Hedge Algebra Approach for Semantics-Based Algorithm to Improve Result of Time Series Forecasting

Loc Vuminh[1(✉)], Dung Vuhoang[2], Dung Quachanh[3], and Yen Phamthe[1,4]

[1] Giadinh University, Ho Chi Minh City, Vietnam
vuminhloc@gmail.com
[2] National University of Singapore, Singapore, Singapore
[3] Vanhien University, Ho Chi Minh City, Vietnam
[4] Graduate University of Science and Technology, Vietnam Academy of Science and Technology, Hanoi, Vietnam

Abstract. During the recent years, many different methods of using fuzzy time series for forecasting have been published. However, computation in the linguistic environment one term has two parallel semantics, one represented by fuzzy sets (computation-semantics) it human-imposed and the rest (context-semantic) is due to the context of the problem. If the latter semantics is not paid attention, despite the computation accomplished high level of exactly but it has been distorted about semantics. That means the result does not suitable the context of the problem. Hedge Algebras, an algebraic Approach to domains of linguistic variables, unifying the above two semantics of each term, is the basis of convenient calculation in the language environment and does not distort the semantics of terms. A new approach is proposed through a semantic-based algorithm, focus on two key steps: partitioning the universe of discourse of time series into a collection of intervals and mining fuzzy relationships from fuzzy time series, which outperforms accuracy and friendliness in computing.

The experimental results, forecasting enrollments at the University of Alabama and forecasting TAIEX Index, demonstrate that the proposed method significantly outperforms the published ones about accurate level, the ease and friendliness on computing.

Keywords: Hedge algebras · Fuzzy time series · Forecasting · Fuzziness intervals

1 Introduction

Fuzzy time series was originally proposed by Song and Chissom [1] and it has been applied to forecast the enrollments at University of Alabama [2, 3]. In Chen [4] opened a new study direction of using fuzzy time series to forecast time series source we have, the better forecasting values we get discourse such as [5] which is the first research confirmed the important role of partitioning the universe of discourse, this employed distribution and average based length as a way to solve the problem. In turn, Jilani et al.

P. C. Vinh and A. Rakib (Eds.): ICCASA 2019/ICTCC 2019, LNICST 298, pp. 188–202, 2019.
https://doi.org/10.1007/978-3-030-34365-1_15

[6] proposed frequency density, Huarng and Yu [7] suggested the ratios and Bas et al. [8] used modified genetic algorithm as basis to improve quality of interval.

The rest of this paper is organized as follows: Sect. 2 briefly introduces some basis concepts of HA; Sect. 3 presents the proposed method; Sect. 4 presents empirical results on forecasting enrollments at University of Alabama, Forecasting AITEX Index and comment; Sect. 5 concludes the paper.

2 Preliminaries

In this section, we briefly recall some concepts associated with fuzzy time series and hedge algebras.

2.1 Fuzzy Time Series

Fuzzy time series was first introduced by Song and Chissom [1], it is considered as this is fuzzy of time series. Formally, fuzzy time series are defined as following definition

Definition 1.
Let $Y(t)$ $(t = \ldots,\ 0,\ 1,\ 2,\ \ldots)$, a subset of R^1, be the universe of discourse on which $f_i(t)$ $(i = 1, 2, \ldots)$ are defined and F(t) is the collection of $f_i(t)$ $(i = 1, 2, \ldots)$. Then F(t) is called fuzzy time series on $Y(t)$ $(t = \ldots,\ 0,\ 1,\ 2,\ \ldots)$.

Song and Chissom employed fuzzy relational equations as model of fuzzy time series. Specifically, we have following definition:

Definition 2.
If for any $f_j(t) \in F(t)$, there exists an $f_i(t-1) \circ F(t-1)$ such that there exists a fuzzy relation $R_{ij}(t, t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-l)$ where 'o' is the max-min composition, then F(t) is said to be caused by F(t − 1) only. Denote this as $f_i(t-1) \rightarrow f_j(t)$ Or equivalently $F(t-1) \rightarrow F(t)$.

In [2, 3], Song and Chissom proposed the method which use fuzzy time series to forecast time series. Based upon their works, there are many studies focus on this field.

2.2 Some Basis Concepts of Hedge Algebras

"In the HA approach, it seems to be essential that the fuzziness measure of words of a variable, which is a quantitative characteristic expressing an essential and key semantic aspect of the fuzzy linguistic information, does play a centric role in the determination of other quantitative characteristics of words, such as the fuzziness intervals of words, the similarity intervals and the semantically quantifying mappings (or the numeric semantics of) words, when providing the values the fuzziness parameters of the variable. In summary, this approach is developed based on a convincing logical and mathematical foundation, as the inherent word semantics and its fuzziness are defined and formalized in an axiomatization manner" [14].

In this section, we briefly introduce some basis concepts in HA, these concepts are employed as basis to build our proposed method. HA are created by Ho Cat Nguyen et al. in 1990. This theory is a new approach to quantify the linguistic terms differing from the fuzzy set approach. The HA denoted by $\mathrm{AX} = (\mathrm{X}, \mathrm{G}, \mathrm{C}, \mathrm{H}, \leq)$, where, $\mathrm{G} = \{\mathrm{c}^{+}, \mathrm{c}^{-}\}$ is the set of primary generators, in which c^+ and c^- are, respectively, the negative primary term and the positive one of a linguistic variable X, C = {0, 1, W} a set of constants, which are distinguished with elements in X, H is the set of hedges, "$\leq$" is a semantically ordering relation on X. For each x $\in$ X in HA, H(x) is the set of hedge u$\in$X that generated from x by applying the hedges of H and denoted $u = h_n \ldots h_1 x$, with $h_n, \ldots, h_1 \in H$. $H = H^+ \cup H^-$, in which H^- is the set of all negative hedges and H^+ is the set of all positive ones of X. The positive hedges increase semantic tendency and vise versa with negative hedges. Without loss of generality, it can be assumed that

$$H^- = \{h_{-1} < h_{-2} < \ldots < h_{-q}\} \text{ and } H^+ = \{h_1 < h_2 < \ldots < h_p\}.$$

If X and H are linearly ordered sets, then $AX = (X, G, C, H, \leq)$ is called linear hedge algebra, furthermore, if AX is equipped with additional operations $\sum$ and Φ that are, respectively, infimum and supremum of H(x), then it is called complete linear hedge algebra (ClinHA) and denoted $AX = (X, G, C, H, \Sigma, \Phi, \leq)$.

Complete linear hedge algebra (ClinHA) There are also following important concepts and properties are present in [12].

- Fuzziness interval of terms in X and its properties.
- Semantically quantifying mapping υ: X → [0, 1] of AX and its identification.
- Function Sign: X → {−1, 0, 1} is a mapping which is defined recursively.

It is user's Linguistic Frame of Cognitive and is basis of calculations on the word makes the calculation method are simple and accurate.

Here we just add two properties related to (ClinHA) which have two factors (one negative and one positive), i.e.

$$\mathbf{H = H- \cup H+;\ H- = \{h-1\},\ H+ = \{h+1\}}.$$

Definition 1. [12] Given AX2 $k \geq 1$ the similar fuzzy space of set X (k) denoted $\zeta_{(k)}$ is a set of similar fuzzy space of all grades from $X_{(k)}$ for $\forall x \in X_{(k)}$, $\Im g(x) \in \zeta_{(k)}, g + l(x) = k$ unchanged i.e., $\forall x \in X_{(k)}$, $\Im g(x)$ made up of the same fuzzy space of level k* and $\zeta_{(k)}$ is a partition of [0, 1]. (See the Fig. 1).

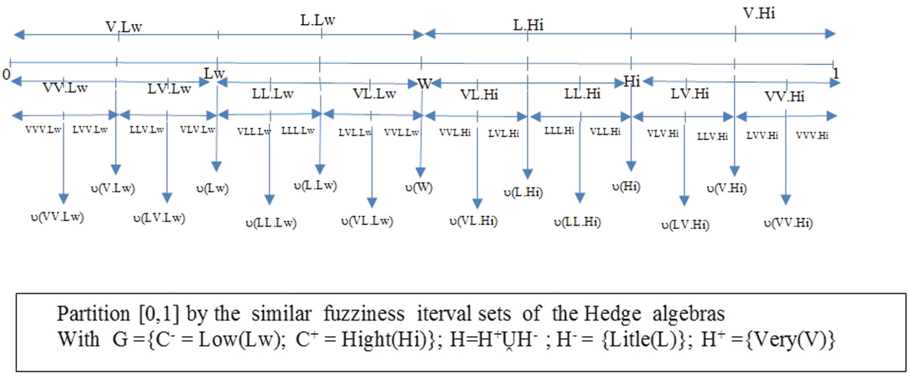

Fig. 1. Shown partition [0, 1] by the similar fuzziness interval sets of the Hedge algebras)

Definition 2. [12] Give AX^2, 1, $k \geq 1$, $\forall x \in X_{(k)}$ identify the similar fuzzy space $\Im g(x) \in \zeta_{(k)}$ definition of the compatibility level $g = k + 2 - l(x)$ of quantitative value v for Grade x to be a mapping S_g: [0,1] x X →[0,1]: determined based on the distance from v to $\upsilon(x)$ and two similar fuzzy space close to $\Im g(x)$ as follows

$$S_g(v, x) = max\left(min\left(\frac{v - \upsilon(x)}{\upsilon(x) - \upsilon(y)}, \frac{\upsilon(x) - v}{\upsilon(z) - \upsilon(x)}\right), 0\right) \quad (1)$$

Where y, z are two grades defining two similar fuzzy space neighbors left and right of $\Im g(x)$. (See the Fig. 2).

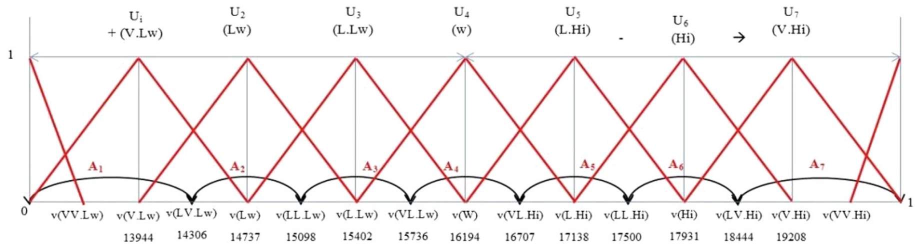

Fig. 2. Shown identify the similar fuzzy space $\Im g(x) \in \zeta_{(k)}$. Where Ai are similar fuzzy intervals and U_i are These are triangular fuzzy sets that represent their membership level function according to expression (1)

3 Proposed Method

3.1 Calculations on the Language Value Apply to the Forecast

In Fig. 2 we have triangular fuzzy sets: Here the set of 3 linguistic values for example the very little low-denominated (VL.Lw) are the vertices of the triangle.

$0\upsilon := (0, 0, \upsilon(VV.Lw))$; $1\upsilon := (\upsilon(VV.Hi\ ), 1, 1)$.
$U1 := (0, \upsilon(V.Lw),\ \ \upsilon(Lw))$
$U2 := (\upsilon(V.Lw),\ \upsilon(Lw),\ \upsilon(L.Lw))$;
$U3 := (\upsilon(Lw),\ \upsilon(L, Lw),\ \upsilon(W))$;
$U4 := (\upsilon(L.Lw),\ \upsilon(W),\ \upsilon(L.Hi))$;
$U5 := (\upsilon(W),\ \upsilon(L.Hi),\ \upsilon(Hi))$;
$U6 := (\upsilon(L.Hi),\ \upsilon(Hi),\ \upsilon(V.Hi))$;
$U7 := (\upsilon(Hi),\ \upsilon(V.Hi),\ 1)$;

They are member functions representing the following similar fuzzy intervals, where in the order U1 corresponds to A1 and …… U7 corresponds to A7 Here the set of 3 linguistic in order from left to right only the left end, the semantic core, and the right end of a similarity Interval for semantic.

$$
\begin{aligned}
&A1 := [0, \upsilon(\mathrm{V.Lw}),\ \upsilon(\mathrm{LV.Lw})]\\
&A2 := [\upsilon(\mathrm{LV.Lw}),\ \upsilon(\mathrm{Lw}),\ \upsilon(\mathrm{LL.Lw})];\\
&A3 := [\upsilon(\mathrm{LL.Lw}),\ \upsilon(\mathrm{L, Lw}), \upsilon(\mathrm{VL.Lw})];\\
&A4 := [\upsilon(\mathrm{VL.Lw}),\ \upsilon(\mathrm{W}),\ \upsilon(\mathrm{VL.Hi})];\\
&A5 := [\upsilon(\mathrm{VL.Hi}),\ \upsilon(\mathrm{L.Hi}),\ \upsilon(\mathrm{LL.Hi})];\\
&A6 := [\upsilon(\mathrm{LL.Hi}),\ \upsilon(\mathrm{Hi}),\ \upsilon(\mathrm{LV.Hi})];\\
&A7 := [\upsilon(\mathrm{LV.Hi}),\ \ \upsilon(\mathrm{V, Hi}),\ 1];
\end{aligned}
$$

3.1.1 For Similar Fuzzy Space and Similar Fuzziness Interval

According to Definition 1. [12], Definition 2. [12] and on Fig. 2 Show: 0_U,, U_1,……. U_7, 1_U are fuzzy triangular sets created Similar fuzzy space of elements {Very Low(V. Low), Low(Lw), LitleLow(L.Lw), W, LitleHigh(L.Hi), High(Hi), VeryHigh(V.Hi)}, It is also the membership function in the order of similar fuzziness interval A_1, …… A_7 In that A_2 := ([υ(LV.Lw) , υ(Lw) , υ(LL.Lw)]) υ(LV.Lw), υ(LL.Lw)), are the left and right border of the linguistic value Low and υ(Lw) is semantically quantifying value. This means that all elements of this interval are similar to Low in the degree of acceptance. Mapping S_g: [0,1] × X → [0,1] determined based on the distance from v to υ(x) and two similar fuzzy space close to $\Im g(x)$ as follows:

$$
\mathrm{S_g}(\mathrm{v},\mathrm{x}) = max\left(min\left(\frac{v-\upsilon(\mathrm{x})}{\upsilon(x)-\upsilon(\mathrm{y})},\frac{\upsilon(\mathrm{x})-\mathrm{v}}{\upsilon(z)-\upsilon(\mathrm{x})}\right),0\right) \tag{2}
$$

Where y, z are two grades defining two similar fuzzy space neighbors left and right of x. This is membership function of similar fuzzy interval A_i (i = 1, …. 7). Easy to deduce that: if v < υ(x) then

$$
\mathrm{S_g}(\mathrm{v},\mathrm{x}) = \frac{v-\upsilon()}{\upsilon(x)-\upsilon(\mathrm{y})}
$$

if $v = \upsilon(\mathrm{x})$ then $\mathrm{S_g}(\mathrm{v},\mathrm{x}) = 1$;

if $v > \upsilon(\mathrm{x})$ then $\mathrm{S_g}(\mathrm{v},\mathrm{x}) = \frac{\upsilon(x)-}{\upsilon(z)-\upsilon(\mathrm{X})}$

In [15] we have clearly stated how to build a HA that matches the context of the problem. In this section we introduce additional expressions for calculating linguistic values according to two HA parameters. This works for problem solving. More importantly, it solves the problem by using the neural network method or the Ge.

3.1.2 Specifying Some Expressions

In Part F.2.1 have $\upsilon(\mathrm{W}) = \theta = \mathrm{fm}(\bar{c})$, $\mathrm{fm}(\mathrm{c}+) = 1-\theta$, $\upsilon(\mathrm{c}-) = \theta - \alpha\mathrm{fm}(\mathrm{c}-) = \beta\mathrm{fm}(\mathrm{c}-)$, $\upsilon(\mathrm{c}+) = \theta + \alpha\mathrm{fm}(\mathrm{c}+)$, $\mathrm{fm}(\mathrm{hx}) = \mu(\mathrm{h})\mathrm{fm}(\mathrm{x})$, $\mathrm{H}+ = \{\mathrm{Very\ (V)}\}$, $\mathrm{H}- = \{\mathrm{Little(L)}\}$, $\mu(\mathrm{V}) = \beta$, $\mu(\mathrm{L}) = \alpha$, $\alpha + \beta = 1 \rightarrow \alpha = 1-\beta$ $\mathrm{fm}(\mathrm{VL.Lw}) = \mathrm{fm}(\mathrm{LV.Lw}) = \mu(\mathrm{V})\mu(\mathrm{L})\mathrm{fm}(\mathrm{Lw})$, form $(\mathrm{Lw}) = \theta\beta)$, $(\mathrm{W}) = \theta$, $(\mathrm{Hi}) = 1-(1-\theta)\beta$ and $\mathrm{fm}(\mathrm{VL.Lw}) = \mathrm{fm}(\mathrm{LV.Lw}) = \mu(\mathrm{V})\ \mu(\mathrm{L})\mathrm{fm}(\mathrm{Lw})$, We calculate the number of fuzzy intervals fm(x) listed in the table below: if $\theta = 0.4563 = \mu(V) = 1 - \mu(L)$ add another lookup at Fig. 1 and results of the table above we have (Table 1):

Table 1. The value of the fuzzy spaces for the calculation.

fm(V. Lw)	fm(L. Lw)	fm(VL. Lw)	fm(LL. Lw)	fm(VV. Lw)	fm(V. Hi)	fm(L. Hi)	fm(VL. Hi)	fm(LL. Hi)	fm(VV. Hi)
0.20820	0.24801	0.13489	0.13488	0.09500	0.24801	0.29561	0.13489	0.16072	0.11320

$$\begin{aligned}\upsilon(\mathrm{LL.Lw}) &= \upsilon(\mathrm{Lw}) + \mathrm{fm}(\mathrm{VLL.Lw}) = \beta * \theta + \mu(\mathrm{L}) * \mathrm{fm}(\mathrm{LL.Lw}) \\ &= (0.4563)^2 + (1 - 0.4563) * 0.16072 = 0.26976\end{aligned}$$

Similarly, for cases we have the numerical result in Fig. 2.

"There is an induced about the trend change in the forecasting in the discourse space into the space of [0, 1] where there is a trend change to the quantitative semantics value due to the impact on the terms of the hedges. That is the basic to we construct the mathematics model for forecasting time series by (HA) approach" [15]. We want to say about μ(h), the single operator impact on the operand (language value) generate new semantics for it - to create an upward or downward direction of the operand, corresponding to the change of time series at a time - is an important factor for the time series forecasting.

"According to the context, semantics of $\bar{F}(t)$ denotes number of the enrollment students at the medium level and W is the normalization value of $\bar{F}(t)$, they are calculated according formulas": [16]

$$\bar{F}(t) = \frac{1}{n}\sum\nolimits_{i=1}^{n} x_i \tag{3}$$

$$W = \frac{\bar{F}(t) - Min.F(t)}{Max.F(t) - Min.F(t)} \tag{4}$$

Determining the two parameters of the HA through the analysis of the relationship of historical values in the properties:

- The average value - the boundary between the main semantic value: what is "high" and what is "low".
- Change of each value "Continue to increase or decrease" or "change in the opposite direction". As a bridge between the two semantics: "inherent" and "represented by fuzzy set imposed by the user" of each word.

On that basis, we propose the following new time series forecasting algorithm. The algorithm emphasizes semantics generated by the problematic context and focuses on two steps:

- Adjust spacing partitioning the universe of discourse.
- Set up a logical relationship group.

3.2 Forecasting Algorithm (Algorithm Based on Semantics)

- For convenience to present proposed method, we name the linguistic values of fuzzy time series as the variables A_i with $i \in N$. Revυ(x) and Rev f_m(x) are the reversed mapping of υ(x) and f_m (x), respectively, from [0, 1] to the universe of discourse of fuzzy time series U. Denote I_k, on U, is the interval corresponding to A_k.
- 7 basic language values: Very Low (V.Low), Low, L.Low, W, Little High(L.Hi), Hi, V.Hi.

Step 1: Constructing the Hedge Algebra (HA)
Constructing the Hedge algebra (HA) is consistent with the context of the forecasting problem by determining the fuzzy parameters set of HA based on the historical values relationship analysis of the time series. Specifically

- Determine the U, the universe of discourse of fuzzy time series $F(t)$.

$U = [\min(F(t)) - D_1, \max(F(t)) + D_2]$, where D_1 and D_2 are proper positive numbers.

$$\bar{F}(t) = \frac{1}{n}\sum\nolimits_{i=1}^{n} x_i$$

$$W = \frac{\bar{F}(t) - Min.F(t)}{Max.F(t) - Min.F(t)}$$

with $x_i (1 \le i \le n)$ are historical values

$$\mu(h) = \frac{2\bar{S}}{(S^+ + S^-)}$$

If $S^+ \ge S^-$ then $h := h_{+1}$ else $h := h_{-1}$.

Step 2: Method for Partitioning the Universe of Discourse, Fuzzifying Historical Data of Time Series and Mining Fuzzy Relationships from Fuzzy Time Series

- Based on the explanation in Fig. 2, constructing 7 similar fuzzy intervals correspond to 7 basic language values to partitioning the universe of discourse
- Based upon the distribution of historical values, put them into the corresponding linguistic term fuzziness interval for fuzzifying historical data of time series.
- Adjust the position of the historical values near the boundaries of the divisors intervals to reach the optimal devise method.
- The logical relation group is established as follows for mining fuzzy relationships from fuzzy time series.

In $A_j \rightarrow A_i$ Who:

– if j > i then the group is forecasting down,
– if j = i then the group forecasting is equal,
– if j < i then the group is forecasting increase.

Step 3: Compute the Forecasting Values
Assume Set the group of fuzzy logical relationships is established in the Step 2 having the same left side:

$A_t \rightarrow A_u(m) \cdots A_v(n), m, \ldots, n$ are the number of iterations of fuzzy logical relationship $A_t \rightarrow A_u$ and $A_t \rightarrow A_v$.

– Suppose that the value of the time series at *(t−1)* have known according to above logical relationship groups if *f*(*t*) belong to *Revfm*(*At*), then
– The forecasting value at *t* is

$$\frac{m * Rer\,\upsilon(\mathrm{Au}) + \cdots n * Rer\,\upsilon(\mathrm{Av})}{m + \cdots .. n}$$

Table 2. Shown left, right and Rve. of A_i.

A_i	Left and right of A_i	Rve(A_i)
A_1	[0, 14096]	13665
A_2	[14096, 15832]	14888
A_3	[15832, 16194]	15832
A_4	[16194, 16625]	16194
A_5	[16625, 17750]	17138
A_6	[17750, 18694]	18263
A_7	[18694, 20000]	19208

4 Experiment Result and Comment

4.1 Enrollment Forecasting

The proposed approach is applied to forecast the enrollments at the University of Alabama from year 1971 to 1992 (n = 22). The result will then be compared with different published methods. To measure the accuracy of the forecasting methods, the following metrics are used for comparison that Defined in [15]

RMSE: The Root Mean Square Error;
NE(%): The Numerical Error (NE) percentage
NEE(%): The Normalized Numerical Error (NNE) percentage

According to [15] we have $\upsilon(\mathrm{W}) = \theta = 0.4563$ and $\mu(\mathrm{V}) = \beta = 0.4563$ are parameter values use to constructing 7 similar fuzzy intervals for partitioning the universe of discourse are illustrated in Fig. 2.

However, in this division at A4 there are two levels of value between the "low" and the "high" as the "elements of meaning" so that the semantic difference in this range is greater than the difference in numeric value. Therefore, these values must be adjusted accordingly. After adjusting the reasonable divisions we have (VL.Lw replace L.Low).

Table 3. Shown left, right and Rve. of A_i

A_i	Left and right of A_i	Rve(A_i)
A_1	[0, 14096]	13665
A_2	[14096, 15832]	14888
A_3	[15832, 16194]	15832
A_4	[16194,16625]	16194
A_5	[16625, 17750]	17138
A_6	[17750, 18694]	18263
A_7	[18694, 20000]	19208

The following is the result of our statistics together with the results of other authors for comparison. The details are shown in Table 4 below.

Table 4. Shown metrics of results of the methods

Metrics	Author			
	Wang et al. 14	Chen et al. 14	Lu et al. 15	Our approach
RMSE	506.0	486.3	445.2	39.6
NE (%)	2.68	2.52	2.30	1.79
NEE (%)	6.93	6.43	5.88	4.56

4.2 Forecasting AITEX Index

In this section, our proposed method is compared to [13]. Chen et al. [13] suggested a method consisting of 6 steps to calculate the forecast TAIEX Index, these steps are listed below:

Propose a method to fuzzify the historical training data of TAIEX into fuzzy sets to from fuzzy logical relationships.

Grouped the logical relationships into fuzzy logical relationship groups (FLRGs) based on the fuzzy variations of secondary factor.

Evaluate the leverage of the fuzzy variations between the main factor and the secondary factor to construct fuzzy variation groups.

Get the statistics of the fuzzy variations appearing in each fuzzy variation group.

Calculate the weight of the statistics of the fuzzy variations appearing in each fuzzy variation group, respectively.

Use the weights of the statistics of the fuzzy variations appearing in the fuzzy variation groups and the FLRGs to perform the forecasting the daily TAIEX.

Chen et al. [13] have applied their proposed method on the experimental data sets TAIEX Index of November and December 2004. The data set consists of 44 items. In the first step, the historical training data of TAIEX is fuzzified into 9 fuzzy sets (h = 9 form A1 to A9), the accuracy metrics of the result are

RSME = 56.86
NE(%) = 0.8
NNE(%) = 12.44.

Table 5. Shown the detailed results of the proposed method and the preceding results.

	With 7 split points (h = 7)							
Year	Actual enrollment	Chen et al. 96	Wang et al. 13	Wang et al. 14	Chen et al. 13	Lu15 et al. 15	Our approach	
							At. [15]	Proposed method
1972	13563	14000	13486	13944	14347	14279	14003	13665
1973	13867	14000	14156	13944	14347	14279	14003	13665
1974	14696	14000	15215	13944	14347	14279	14003	14888
1975	15460	15500	15906	15328	15550	15392	15510	14888
1976	15311	16000	15906	15753	15550	15392	15510	14888
1977	15603	16000	15906	15753	15550	15392	15510	14888
1978	15861	16000	15906	15753	15550	16467	15510	15832
1979	16807	16000	16559	16279	16290	16467	17138	17138
1980	16919	16833	16559	17270	17169	17161	17186	17138
1981	16388	16833	16559	17270	17169	17161	17186	16194
1982	15433	16833	16559	16279	16209	14916	15402	14888
1983	15497	16000	15906	15753	15550	15392	15510	14888
1984	15145	16000	15906	15753	15550	15392	15510	14888
1985	15163	16000	15906	15753	15550	15392	15510	14888
1986	15984	16000	15906	15753	15550	15470	15510	15832
1987	16859	16000	16559	16279	16290	16467	17138	17138
1988	18150	16833	16559	17270	17169	17161	17186	18263
1989	18970	19000	19451	19466	18907	19257	19207	19208
1990	19328	19000	18808	18933	18907	19257	19207	19208
1991	19337	19000	18808	18933	18907	19257	19207	19208
1992	18876	19000	18808	18933	18907	19257	19207	19208
	RMSE	638.4	578.3	506.0	486.3	445.2	400.4	339.6
	NE (%)	3.11	2.76	2.68	2.52	2.30	1.95	1.79
	NNE (%)	7.94	7.17	6.93	6.43	5.88	4.85	4.56

Our proposed method is applied to the same TAIEX datasets. The process is as follows According to Algorithm Based on semantics. According to [15] we have $\upsilon(W) = \theta = 0.52$ and $\mu(V) = \beta = 0.29$ are parameter values use to constructing 7 similar fuzzy intervals for partitioning the universe of discourse. Then perform the remaining steps of the algorithm. Forecast results according to the metrics as shown below with previous results for Compare. The details are shown in Table 6 below (Table 7).

Table 6. Shown Metrics of results of the methods

Metrics	Chen's	Our in [1, 6]	Proposed method
RMSE	56.86	48.02	26.88
NE(%)	0.80	0.5	0.37
NEE(%)	12.44	9.17	0.059

Table 7. Shown the detailed results of the proposed.

Date	Actual index	Chen' forecasted index	Our forecasted index at [15]	Our proposed method
		h = 9	h = 9	h = 7
2/11/2004	5759.61	5674.81	5743	
3/11/2004	5862.85	5768.14	5852	5886
4/11/2004	5860.73	5854.81	5876.04	5886
5/11/2004	5931.31	5908.26	5876.04	5934
8/11/2004	5937.46	5934.81	5912.05	5934
9/11/2004	5945.2	5943.81	5912.05	5934
10/11/2004	5948.49	5934.81	5912.05	5978
11/11/2004	5874.52	5937.12	5912.05	5886
12/11/2004	5917.16	5908.26	5919.27	5934
15/11/2004	5906.69	5934.81	5919.27	5934
16/12/2004	5910.85	5934.81	5919.27	5934
17/11/2004	6028.68	5937.12	5919.27	5978
18/11/2004	6049.49	6068.14	5979.18	5978
19/11/2004	6026.55	6068.14	5979.18	5978
22/11/2004	5838.42	5976.47	5979.18	5886
23/11/2004	5851.1	5854.81	5876.04	5886
24/11/2004	5911.31	5934.85	5876.04	5934
25/11/2004	5855.24	5934.81	5919.27	5886
26/11/2004	5778.65	5854.81	5876.04	5768
29/11/2004	5785.26	5762.12	5797.89	5768
30/11/2004	5844.76	5762.12	5852	5886
1/12/2004	5798.62	5834.85	5876.04	5768
2/12/2004	5867.95	5803.26	5797.89	5886
3/12/2004	5893.27	5854.81	5876.04	5886
6/12/2004	5919.17	5854.81	5919.27	5934
7/12/2004	5925.28	5937.12	5912.05	5934
8/12/2004	5892.51	5876.47	5912.05	5886
9/12/2004	5913.97	5854.81	5919.27	5934
10/12/2004	5911.63	5934.81	5919.27	5934
13/12/2004	5878.89	5937.12	5919.27	5886
14/12/2004	5909.65	5854.81	5919.27	5934

(*continued*)

Table 7. (*continued*)

Date	Actual index	Chen' forecasted index	Our forecasted index at [15]	Our proposed method
		h = 9	h = 9	h = 7
15/12/2004	6002.58	5934.81	5919.27	5978
16/12/2004	6019.23	6068.14	5979.18	5978
17/12/2004	6009.32	6062.12	5979.18	5978
20/12/2004	5985.94	6062.12	5979.18	5978
21/12/2004	5987.85	5937.12	5979.18	5978
22/12/2004	6001.52	5934.81	5979.18	5978
23/12/2004	5997.67	6068.14	5979.18	5978
24/12/2004	6019.42	5934.81	5979.18	5978
27/12/2004	5985.94	6068.14	5979.18	5978
28/12/2004	6000.57	5937.12	5979.18	5978
29/12/2004	6088.49	6068.14	5979.18	6087
30/12/2004	6100.86	6062.12	6119.36	6087
31/12/2004	6139.69	6137.12	6143.57	6144
RSME		56.86	48.02	26.88
NE (%)		0.80	0.59	0.37
NNE (%)		12.44	9.17	0.059

4.3 Comment

The First of all, this proposed method is an improvement of the method already in [15], so it has advantages over the methods of the previously published authors, we briefly recall: We compare our approach with the method Wei Lu et al. published in [11] to illustrate our superior efficiency. Conclusion of his methodological advantages of semantic assurance due to the context of Wei Lu stated "Interval information granules are always run through the whole process of finding optimal intervals, which make the partitioned intervals carry apparent semantics" [11]. As so, in the method of Wei Lu also pay attention to the balance between accuracy and semantics which is suitable with context in the calculation that is why we compare it with our approach. Our comparison focuses on two aspects: the calculation convenience and the forecasting accuracy.

- First, the convenience in calculations: only with the simple calculations using Method for partitioning the universe of discourse and Algorithm for forecasting, we have obtained results about group of logical relationship like the results from Lu [11]. However, our calculation is much simpler than theirs as shown in Table 2. [15]
- Second, the forecasting accuracy: Table 3. Shows our proposed method is about 10% better in term of accuracy (all metrics) compared to Lu et al. approach [11].

[15] The Second, As its name implies, semantic-based algorithms for performing two important steps are for partitioning the universe of discourse and data mining through logical relational grouping. Thus, a more convenient and efficient method of adjusting the divide interval and predicting more accurately than similar functional methods show in [15] (Table 5).

In terms of accuracy, the greater the number of divisions, the higher the accuracy. In both empirical problems forecasting enrollments at the University of Alabama forecasting (forecasting enrollments) and forecasting TAIEX Index (forecasting TAIEX) we used the divisor of 7 and compared the results of other methods with the number of divisions equal (7 for forecasting enrollments) and larger (9 for forecasting TAIEX). The accuracy of the method proposed (for metrics RMSE: The Root Mean Square Error) - for the forecasting enrollments problem was 20.23% higher than that of Lu and 15.25% higher than our results at [15] - for the forecasting TAIEX problem The accuracy of the proposed approach to the problem was 52.72% higher than that of Chen and 36.12% higher than our results at [15]. The numbers are very impressive and very convincing demonstrates the superiority of the accuracy of the proposed method! (Figs. 3 and 4).

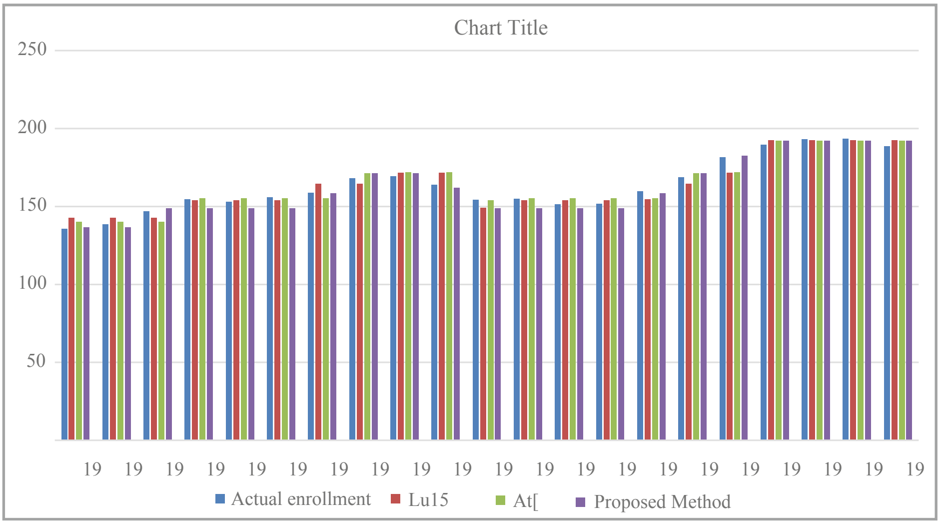

Fig. 3. Chart illustrates Table 4

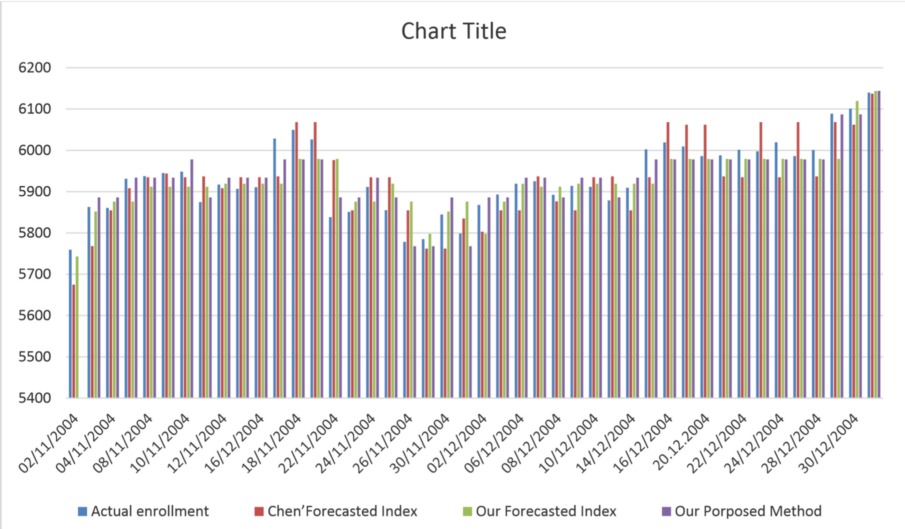

Fig. 4. Chart illustrates Table 6

5 Conclusion

In this approach, each linguistic domain can be considered as a hedge algebra (HA for short) and based on the structure of HAs, a notion of fuzziness measure of linguistic hedges and terms can be defined. It is highlights in proposed method:

Analyze the data of forecasting problem, special for historical values and their relationship to determine the fuzzy parameter set of hedge algebras.

Thereby the context-semantic of terms has preserved in the calculation. which concentrated on key steps: partitioning the universe of discourse of time series into a collection of intervals, mining fuzzy relationships from fuzzy time series,

Forecasting outputs it do not have to choose fuzzy sets for linguistics terms and defuzzifying output, which are the required steps in method based on fuzzy set theory.

This is subjective imposition and so it is the reason for separating the two types of semantics mentioned above. So this algorithm is the process of determining the parameters of hedge algebra. Naturally, the next research problem is to optimize the process. We continue to study this problem with Ge algorithm and Neuron network.

References

1. Song, Q., Chissom, B.S.: Fuzzy time series and its models. Fuzzy Sets Syst. **54**(3), 269–277 (1993)
2. Song, Q., Chissom, B.S.: Forecasting enrollments with fuzzy time series part I. Fuzzy Sets Syst. **54**(1), 1–9 (1993)
3. Song, Q., Chissom, B.S.: Forecasting enrollments with fuzzy time series part II. Fuzzy Sets Syst. **62**(1), 1–8 (1994)

4. Chen, S.-M.: Forecasting enrollments based on fuzzy time series. Fuzzy Sets Syst. **81**(3), 311–319 (1996)
5. Huarng, K.: Effective lengths of intervals to improve forecasting in fuzzy time series. Fuzzy Sets Syst. **123**(3), 387–394 (2001)
6. Jilani, T.A., Burney, S.M.A., Ardil, C.: Fuzzy metric approach for fuzzy time series forecasting based on frequency density based partitioning. World Acad. Sci. Eng. Technol. Int. J. Comput. Electr. Autom. Control Inf. Eng. **4**(7), 1194–1199 (2007)
7. Huarng, K., Yu, T.H.-K.: Ratio-based lengths of intervals to improve fuzzy time series forecasting. IEEE Trans. Syst. Man Cybern. Part B (Cybern.) **36**(2), 328–340 (2006)
8. Bas, E., Uslu, V.R., Yolcu, U., Egrioglu, E.: A modified genetic algorithm for forecasting fuzzy time series. Appl. Intell. **41**(2), 453–463 (2014)
9. Wang, L., Liu, X., Pedrycz, W.: Effective intervals determined by information granules to improve forecasting in fuzzy time series. Expert Syst. Appl. **40**(14), 5673–5679 (2013)
10. Wang, L., Liu, X., Pedrycz, W., Shao, Y.: Determination of temporal information granules to improve forecasting in fuzzy time series. Expert Syst. Appl. **41**(6), 3134–3142 (2014)
11. Wei, L., Chen, X., Pedrycz, W., Liu, X., Yang, J.: Using interval information granules to improve forecasting in fuzzy time series. Int. J. Approx. Reason. **57**, 1–18 (2015)
12. Ho, N.C., Sơn, T.T. and Long, D.T.: Hedge algebras with limited number of hedges and applied to fuzzy classification problems. J. Sci. Technol. **48**(5) (2012)
13. Chen, S.-M., Chen, C.-D.: Taiex forecasting based on fuzzy time series and fuzzy variation groups. IEEE Trans. Fuzzy Syst. **19**(1), 1–12 (2011)
14. Nguyen, C.H., Hoang, V.T., Nguyen, V.L.: A discussion on interpretability of linguistic rule based systems and its application to solve regression problems. Knowl.-Based Syst. **88**, 107–133 (2015)
15. Vu, M.L., Vu, H.D., Pham, T.Y.: The context-aware calculating method in language environment based on hedge algebras approach to improve result of forecasting time series. In: Cong Vinh, P., Ha Huy Cuong, N., Vassev, E. (eds.) ICCASA/ICTCC-2017. LNICST, vol. 217, pp. 110–123. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-77818-1_11