Optimal Control of Navigation Systems with Time Delays Using Neural Networks

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Abstract. In this paper, an online adaptive dynamic programming (ADP) scheme is proposed to achieve the optimal regulation control of navigation control systems subject to time delays with input constraints. The optimal control strategy is developed in virtue of Lyapunov theories and neural networks (NNs) techniques. From a robust control perspective, we investigate the stability on navigation time delay systems concerning input constraints by means of linear matrix inequalities (LMIs) and set up the optimal control policy, on which basis that a novel NN-based approach is proposed. A single NN is used to estimate the performance function, the constrained control and consequently the optimal control policy with the weights online tuned. Finally, numerical examples are demonstrated to illustrate our results.

Keywords: Online ADP · Optimal control · Time delay · Neural network · Nonlinear control · Navigation control system

1 Introduction

It is not exaggerated to say that the history of navigation control systems is a reflection of the history of human civilization. Many scientific discoveries and technological inventions are developed by the need of navigation control, such as meteorology [5], Kalman filter method [9], satellite technology [12], micro-electromechanical technology [27], to name a few, which greatly promote the development of navigation control technology. Navigation control algorithm is a typical adaptive method, which can be effectively applied in the intelligent environment. In [20], optimal control of an UAV

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autonomous navigation system was developed by using only on-board visual and internal sensing information. [8] introduces a navigation planning algorithm for a robot which is capable of autonomous navigation in a structured, partially known and dynamic environment. [23] develops a nonlinear optimal control method to solve the autonomous navigation of a truck and trailer system. Although various technologies have been widely developed in navigation control case by case, the fundamental issue on the robustness and control has been seldom studied.

Nowadays, optimal control theory has gained lots of progress with the development of robust control theories and numerical methods (see [6, 24, 26, 29], and the references therein). On one hand, the optimal control of linear systems has been well investigated and numerous results are readily applicable, such as maximum principle [1], dynamic programming [13], and convex optimization [4], to name a few. On the other hand, the optimal control of nonlinear systems mainly relies on the solution of Hamilton-Jacobi-Bellman (HJB) equation [22], which is difficult to solve analytically. For few special nonlinear systems, analytical solutions such as state dependent Riccati equation approach [21], alternative frozen Riccati equation method [15], etc., have been derived. A majority of current results on nonlinear optimal control, nevertheless, are carried out resorting to approximation linearization theory. In [14], an iterative ADP algorithm was adopted to deal with the optimal control of nonlinear system with time-delay, rendering a series of remarkable developments on optimal control of nonlinear systems.

In addition, the predominating studies of optimal control are investigated without considering actuator limitation. As for the navigation control systems, however, neglecting such constraint may cause undesirable transient response and even system instability [7, 30]. In the recent work [28] and [25], the optimal control method is proposed for linear system with saturation actuator, while in [26] and [19], an iterative heuristic dynamic programming (HDP) algorithm was introduced to solve the optimal control for a class of nonlinear discrete system with control constraints. The optimal control results refer to nonlinear navigation control time-delay systems with control constraints still remain relatively minor. As a consequence, it is of fundamental significance to study the optimal control of navigation control systems with control constraints.

In this paper, we consider the nonlinear optimal control of navigation control time delay systems with input constraints based on ADP algorithm. The navigation parameters, namely position, velocity, and attitude, is framed into a nonlinear state-space model on the basis of [2]. Large mathematical tools in robust control and optimal control theories are thus readily available. To achieve the optimal control goal, the appropriate performance index function under given constraints is constructed. A non-quadratic index function is adopted to measure the constrained control input, and consequently an approximate NN is introduced to estimate the performance function and calculate the optimal control at the same time. Other than using two NNs as most work does, we propose a novel NN-based optimal control strategy by using only one NN. The tuning weights estimation errors of NN is proved to convergent to zero, thus indicating the approximated NN-based optimal control policy for the navigation control system with time delays actually converges to the real optimal control policy. It is worth noting that the stability of the closed-loop navigation control system subject to time delays is guaranteed according to the Lyapunov stability theory. Numerical examples show the effectiveness of the proposed method.

This paper is organized as followed. In Sect. 2, we formulate the optimal control problem of nonlinear time-delay systems with control constraints. In Sect. 3, we present the optimal control solution is obtained by NN. Section 4 presents two illustrative examples and Sect. 5 concludes the paper.

2 Problem Formulation

In this paper, $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^n_+$, $\mathbb{R}^{l\times p}$ refer to the space of real numbers, n-dimensional real vectors, n-dimensional of positive real vectors, and $1 \times$ p-dimensional real vectors, respectively. For any real matrix and real function, $(\cdot)^T$ denotes its transpose. If A is a Hermitian matrix, let $\lambda(A)$ denote its largest eigenvalue. The expression $A \geq 0$ means A is non-negative definite, and $A > 0$ means it is positive definite. Let $||x(t)||$ denote the Euclidean norm of x(t), which is defined as

$$||x(t)|| = \sqrt{x^t(t)x(t)}$$

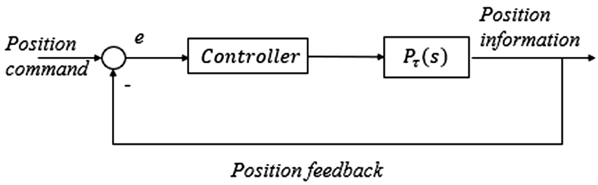


Fig. 1. A subsystem of navigation control systems.

Navigation control systems usually consist of several subsystems, the framework is as shown in Fig. 1. Based on [2] and [23], the state space expression of the navigation control system is derived upon kinetics equations, and can be further represented by the following nonlinear model

$$\dot{x}(t) = Ax(t-\tau) + f(t, x(t), x(t-\sigma)u(t)) \quad (1)$$

where $f(\mathrm{t}, \mathrm{x}(\mathrm{t}), \mathrm{x}(\mathrm{t}-\sigma)) \in \mathbb{R}^{n\times n}$ is the nonlinear process model and satisfies Lipschitz continuous with $f(\mathrm{t}, 0, 0) = 0$. $x(\mathrm{t}), \in \mathbb{R}^n$ is the system state vector and $u(t)$ *is* the input vector, consisting of navigation parameters. τ and σ are constant but unknown time delays. Assume that A is a positive definite constant real matrix with appropriate dimension. To achieve high accuracy of attitude error, we use the second-order divided difference filter (DDF2) proposed in [2]. The objective in this paper is to find the constrained optimal control signal $u^*(t)$ that drives the system states to zero as well as to minimize the following performance index function of the system state.

$$J(x(\mathrm{t}), u(\mathrm{t})) = \int_t^\infty L(x(s), u(s)ds \tag{2}$$

where

$$L(x(\mathrm{s}), u(\mathrm{s})) = x^T(s)Qx(s) + U(u_t) \tag{3}$$

and Q is symmetric and positive definite and $\mathrm{U}(u_t)$ is a quadratic and positive definite function without control constraints. Concerning the control constraint, we adopt the following function [21]

$$\mathrm{U}(u_t) = 2\int_0^u \beta^{-T}\left(\overline{U}^{-1}v\right)\overline{U}Rdv \tag{4}$$

where v, $R \in \mathbb{R}^n$ is positive definite, and

$$\beta^{-1}(u_t) = \left[\varphi^{-1}(u_{1t}), \varphi^{-1}(u_{2t}), \cdots \varphi^{-1}(u_{mt})\right]^{-T}$$

$\varphi^{-1}(\cdot)$ refers to the inverse of $\varphi(\cdot)$, $\beta^{-1}(\cdot) = (\beta^{-1})^T$. $\varphi(\cdot)$ is a bounded monotonic odd function with

$$|\varphi(\cdot)| < 1$$

and

$$|\varphi'(\cdot)| < \mu,\ \mu \text{ is a positive constant}$$

In this paper, we choose $\varphi(\cdot) = \tanh(\cdot)$, and $\mathrm{R} = \mathrm{diag}(\gamma_1, \gamma_2, \cdots \gamma_m)$, with $\gamma_i > 0, i = 1, 2, \cdots \mathrm{m}$ to simplify the subsequent analysis.

The Hamilton function is given as

$$\mathrm{H(x, u, t) = L(x(t), u(t))} + \nabla J^T(x)(Ax + fu) \tag{5}$$

with $\nabla \mathrm{J(x)} = \frac{\partial \mathrm{J}}{\partial \mathrm{X}}$ referring to the partial derivative of the performance index function. Obviously, based on the Bellman's principle of optimal control theory [10], the optimal performance index function $J^*(x(t), u(t))$ satisfies the following HJB equation

$$J^*(x(t), u(t)) = \min\left\{\int_t^\infty x^TQx + 2\int_0^u \beta^{-T}\left(\overline{U}^{-1}v\right)\overline{U}Rdv\right\} \tag{6}$$

It can be solved d by taking partial differential of $J^*(x(t), u(t))$ with respect to $u(t)$.

$$\frac{\partial J^*(x(t), u(t))}{\partial u(t)} = 2tanh^{-1}\left(\overline{U}u\right)\overline{U}R + f^T\frac{\partial J}{\partial x} = 0 \tag{7}$$

Hence, the optimal input is obtained as

$$u^*(t) = -\overline{U}\tanh\left(\frac{1}{2}\left(\overline{U}R\right)^{-1}f^T\frac{\partial J^*}{\partial x}\right) \tag{8}$$

Consequently, the system (1) becomes the following when the optimal control is achieved.

$$\dot{x}(t) = Ax(t-\tau) - f(t, x(t), x(t-\sigma)\overline{U}\tanh\left(\frac{1}{2}\left(\overline{U}R\right)^{-1}f^T\frac{\partial J^*}{\partial x}\right) \tag{9}$$

By employing the so-called model transformation [18], the original system (1) can be rewritten as

$$\frac{d}{dt}\left(x(t) + A\int_{t-\tau}^{t} x(u)du\right) = Ax(t) + f(t, x(t), x(x-\sigma))u(t) \tag{10}$$

Moreover, system (10) can be further equivalent to

$$\frac{d}{dt}\left(x(t) + A\int_{t-\tau}^{t} x(u)du\right) = Ax(t) + F(t, x(t), x(x-\sigma)) \tag{11}$$

where F(·) denotes a nonlinear function of x(t) with

$$F(x, x(t), x(t-\sigma)) = -f(t, x(t), x(t-\sigma)\overline{U}\tanh(\frac{1}{2}\left(\overline{U}R\right)^{-1}f^T\frac{\partial J^*}{\partial x}$$

In what follows, we shall examine the stability of (9) when the optimal control law is achieved. To this end, the following assumption is required.

Assumption 1. *The nonlinear function $F(t, x, y)$ satisfies*

$$||F(t, x, y) - F(t, x_1, y_1)|| \leq \alpha||x - x_1||^2 + \beta||y - y_1||^2, \tag{12}$$

where $t, x, y, x_1, y_1 \in \mathbb{R}^n$ and α, β are some positive scalar.

In virtue of the famous contraction mapping theorem [3], we can conclude that the original nonlinear navigation control system (1) has unique equilibrium if the following in equation satisfies

$$\sqrt{\alpha+\beta}||A^{-1}|| \leq 1. \tag{13}$$

3 Main Results

3.1 Stability Analysis on Nonlinear Time Delay Systems

In this section, drawing upon Lyapunov stability theory, we present sufficient stability condition for navigation control time delay systems in terms of LMIs. The following theorem is derived.

Theorem 1. T*he navigation control system* (1) *subject to input constraint* (8) *is asymptotically stable if for some positive real scalar* α, β, η_1 *and* η_2, *there exists a real matrix* $P = P^T > 0$ *such that the following LMIs hold*

$$\Phi < 0, \tag{14}$$

where

$$\Phi = Q_1 + \tau Q_2 + Q_3$$

$$Q_1 = PA + A^T P + \eta_1^{-1} P^2 + \alpha(\eta_1 + \tau\eta_2)I + \tau A^T PA$$

$$Q_2 = A^T PA + \eta_2^{-1} A^T P^2 A, Q_3 = \beta(\eta_1 + \tau\eta_2)I$$

and I is the identity matrix with appropriate dimension.

Proof: Construct the Lyapunov-Krasovskii functional $V(\mathrm{t})$ as

$$V(\mathrm{t}) = \psi_1(t) + \psi_2(t),$$

where

$$\psi_1(t) = \left(x(t) + A\int_{t-\tau}^{t} x(u)du\right)^T P\left(x(t) + A\int_{t-\tau}^{t} x(u)du\right),$$

and

$$\psi_2(t) = \int_{t-\tau}^{t}\int_{s}^{t} x^T(u)Q_2 x(u)duds + \int_{t-\sigma}^{t} x^T(u)Q_3 x(u)du$$

In light of model transformation and Eq. (11), we are led to

$$\dot{\psi}_1(t) = 2\left(x(t) + A\int_{t-\tau}^{t} x(u)du\right)^T P(Ax(t) + F(t, x(t), x(t-\sigma))$$

which is further bounded by

$$\dot{\psi}_1(t) \le x^T(t)Q_1 x(t) + \int_{t-\tau}^{t} x^T(u)Q_2 x(u)du + x^T(t-\sigma)Q_3 x(t-\sigma),$$

In the similar manner of [16], the time derivati ve of $V_2(t)$ is computed as

$$\dot{\psi}_2(t) = x^T(t)(\tau Q_2 + Q_3)x(t) - \int_{t-\tau}^{t} x^T(u)Q_2 x(u)du - x^T(t-\sigma)Q_3 x(t-\sigma),$$

Consequently, it yields to

$$\dot{V}(t) \le x^T(t)\Phi x(t)$$

According to Lyapunov stability theories, $\dot{V}(t)$ is negative definite if Φ is negative definite. As such, the nonlinear time-delay system with input constraint is stable if condition (14) satisfies. This completes the proof. ■

3.2 NN-Based Online ADP Algorithm

In the following section, we shall develop the NN-based optimal control scheme with the aid of online ADP technique. As mentioned previously, most of past literatures use two NNs to approximate the performance function and optimal control input respectively so as to achieve the so-called optimal control goal. In this paper, however, we propose a novel NN-based optimal control strategy for navigation control systems by utilizing merely single NN other than two NNs, thus largely simplifies the whole structure and decreases the running time. In addition, the performance index function and optimal input are tuned at the same time.

Let $W_c \in \mathbb{R}^{l \times p}$ refer to the ideal weight matrix of NN, $\phi_c(x)$ refer to the activation function and $\varepsilon_c(x)$ the approximation error. It is necessary to make the following assumption.

Assumption 2. *(a) The approximation error of NN has positive upper bound as* $||\varepsilon_c|| \leq \varepsilon_{cM}$;

(b) The residual error ε_H *has a positive upper bounded as* $||\varepsilon_H|| \leq \varepsilon_{HM}$;

(c) The activation function of the NN has a positive lower and upper bound as $\phi_m \leq ||\phi_c|| \leq \phi_M$.

Upon above assumption, the performance index function J(x) is nearly approximated by

$$\mathrm{J(x)} = W_x^T \phi_c(x) + \varepsilon_c(x) \tag{15}$$

Be reminiscent of (5) and (15), we obtain the following equation

$$\mathrm{H(x(t), u(t)}, W_c) = W_c^T \nabla \phi_c(x)\dot{x} + x^T Qx + 2\int_0^u \tanh(\frac{1}{2}\left(\overline{U}v\right)^{-1}\overline{U}Rdv + \nabla\varepsilon_c(x)\dot{x} \tag{16}$$

where $\nabla\phi_c(x)$ is the partial derivative of $\phi_c(x)$ with respect to x. That is, $\nabla\phi_c(x) = \partial\phi_c/\partial x$. $\nabla\varepsilon_c(x)$ is the partial derivative of $\varepsilon_c(x)$. Thus, the Hamilton function is alternatively expressed by

$$W_c^T \nabla \phi_c(x)\dot{x} + x^T Qx + 2\int_0^u \tanh(\frac{1}{2}\left(\overline{U}v\right)^{-1}\overline{U}Rdv = -\nabla\varepsilon_c(x)\dot{x} = e_H \tag{17}$$

where e_H is the residual error caused by NN approximation. Let $\widehat{W}_c \in \mathbb{R}^{l \times p}$ refer to the real weight matrix of NN. Then the estimation of performance index function is

$$\hat{J}(\mathrm{x}) = \widehat{W}_c^T \phi_c(x) \tag{18}$$

As a result, the gradient of $\hat{J}(\mathrm{x})$ is computed by

$$\nabla\hat{J}(\mathrm{x}) = (\nabla\phi_c(x))^T\widehat{W}_c \tag{19}$$

Meanwhile, the corresponding Hamilton function can be expressed as

$$\mathrm{H}\left(\mathrm{x(t), u(t)}, \widehat{W}_c\right) = W_c^T\nabla\phi_c(x)\dot{x} + x^TQx + 2\int_0^u \tanh(\frac{1}{2}\left(\overline{U}v\right)^{-1}\overline{U}Rdv = e_c$$

Let $\tilde{W}_c$ refer to weight estimation error

$$\tilde{W}_c = W_c - \widehat{W}_c$$

We define the objective function $E_c(t)$ by

$$E_c(t) = \frac{1}{2}e_c^2 \tag{20}$$

We seek to find the optimal weight update law such that $E_c(t)$ is minimized. The gradient of objective function with respect to the NN weight estimate is given by

$$\frac{\partial E}{\partial\widehat{W}_c} = e_c\frac{\partial e_c}{\partial\widehat{W}_c} = e_c(\dot{x})^T(\nabla\phi_c)^T \tag{21}$$

On the basis of back propagation (BP) neural network algorithm [32], the weight update law of NN is derived as

$$\dot{\hat{W}}_c = -\xi\frac{\partial e_c}{\partial\widehat{W}_c} = e_c(\dot{x})^T(\nabla\phi_c)^T \tag{22}$$

With the learning rate $\xi > 0$. As such, the ideal optimal control can be approximated achieved as

$$\hat{u}^*(t) = -\overline{U}\tanh\left(\frac{1}{2}\left(\overline{U}R\right)^{-1}f^T(\nabla\phi_c)^TW_c\right) \tag{23}$$

Theorem 2. *Consider nonlinear nagivation control system* (1) *with input constraint* (8)*. Assume the weight update law of NN is given by* (22) *and the control signal* (23) *is applied to the nominal system* (1)*. Then the system state $x(t)$ and the NN estimation errors $\widehat{W}_c$ are uniformly ultimately bounded (UUB) respectively.*

The proof herein shares the similarity as the proof in Theorem 1, thus is omitted [11, 17, 31].

4 Examples

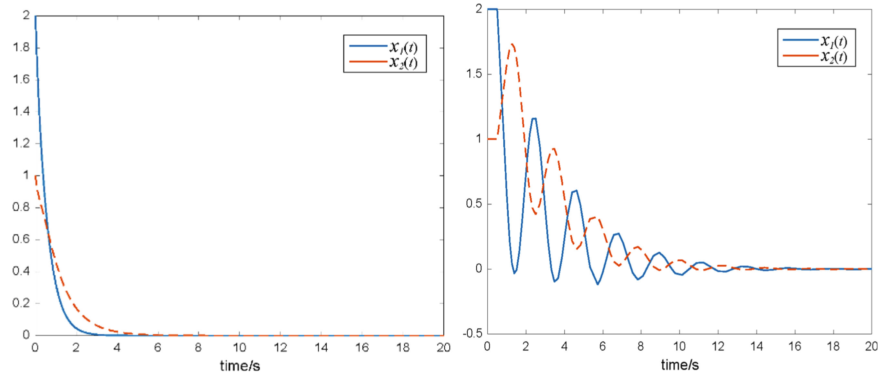
In this section, we illustrate our results by two numerical examples.

Example 1. Consider the following second-order navigation control system

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} x(t-\tau) + \begin{bmatrix} sinx_2(t-\sigma) \\ sinx_1(t-\sigma) \end{bmatrix} u(t) \tag{24}$$

with initial state $x(0) = [2 \quad 1]^T$.

We first examine the stability condition. As suggested in Theorem 1, the nominal system (24) is asymptotically stable if $\tau \leq 0.5$ with σ being arbitrarily large. From Fig. 2 (a)–(b) we can see the state response of the system (24) converges to zero when there is no delay and $\tau = \sigma = 0.5$, where the state response of the delay free system converges to 0 after 7 s, slightly shorter than that of the latter system. In contrast, from Fig. 3 we can see the nominal plant becomes unstable when $\tau = 0.7$.



(a) $\tau = \sigma = 0$ **(b)** $\tau = \sigma = 0.5$

Fig. 2. State response of system (24).

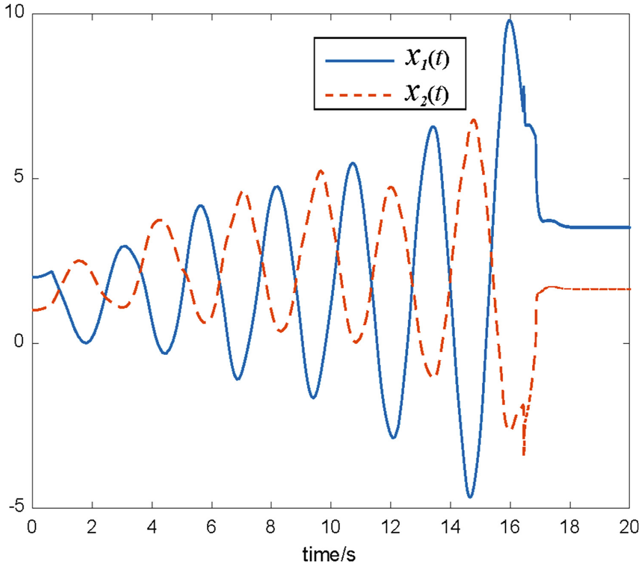


Fig. 3. State response of system (24) with $\tau = \sigma = 0.7$.

Example 2. Specifically, we examine the NN-based optimal control law for the case $\tau = \sigma = 0.5$. We consider the system (24) with input constraints

$$|\mathrm{u(t)}| \leq \overline{U} = 0.1 \tag{25}$$

The optimal control objective is to drive the system state x(t) to zero quickly as well as to minimize the performance function. By selecting Q = R = 1, the activation function of NN is equal to

$$\phi_c(x) = \begin{bmatrix} x_1^2 & x_1 x_2 & x_2^2 \end{bmatrix}^T$$

Assume the initial weights are

$$W_c = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}^T$$

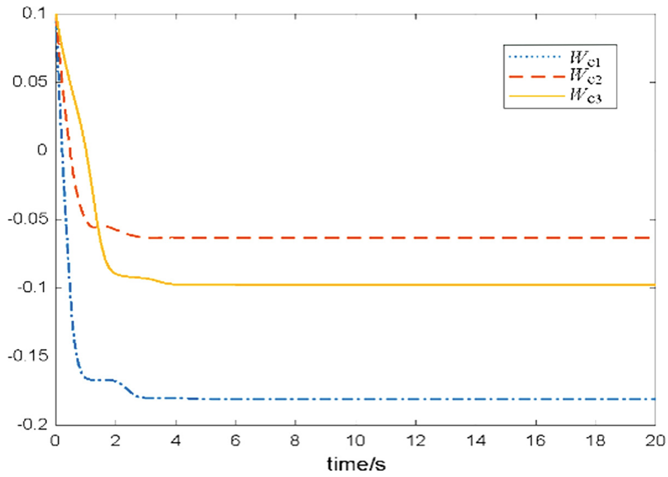


Fig. 4. Trajectories of NN weights for system (24) with input constraint (25)

Let the learning rate of NN be $\xi = 0.5$. The convergent trajectories of NN weights are shown in Fig. 4.

Besides, the optimally controlled signals for system (24) without control constraint, with input constraint (25), and with input constraint

$$|\mathrm{u(t)}| \leq \overline{U} = 0.2 \tag{26}$$

are respectively drawn in Fig. 5, from which we can see that for the circumstance without constraint, control signal u varies within a relatively large range, while the proposed scheme regulates the input signal effectively for both cases of input constraint (25) and (26).

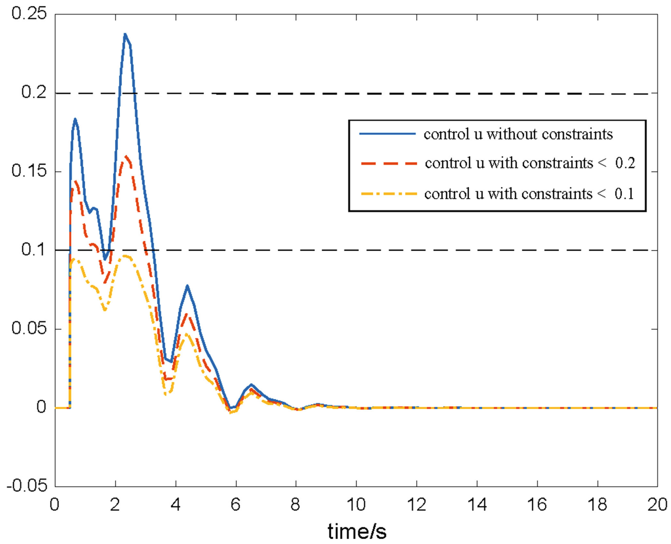


Fig. 5. Unconstrained and constrained control signals.

5 Conclusion

In this paper we propose a novel NN-based optimal control policy for navigation control systems subject to time delay. Optimal control strategy is proposed under the consideration of control constraints by applying a non-quadratic performance function. We investigate the stability on nonlinear time delay systems in terms of LMIs in the first. Afterwards, a novel NN-based optimal control policy is introduced to approximate the optimal cost function and obtain the optimal constrained control signal using only one NN. Finally, Numerical simulation shows the effectiveness of our results.

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