



# Multiple Tasks Assignment for Cooperating Homogeneous Unmanned Aerial Vehicles

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**Abstract.** Using multiple unmanned aerial vehicles (UAVs) to perform some tasks cooperatively has received growing attention in recent years. Task assignment is a difficult problem in mission planning. Multiple tasks assignment problem for cooperating homogeneous UAVs is considered as a traditional combinatorial optimization problem. This paper addresses the problem of assigning multiple tasks to cooperative homogeneous UAVs, minimizing the total cost and balancing the cost of each UAV. We propose a centralized task assignment scheme which is based on minimum spanning tree. This scheme involves two phases. In the first phase, we use the Kruskal algorithm and the breadth first search algorithm to assign all tasks to UAVs and get a proper initial task assignment solution. The second phase involves the Pareto optimization improvement in the solution generated from the first phase. For a single UAV, we use the dynamic programming algorithm to calculate the total cost of completing all assigned tasks. The performance of the proposed scheme is compared to that of heuristic simulated annealing algorithm. The simulation results show that the proposed scheme can solve the homogeneous multi-UAV cooperative task assignment problem effectively.

**Keywords:** Unmanned aerial vehicle · Task assignment · Minimum spanning tree · Pareto optimization

## 1 Introduction

In recent years, the advantage of using UAV to perform various military and civilian missions in the air, sea, and on the ground has become more and more obvious. Compared with manned aircraft, UAV has the advantages of small size,

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light weight, low cost, no risk of casualties, good concealment, strong survivability, strong self-control ability, and ability to fly in some high-risk areas. In the military field, UAV can be used to complete battlefield reconnaissance and surveillance, deceive enemy decoys, locate shots, strike against the ground, and also serve as targets for artillery and missiles. In the civilian sector, UAV can be used for search rescue, disaster monitoring, meteorological detection, communication relay, pesticide spraying, etc. However, the role that a single UAV can play is very limited. Especially, it is often impossible to complete some complicated tasks. An effective method is to use multiple UAVs to accomplish some tasks collaboratively. It is expected that the capabilities of a joint system far exceeds the sum of its individual parts. Most tasks can be accomplished more effectively by cooperation and coordination of multiple UAVs.

Multi-UAVs collaborative mission planning is usually divided into two major parts: task assignment and path planning. Due to the certain coupling between task assignment and path planning, the current research on multi-UAVs collaborative mission planning usually has two methods. One is to research separately and the other is to research together. In this paper, we only concentrate on task assignment problem.

Multiple tasks assignment problem is an NP-hard problem. Numerous exact and heuristic methods have been proposed for solving this problem [2]. Exact methods include mixed integer linear programming (MILP) [3,10], branch and bound [9], network flow [7], iterative CTP algorithm [11], and so on. Nygard et al. [7] propose a network flow optimization model for solving task assignment. The network optimization problem is formulated as a linear programming problem. Mixed Integer Linear Programming (MILP) is an effective method for task assignment because it can use discrete decision variables to deal with dynamic system. Bellingham et al. [3] use MILP for task assignment to deal with waypoint visiting problem. However, the complexity of these exact algorithm rapidly grows as the number of UAVs and tasks increase. Therefore exact algorithm is often suitable for some small-size problems. Recently, several heuristic algorithms have been used to solve task assignment problem including ant colony optimization (ACO) [1,15,16], genetic algorithm (GA) [8,12], and particle swarm optimization [4,6,13,14]. The ACO was introduced in early 90's and simulates the process of ants foraging. Zaza et al. [16] propose an enhanced version of ACO for solving UAV task allocation and route planning. The ACO adopts a multi-colony approach to incorporate variable loitering times. In comparison with MILP, the method has been shown to offer near-optimal solutions in faster time and with better scalability with the number of tasks. The GA is a stochastic search method which is described in many papers. Shima et al. [12] use genetic algorithm for assigning the multiple agents to perform multiple tasks on multiple targets. Monte Carlo simulations demonstrate the viability of the genetic algorithm, providing good feasible solutions quickly. Heuristic algorithms can effectively solve some hard large-size optimization problems, but they have a large amount of calculation and their convergence is challenging in some cases.

This paper investigates the multi-objective multiple-UAV multiple tasks assignment problem, optimizing two objectives: the total cost of all UAVs and the workload among the UAVs. We propose a MST-based method to solve the above problem effectively. Firstly, we use Kruskal algorithm to obtain a minimum spanning tree (MST) for UAV and task point. Then, based on the MST, BFS algorithm is used to obtain a proper initial task assignment solution. Finally, Pareto optimization is used to improve the initial solution. On the one hand, we consider the multi-objective optimization problem. On the other hand, compared with heuristic algorithm, our method has good convergence.

The rest of the paper is organized as follows. Section 2 describes the system model. Section 3 presents the MST-based task assignment algorithm. Section 4 presents some experimental results to validate the proposed method. Finally, we conclude the paper in Sect. 5.

## 2 System Model

In this paper, we are interested in solving the multi-objective multiple-UAV multiple tasks assignment problem. We assume that each UAV has the same capabilities and can perform every task. Task can be defined as reconnaissance, attack, or assessment and so on. Without loss of generality, we define the task as visiting a waypoint. Also, each task must have at least one UAV to execute. The total environment is modeled as a two-dimensional (2D) Euclidean space. Our goal is to accomplish all tasks at the lowest cost. Since we use multiple UAVs to perform tasks, we usually don't let one UAV accomplish tasks too many or even tens of times that of other UAVs. In this case, UAV may not be able to accomplish all tasks due to its own energy constraints. Therefore, balancing the cost of UAVs is an important issue with practical significance in multiple tasks assignment. So the other objective is to balance the cost of each UAV.

The problem can be formulated as follows. Consider a set of  $m$  UAVs  $\{r_1, \dots, r_m\}$ , initially located at different positions  $\{p_1, \dots, p_m\}$ , which must visit a set of  $n$  ( $m < n$ ) task locations  $\{t_1, \dots, t_n\}$  and return to their initial positions after task completion. We define  $tour_i$  as the tour of UAV  $r_i$  starting from and ending at its initial position  $p_i$  and going through the list of its all allocated tasks  $\{t_{i_1}, \dots, t_{i_k}\}$  in some order. The tour cost of the UAV  $r_i$  is defined as:

$$cost(tour_i) = cost(p_i, t_{i_1}) + \sum_{j=1}^k cost(t_{i_j}, t_{i_{j+1}}) + cost(t_{i_k}, p_i) \quad (1)$$

where  $k$  is the number of task assigned to UAV  $r_i$ .  $cost(t_{i_j}, t_{i_{j+1}})$  represents the cost between task  $t_{i_j}$  and  $t_{i_{j+1}}$ . The tour cost of UAV  $r_i$  may be any of several things, including consumed energy, time, or distance. In this paper, we use Euclidean distance to represent the cost between tasks.  $t_{i_1}$  and  $t_{i_k}$  are the first and the last task locations for UAV  $r_i$ .  $cost(p_i, t_{i_1})$  represents the cost between the initial position of UAV  $r_i$  and the first task location, and  $cost(t_{i_k}, p_i)$

represents the cost between the last task location and the initial position of UAV  $r_i$ .

In the context of the multi-objectives optimization problem, the goal is to generate a solution that provides a good trade-off between the objectives. In this paper, we have two objectives. One is to minimize the cost of completing all tasks and the other is to balance the cost of each UAV. Because the cost of each UAV is not necessarily balanced when the total cost is the smallest, it is necessary to put forward some requirements for balancing. The objective function is defined as:

$$\text{minimize } \sum_{i=1}^m \text{cost}(\text{tour}_i) \quad (2)$$

$$\text{minimize } \max_{i \in 1 \dots m} \text{cost}(\text{tour}_i) \quad (3)$$

### 3 Minimum Spanning Tree-Based Task Assignment

From the perspective of graph theory, our method abstracts UAVs and task points as vertices, and paths between points as edges in undirected graph. Moreover, the cost of the path is used as the weight of the edge in the graph. Since we have a goal to accomplish all tasks at the lowest cost, we associate the minimum spanning tree in graph theory. Using the minimum spanning tree, we can get the lower bound of the total cost, that is, the sum of the weights of the minimum spanning tree. Based on the MST, our method includes two steps: generating a proper initial task assignment solution and Pareto optimization improvement.

When task assignment problem is abstracted into graph theory problem, we can find that the final result of modeling is an undirected fully connected graph. Because when an obstacle is encountered in the process of connecting two points in a straight line, the two points can be connected in a curved manner around the obstacle. Therefore, the points can be interconnected to form an undirected fully connected graph. In this paper, the weight between points is represented by the distance between two points. Based on the graph generated above, we use the Kruskal algorithm which is one of the most popular methods used to generate MST [5]. Then we can calculate the sum of the weights of the edges of the minimum spanning tree which is taken as the lower bound of the total cost.

According to the lower bound and the number of UAVs, we can get the lower bound of the cost of each UAV. Starting with a UAV, we use the BFS algorithm to assign task to each UAV. The sum of the weights is calculated while assigning. When a task is assigned, the sum of the weights exceeds the lower bound of the UAV and the allocation is stopped. Each UAV repeats the above process until all UAVs are assigned. Finally, there may be some remaining tasks that are not assigned. We assign each remaining task to the UAV that is closest to the task. In the end, we will get a initial task assignment solution.

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**Algorithm 1 (Task Assignment)**


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1. **Inputs:** UAVs  $r_i$  ( $1 < i < m$ ), Tasks  $t_j$  ( $1 < j < n$ ), Minimum spanning tree  $T$ , Distance matrix  $G$
  2. Calculate the sum of the weights of the minimum spanning tree  $T$
  3. Divide the total weight by the number of UAVs to get the lower bound of each UAV
  4. **For** each UAV  $r_i$  **do**
  5.     **while** isEmpty(queue)==false&&sum<lower bound **do**
  6.         Assign a task to UAV  $r_i$  using breadth-first search algorithm
  7.         Calculate the sum of weights
  8.     **end**
  9. **end**
  10. **For** each task  $t_i$  **do**
  11.     **If**  $t_i$  is not assigned **do**
  12.         Assign it to the nearest UAV
  13.     **end**
  14. **end**
  15. **Outputs:** Assignment( $r_i, t_j$ )  $1 < i < m$   $1 < j < n$
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The Pareto optimization improvement step is to adjust tasks between UAVs in order to optimize our two objectives, as shown in Algorithm 2. Based on the initial task assignment solution obtained in the first step, for each UAV, we use the dynamic programming algorithm to calculate the cost for completing all assigned tasks. Further, the total cost can be obtained. From all UAVs, we first select the UAV with the largest and smallest cost which are denoted as UAV  $r_{max}$  and  $r_{min}$  respectively. Then, we select a task from the task assigned to the UAV  $r_{max}$ , which is farthest from the UAV  $r_{max}$  and closest to the UAV  $r_i$ , and assign it to the UAV  $r_i$ . If such a task can not be found, we look for a UAV closest to the UAV  $r_{max}$ , denoted  $r_j$ . The task closest to the UAV  $r_j$  in the

task of UAV  $r_{max}$  is assigned to  $r_j$ . After the end of one adjustment, the above process is repeated for the UAV  $r_i$  or  $r_j$ , while it encounter a bad solution or UAV  $r_{min}$ . After the end of a round of cycles, a new task assignment solution is obtained. According to the total cost of completing all tasks and the maximum cost of all UAVs, we decide whether to add the new solution to the candidate set of Pareto solution. The above process is repeated until the Pareto candidate set remains unchanged regardless of which task in the UAV  $r_{max}$  is adjusted.

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### Algorithm 2 (Pareto Optimization Improvement)

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1. **Inputs:** UAVs  $r_i$  ( $1 < i < m$ ), Tasks  $t_j$  ( $1 < j < n$ ), Assignment( $r_i, t_j$ )  
 $1 < i < m \quad 1 < j < n$
  2. Calculate the cost of each UAV  $r_i$  and total cost  $d_1$  of the initial task assignment solution using dynamic programming algorithm
  3. **Repeat** until  $r_{max}$  remains unchanged **do**
  4.     Select  $r_{max}$  and  $r_{min}$
  5.     **while**  $r_{max} \neq r_{min}$  **do**
  6.         Select a task from UAV  $r_{max}$  and assign it to  $r_i$
  7.         Calculate the cost of each UAV  $r_i$  and total cost  $d_2$  of the new task assignment solution
  8.         Update the Pareto candidate set
  9.         **If** the Pareto candidate set remains unchanged **do**
  10.             **break**
  11.         **end**
  12.          $r_{max} = r_i$
  13.     **end**
  14. **end**
  15. **Outputs:** Assignment( $r_i, t_j$ )  $1 < i < m \quad 1 < j < n$ , total distance
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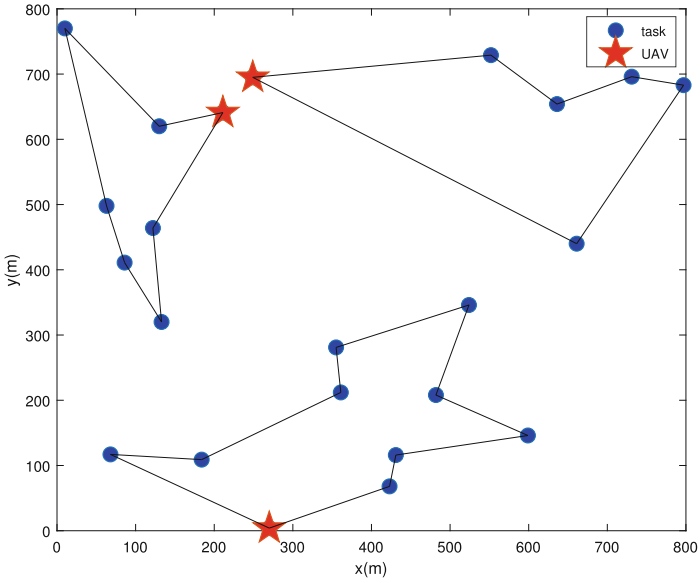
## 4 Performance Evaluation

In this section, we evaluate the performance of the proposed approach to solve multiple tasks assignment problem for cooperating homogeneous UAVs. The UAVs are initially located at different positions and the number of UAVs is less than the number of tasks. We evaluate the total cost and the maximum cost. The proposed scheme has been implemented and tested in MATLAB. We set up two test scenarios in which the location of the UAVs and tasks are randomly placed in a predefined area.

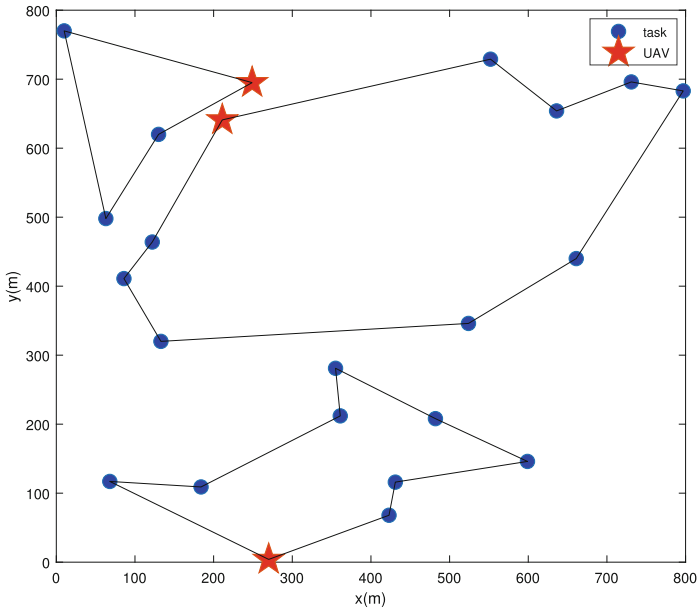
In order to validate the effectiveness of the proposed scheme, we compare it with the simulated annealing algorithm. The first test scenario contains three UAVs and twenty tasks. The size of the predefined area is  $800\text{ m} \times 800\text{ m}$ . Figure 1 shows the experimental results of the MST-based and the simulated annealing algorithm. Another test scenario consists of five UAVs and thirty-five tasks. The size of the operational area is  $3000\text{ m} \times 3000\text{ m}$ . Figure 2 shows the experimental results of the two kinds of algorithms. Detailed task assignment results of the two algorithms in Figs. 1 and 2 are recorded in Table 1. From the data in Table 1, we can find that the total and maximum cost of the proposed algorithm is smaller than that of the simulated annealing algorithm. Therefore, the proposed MST-based algorithm can provide a better balance between two objectives, compared to simulated annealing algorithm.

**Table 1.** The task assignment results of the MST-based and the simulated annealing algorithm

Algorithm	The number of UAV	The number of task	Operation area	Total cost ( $m$ )	The cost of each UAV ( $m$ )		The maximum cost ( $m$ )
MST-based algorithm	3	20	$800\text{ m} \times 800\text{ m}$	3904.2	1464.7 1087.8	1351.6	1464.7
Simulated annealing algorithm	3	20	$800\text{ m} \times 800\text{ m}$	3930.3	1837.0 807.5	1285.8	1837.0
MST-based algorithm	5	35	$3000\text{ m} \times 3000\text{ m}$	18075	4076.5 3591.9 3414.8	3564.4 3427.3	4076.5
Simulated annealing algorithm	5	35	$3000\text{ m} \times 3000\text{ m}$	18278	5706.0 2636.4 2144.7	5690.1 2100.5	5706.0



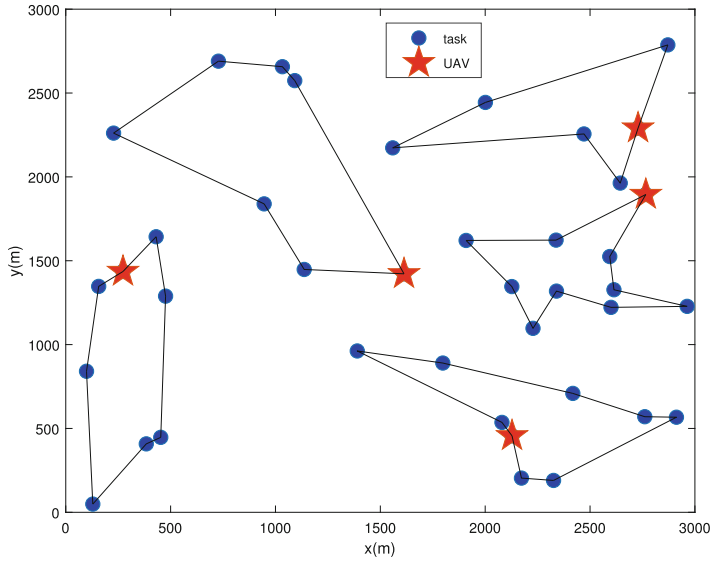
(a) MST-based algorithm



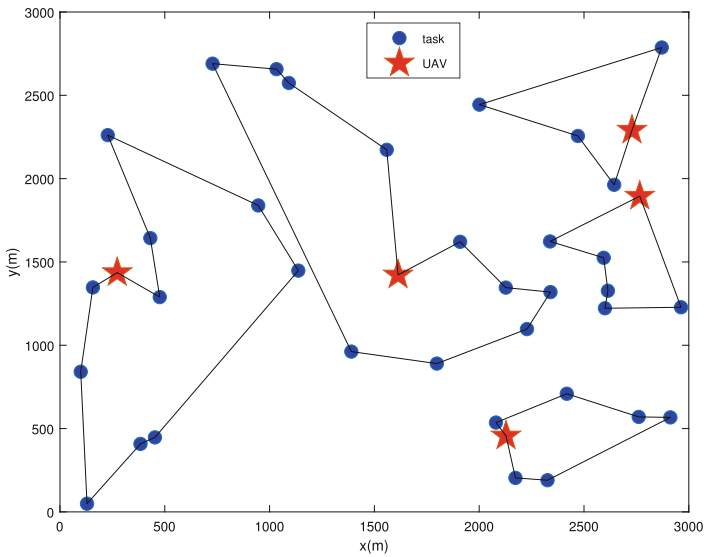
(b) Simulated annealing algorithm

**Fig. 1.** The task assignment result of 3 UAVs and 20 tasks of the MST-based and the simulated annealing algorithm





(a) MST-based algorithm



(b) Simulated annealing algorithm

**Fig. 2.** The task assignment result of 5 UAVs and 35 tasks of the MST-based and the simulated annealing algorithm

## 5 Conclusion

We decouple task assignment and path planning and only concentrate on task assignment problem. We consider multiple tasks assignment problem for multiple cooperating homogeneous UAV. In order to solve this problem, we propose a minimum spanning tree-based method. Our objective is to minimize the total cost and the maximum cost of all UAVs. In the performance evaluation part, our comparison of the MST-based method with existing simulated annealing algorithm shows that our method outperforms the simulated annealing algorithm, and provides a good trade-off between two objectives.

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