



Joint Transmit Power Allocation and Power Splitting for SWIPT System with Time Reversal

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Abstract. In the simultaneous wireless information and power transfer (SWIPT) system, time reversal (TR) is a special signal processing technology used to enhance the signal strength at the receiving end and improve the signal-to-noise ratio. In this paper, we study the joint transmit power allocation and power splitting algorithm for SWIPT system with TR. Considering a multi-input and single-output (MISO) SWIPT communication system, TR is introduced to construct a novel channel model, and the closed-form expression of energy efficiency (EE) is analyzed. An important problem is to maximize the energy efficiency by jointly transmit power allocation and power splitting, which is a two-element fractional non-convex problem. To solve it, we transform this problem to a convex optimization problem by a parametric method, and then solve it by one-dimensional search and CVX. Numerical results are provided to validate our proposed algorithm.

Keywords: Time reversal · Simultaneous wireless information and power transfer · Energy efficiency optimization

1 Introduction

With the development of energy revolution, the focus of people is no longer limited to oil, heat and so on, but for the renewable energy explored by advanced science and technology. The radio-frequency signals is a new renewable energy, which can be used to charge the device and send information synchronously, and thus it becomes one of the research hotspots in information transmission and security communication [1–3].

Simultaneous wireless information and power transfer (SWIPT) makes full use of these two characteristics of radio-frequency signals and attracts scholars' attention. The concept of SWIPT was first proposed by Varshney [4], who had made great progress in the entire communication industry. As data traffic grows and energy consumption increases, future wireless networks will pursue a communication method with higher energy efficiency [5–8]. However, most of the existing SWIPT literatures focus on the study of maximizing throughput, maximizing sum rate, and maximizing spectral efficiency [9–13], the maximization of energy efficiency (EE) got little attention. In the

case of satisfying the corresponding constraints, the maximization EE algorithm of SWIPT was studied only in broadcast channel, multiple access, relay network and wireless sensor network respectively [14–17]. Although these optimization algorithms improve the EE, it is limited by the signal strength of power splitter (PS) to some extent. The principle of time reversal (TR) is to use the spatio-temporal focusing effect generated by multipath propagation to enhance the signal strength at the receiving end, and simplify the receiver structure [18–20]. In [21], some researchers introduced TR into the SWIPT eavesdropping system, and proposed a new physical layer security transmission scheme, which greatly improved the transmission security of the system.

For the problem of maximizing EE in TR-SWIPT system, we consider to use TR to improve the achievable information rate and energy harvested firstly, and then propose an EE problem with transmit power allocation and power splitting. The objective function of the problem is a two-dimensional non-linear non-convex programming problem, which can be transformed into a linear convex optimization problem by the transformation of the objective function, and then the optimal solution is obtained by one-dimensional search and CVX. By means of numerical simulations, the effects of the signal-to-noise ratio (SNR), the number of antenna, the number of multipath, and the search accuracy on energy efficiency are analyzed, and the comparison of EE with other systems [1, 21] will be also discussed in the later.

The remainder of this paper is organized as following

- Section 2 to describe the system model and communication process of TR-SWIPT.
- Section 3 to present the problem planning and algorithm design.
- Section 4 to show numerical results, and Sect. 5 shows conclusion.

2 System Model

This section describes the specific communication process of the TR-SWIPT system, and analyzes the corresponding transmission signals and EE expressions.

Consider a point-to-point TR-SWIPT system with only one transmitter and one receiver, where the transmitter is equipped with M transmits antennas and the receiver is equipped with one antenna. For convenience, the PS adopts a separate structure and a dynamic power splitting mode, and the channel state information (CSI) of the system remains unchanged during the complete communication process. In addition, perfect CSI has been known to the system.

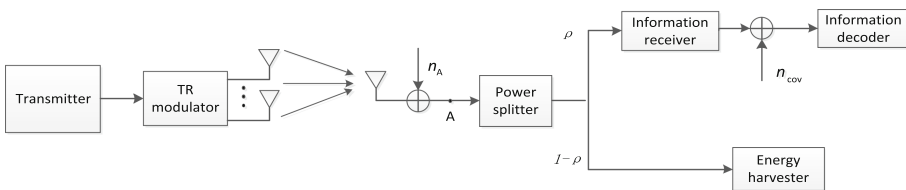


Fig. 1. TR-SWIPT system model

The system model of TR-SWIPT is shown in Fig. 1, and the communication process on the frequency selective fading channel can be divided into three steps, as shown below.

- Step 1. The information receiver or energy receiver in the PS transmits the sounding signal, and then the transmitter receives the sounding signal and analyzes the CSI.
- Step 2. The TR modulator modulates the signal according to the CSI, and then the transmitter transmits the modulated signal to the PS via multiple antennas.
- Step 3. The PS receives the transmitted signal, and then splits the received signal into two parts, one for the information decoder (ID), and the other for the energy harvester (EH). At this point, a completed communication process is implemented.

Based on the above communication process, we will first analyze the signal received at point A. Let P represent the average transmit power of the antenna, and let s ($E[|s|^2] = 1$) denote the data symbol to be transmitted. Modeling a frequency selective fading channel into a linear tap delay model, $\mathbf{h}_m \in \mathbb{C}^L$ ($E[|h_m[l]|] = 0$, $E[|h_m[l]|^2] = \sigma_{m,l}^2$, $l = 1, 2, \dots, L$ is the number of multi-paths) denotes channel vector between the m th ($m = 1, 2, \dots, M$) transmission antenna and receiver, and antennas are independent of each other. Moreover, \mathbf{n}_A is the additive white Gaussian noise introduced by the antenna with zero mean and variance σ_A^2 . Thus the signal at point A is

$$\mathbf{y}_A = \sqrt{P}s \sum_{m=1}^M \mathbf{h}_m * \mathbf{g}_m + \mathbf{n}_A \quad (1)$$

where $\mathbf{g}_m \in \mathbb{C}^L$ represents TR pre-modulated vector for the signal, and can be written as

$$g_m[l] = h_m^*[L+1-l] / \sqrt{\sum_{m=1}^M \|h_m\|^2} \quad (2)$$

where $h_m^*[L+1-l]$ denotes the complex conjugation of $h_m[l]$, and $g_m[l]$ has been normalized.

As the signal is transmitted to point a, we can see from the literature [22] that for the signal transmitted after time-reversed modulation, the signal power is less lost during transmission, and most of the power is concentrated in the center tap. So, we only need to take the value of L th tap to get the ideal signal power. Let p_A , p_{ID} and p_{EH} represent the power of received signal, the power of ID and the power of EH, respectively, as indicated in (3)–(5).

$$p_A = p_{\text{sig}} + \sigma_A^2 \quad (3)$$

$$p_{\text{ID}} = \rho p_{\text{A}} + \sigma_{\text{cov}}^2 = \rho \left(P \left| \sum_{m=1}^M g_m[L] * h_m[L] \right|^2 + \sigma_{\text{A}}^2 \right) + \sigma_{\text{cov}}^2 \quad (4)$$

$$p_{\text{EH}} = (1-\rho)p_{\text{A}} = (1-\rho) \left(P \left| \sum_{m=1}^M g_m[L] * h_m[L] \right|^2 + \sigma_{\text{A}}^2 \right) \quad (5)$$

where p_{sig} denotes ideal signal power, and $\rho \in [0, 1]$ represents power splitting. Moreover, conversion noise \mathbf{n}_{cov} ($E[\mathbf{n}_{\text{cov}}] = 0$, $E[\mathbf{n}_{\text{cov}}^2] = \sigma_{\text{cov}}^2$) is the additive noise introduced by the wireless signal to baseband signal conversion.

3 Problem Planning and Algorithm Design

In this section, we propose a two-element fractional non-convex problem with joint transmit power allocation and power splitting, and find optimal solution by a parametric method, one-dimensional search and CVX.

3.1 Problem Planning

Based on the TR-SWIPT system, we define $\text{SNR}(P, \rho)$, $R(P, \rho)$ and $Q(P, \rho)$, i.e. (6)–(8), as SNR, achievable information rate and harvested energy, respectively.

$$\text{SNR}(P, \rho) = \frac{\rho p_{\text{sig}}}{\rho \sigma_{\text{A}}^2 + \sigma_{\text{cov}}^2} = \frac{\rho P \left| \sum_{m=1}^M g_m[L] * h_m[L] \right|^2}{\rho \sigma_{\text{A}}^2 + \sigma_{\text{cov}}^2} \quad (6)$$

$$R(P, \rho) = B \log_2 \left(1 + \left(\rho P \left| \sum_{m=1}^M g_m[L] * h_m[L] \right|^2 \right) / (\rho \sigma_{\text{A}}^2 + \sigma_{\text{cov}}^2) \right) \quad (7)$$

$$Q(P, \rho) = \zeta(1-\rho)(p_{\text{sig}} + \sigma_{\text{A}}^2) = \zeta(1-\rho) \left(P \left| \sum_{m=1}^M g_m[L] * h_m[L] \right|^2 + \sigma_{\text{A}}^2 \right) \quad (8)$$

where ζ represents the conversion efficiency of energy which is not the focus of this article, and for the convenience of simulation we set $\zeta = 0.8$, namely linear energy conversion. B denotes the system bandwidth. Note that the above performance indicators are all within the unit time.

Then, assume the circuit power consumption of TR-SWIPT system is P_{C} . As we use the harvested energy as compensation for P_{C} , the total energy expense of entire system $Q_{\text{total}}(P, \rho)$ is derived as Eq. (9), and EE η_e as Eq. (10).

$$Q_{\text{total}}(P, \rho) = P_C + P - Q(P, \rho) \quad (9)$$

$$\eta_e = \frac{R(P, \rho)}{Q_{\text{total}}(P, \rho)} \quad (10)$$

In Eq. (10), whether it is increasing $R(P, \rho)$ or reducing $Q_{\text{total}}(P, \rho)$ by the system, the EE can be increased. However, for practical reasons, the system can obtain a maximum EE only when constraint is satisfied. So, this problem P1 of EE can be formulated as

$$\max_{\rho, P} \eta_e = R(\rho, P)/Q_{\text{total}}(\rho, P) \quad (11)$$

$$\text{s.t. } R(\rho, P) \geq R_{\min} \quad (11a)$$

$$Q(\rho, P) \geq Q_{\min} \quad (11b)$$

$$0 \leq P \leq P_{\max} \quad (11c)$$

$$0 \leq \rho \leq 1 \quad (11d)$$

where R_{\min} is the minimum achievable information rate requested data rate for the QoS, P_{\max} denotes the maximum transmit power for antenna, and Q_{\min} represents the minimum harvested energy.

3.2 Algorithm Design

After some mathematical analysis with Hessian matrix of Eq. (11), we can know that the P1 is a two-element fractional problem with joint transmit power allocation and power splitting, and a non-convex problem at the same time [23]. It is difficult to solve a non-convex problem, so we consider a parametric method as Eq. (12), as $F(\eta_e)$ is equal to zero, the optimal solution of Eq. (12) is also the optimal solution of problem P1, which is specifically proved in [23].

$$F(\eta_e) = \max_{\rho, P} \{R(\rho, P) - \eta_e^* Q_{\text{total}}(\rho, P)\} \quad (12)$$

where η_e^* is the global optimal solution to the problem.

So, the problem P1 can be converted into the problem P2 through above analysis, as shown in the formula (13).

$$\max_{\rho, P} R(\rho, P) - \eta_e Q_{\text{total}}(\rho, P) \quad (13)$$

$$\text{s.t. } R(\rho, P) \geq R_{\min} \quad (13a)$$

$$Q(\rho, P) \geq Q_{\min} \quad (13b)$$

$$0 \leq P \leq P_{\max} \tag{13c}$$

$$0 \leq \rho \leq 1 \tag{13d}$$

Continue to analyze the problem P2, we find that the Hessian matrix of Eq. (13) is always less than or equal to 0 as Eq. (14). In other words, it means that the objective function of P2 is concavity, and the in-Eq. (13a) also is a concavity constraint by the same way. Moreover, Eqs. (13b), (13c) and (13d) are linear, that's to say, the problem P2 can be seen as a convex problem, and can be solved by any convex optimization methods. Based on the above analysis, the key point of solving the problem P2 is that the value of η_e^* is not known, and fortunately the literature [24] proposes a method of solving the optimal value by iteratively updating η_e .

$$\partial^2[R(\rho, P) - \eta_e Q_{\text{total}}(\rho, P)]/\partial P^2 \leq 0 \tag{14}$$

CVX is a modeling system for constructing and solving convex optimization, so we build an optimization algorithm based on CVX and iteratively updating algorithm [24], the specific algorithm is shown in Table 1, where k is the search precision, and n represents the number of iterations.

Table 1. EE algorithm

Optimization algorithm for EE

- (1) System initialization, set $\eta_e = 0$ and $n = 1$, assume δ is a small enough positive number, and $S \in \mathbb{C}^{(1/k) \times 3}$ is a zero matrix.
- (2) Loop $\rho = 0 : k : 1$
 - If The constraints of P2 are satisfied
 - (a) Use CVX to find the optimal solution of $F(\eta_e)^\#, \rho^\#$ and $P^\#$,
 - (b) Store the optimal solution of $F(\eta_e)^\#, \rho^\#$ and $P^\#$ in the n th line of S.
 - End if. End loop.
- (3) Search for the largest $F(\eta_e)$ in the first column of S, and get the corresponding ρ_L and P_L .
- (4) If $R(\rho_L, P_L) - \eta_e Q_{\text{total}}(\rho_L, P_L) > \delta$
 - (a) Get $\eta_e = R(\rho_L, P_L) / Q_{\text{total}}(\rho_L, P_L)$,
 - (b) Set $n = n + 1$, and reset S to zero matrix. Then go back to step (2).
- Else
 - Get the optimal solution and end algorithm. End if.

4 Numerical Results

In this section, we numerically evaluate the performance of optimization algorithm for EE in link level simulation platform. The general simulation parameters are shown in Table 2.

Table 2. Simulation parameters

Simulation parameters	Values
Channel bandwidth B	10 MHz
Threshold of end δ	0.001
Circuit power consumption P_C	25 W
Minimum harvested energy Q_{\min}	20 dBm
Delay spread	14 μ s

In Fig. 2, we can observe that the performance of EE can achieve improvement at high SNR region for $k = 0.01$ compared to $k = 0.1$, $k = 0.2$ and $k = 0.02$, this is because the optimal solution may be hidden in the unsearched value of ρ . Therefore, the more comprehensive the search value k , the larger the value of EE. However, when the SNR is small, the search accuracy has little effect on EE. In addition, with the increase of SNR, $R(\rho, P)$ is more dominant than $Q_{\text{total}}(\rho, P)$, thus improving the EE of system.

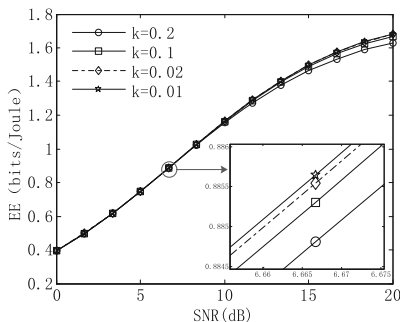


Fig. 2. Effect of SNR and search accuracy on EE when $R_{\min} = 100$ kbps, $P_{\max} = 30$ dBm, and $P_{\text{cov}} = -10$ dBm

In Fig. 3, we change the antennas number M and multi-paths number L , and then calculate the value of EE correspondingly. As M and L increase, EE of the system increases also. From the formula analysis, when M and L increase, the ideal signal

power $\sum_{m=1}^M g_m[L] * h_m[L]$ to be split is increasing, thereby further increasing the factor of system energy efficiency.

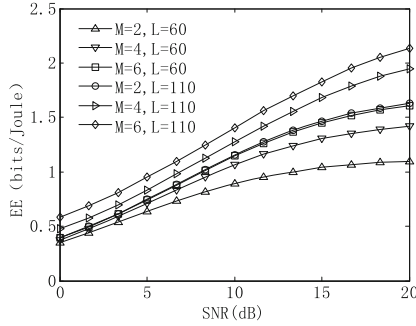


Fig. 3. Effect of antennas number and multi-paths number on EE when $R_{\min} = 120$ kbps, $k = 0.1$, $P_{\max} = 30$ dBm, and $P_{\text{cov}} = -20$ dBm

In Fig. 4, we compare the EE performance with three systems, namely conventional SWIPT system [1], non-optimized TR-SWIPT system [21] and optimized TR-SWIPT system. In more detail, as in the lower SNR, the ideal signal power is equal to the noise power, so the constraint of problem P2 is not satisfied, resulting the energy efficiency is zero. Furthermore, when problem P2 is feasible, the EE performance is better than the other two systems. It is worth noting that no matter how the SNR changes, the non-optimized TR-SWIPT system is always superior to conventional SWIPT, which indicating that the TR itself has good optimization performance.

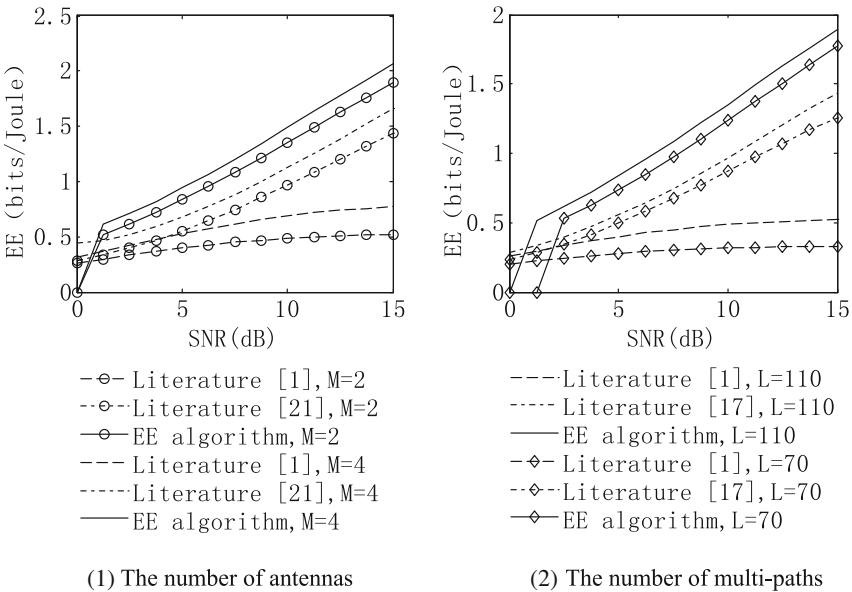


Fig. 4. Comparison of EE of different transmission schemes when $R_{\min} = 120$ kbps, $k = 0.1$, $P_{\max} = 30$ dBm, and $P_{\text{cov}} = -20$ dBm

5 Conclusion

This paper considers a point-to-point transmission communication system of broadband TR-SWIPT. Firstly, the specific communication process is analyzed in frequency selective fading channel, then, the mathematical model of TR-SWIPT system is established and the EE expression is also derived, finally, a non-convex fraction of maximize EE is converted into convex by function transformation and an EE optimization algorithm by jointly transmit power allocation and power splitting is proposed. From the simulation results, some conclusions can be drawn as below

- (1) The value of EE will increase as the SNR, number of antennas, and number of multi-paths increase.
- (2) In the higher SNR area, the search accuracy has a greater impact on energy efficiency.
- (3) TR can improve the EE of SWIPT, therefore, the optimization algorithm proposed in this paper performs better than conventional SWIPT.

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