbb{LCM}_{\mathcal{H}}]$. A graph language $\mathscr{L}$ is a subset $\mathscr{L} \subset [\mathbb{LCM}_{\mathcal{H}}]$.

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