Research on Energy Efficiency in Wireless Powered Communication Network with User Cooperative Relay

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Abstract. A user collaborative relay wireless powered communication network (UCR-WPCN) is studied in this paper, where users can harvest energy from the dedicated power device, named hybrid access point (HAP), and then transmit the information to HAP. Our goal is to study the total energy efficiency (EE) maximization of users in UCR-WPCN via joint time allocation and power control while meeting minimum rate requirements. However, because this problem is a non-convex, it is difficult for us to solve it. Then, we can use fractional programming principle theory and variable substitution to convert it into a standard convex optimization problem. Finally, we proposed an efficient optimization iterative algorithm in order to find the optimal solution. The simulation results show that the UCR plan can improve the user's information transmission rate and significantly improve the user's total energy efficiency in the system, compared with the non-cooperative relay transmission scheme.

Keywords: Wireless powered communication network · Energy efficiency · Harvesting energy · Optimization iterative algorithm

1 Introduction

With the rapid development of wireless communication, 5G, as the next generation of communication systems, will provide ubiquitous connectivity services for unprecedented devices. It is predicted that by 2020, there will be more than 50 billion connected devices [1], and the installed capacity of global Internet of Things (IoT) devices will reach 28.1 billion. The emergence of 5G networks has revolutionized the development of the IoT. The IoT will achieve ultra-low latency, efficient connectivity, low cost, low power consumption, high reliability, and full geographical coverage, which will completely reshape and change the world. Traditionally, energy constrained wireless networks (such as sensor networks) are powered by fixed sources such as batteries. However, due to the limitation of power, the operation time of the network is limited. Although the battery can be replaced or recharged to extend the life of the network, frequent battery charging is not only waste of resources, but also inconvenient (such as environmental monitoring, forest fire prevention, etc.), expensive, dangerous (such as in the toxic environment), even impossible [2] (such as sensor implanted in the

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human body). However, as an emerging green communication technology, energy harvesting can harvest energy from the surrounding environment to extend the service life of energy-limited communication networks [3]. Traditional natural renewable resources (such as solar and wind energy) can provide a green and renewable energy supply for wireless communication systems. However, those resources are indirect and uncontrollable, which makes the wireless devices not collect energy efficiently. Compared with the traditional natural renewable resources, radio frequency (RF) energy harvesting technology will be more stable and controllable. According to reports, using the Powercast RF energy harvester at 915 MHz is possible to acquire 3.5 mW and 1 μW of wireless power from 0.6 m and 11 m RF signals respectively [4]. Wireless devices can harvest energy from RF signals generated by special equipment, which is also known as wireless energy transfer [5, 6].

There are two different research lines about WET. One is simultaneous wireless information and power transfer (SWIPT). The information received by the wireless device is divided into two parts. One is for information decoding and the other for harvesting energy [7–9]. In [7, 8], it is assumed that an ideal SWIPT receiver can acquire energy and receive information from the same signal. In order to make SWIPT feasible, several practical receiver architectures have been proposed in [9]. SWIPT has been extensively studied in various wireless systems [10–12]. The other is wireless powered communication networks (WPCN). This system is divided into two phases, wireless energy transfer (WET) and wireless information transfer (WIT), where the wireless devices are first powered in WET and then use the harvested energy to transmit data signals in WIT [13–17]. The maximum weighted rate summation problem in the relay system is studied in [13] and [14]. The rate of the relay node is treated as an optimization target in [15]. [16] investigates the maximum system rate problem in cognitive radio environment. And the authors in [17] study the maximum system rates problem by optimizing time allocation. However, the purpose of all those work focus on the maximization of the system rate and ignores the system's energy consumption. However, in the next generation communication network (5G), the energy consumption of communication system is a very critical problem. Energy efficiency (EE), measured in bit/s per joule, has been gradually becoming an important metric for future communication system design under the rapid growth of energy consumption and significant carbon emission in the existing system [18]. In addition, the attenuation of the signal is related to the transmission distance. Therefore, the problem of "doubly near-far" is studied in [17], i.e. compared the users who are close to the special base station that can transmit wireless energy and receive information, the users that are far away the special base station can not only collect less energy in the WET stage and consume more power in order to ensure reliable transmission of information in the WIT phase, which results in unfairness between users. And this problem is improved by maximizing the constraint rate with increasing the same rate constraint for all users. Based on [17, 19] adopts the separation form of base stations (i.e., the special base station that combines the dual functions of the energy transmission and the signal reception is separated into two base stations with single functions, and assume that the two base stations are distributed in both sides of the user), which can avoid the "doubly near-far" problem by the way of distance balancing (i.e., if a user is close to the signal receiving base station, it will far away from the wireless energy harvesting base station, and the reverse is also true) and studies the maximization of the system energy efficiency. But compared to [17], the system model does not have a breakthrough improvement.

The main contributions of this paper are summarized as follows.

In order to solve the problem of "doubly-near-far", we propose a plan, named user cooperative relay (UCR). Based on UCR plan, we formulate the energy efficiency maximization problem of users in WPCN system by jointly optimizing time and power allocation while taking into the account the minimum rate requirements of each user. Meanwhile, the system model takes into account the circuit energy consumption of the user terminals, which is more realistic.

The optimization problem is a fractional structure, which is a non-convex problem. Hence, it is difficult to solve this problem. Thus, we can use fractional programming principle theory and variable substitution to convert it into a standard convex optimization problem.

Using the optimal structure of time allocation and power control, an optimization iterative algorithm is proposed to solve the optimization problem. And the simulation results are compared with the benchmark scheme, i.e. no cooperative relay system.

The rest of this paper is organized as follows. The system model is described in Sect. 2. The Sect. 3 presents the problem formulation and transformation. The Sect. 4 shows the simulation results. Finally, the Sect. 5 concludes the paper.

2 System Model

2.1 System Rate Model

As is shown in Fig. 1, we formulate a user collaborative relay WPCN system (UCR-WPCN), which consists of one hybrid access point (HAP) and two users denoted by U_1 and U_2. In this model, HAP has a stable power supply. In order to simplify the model,

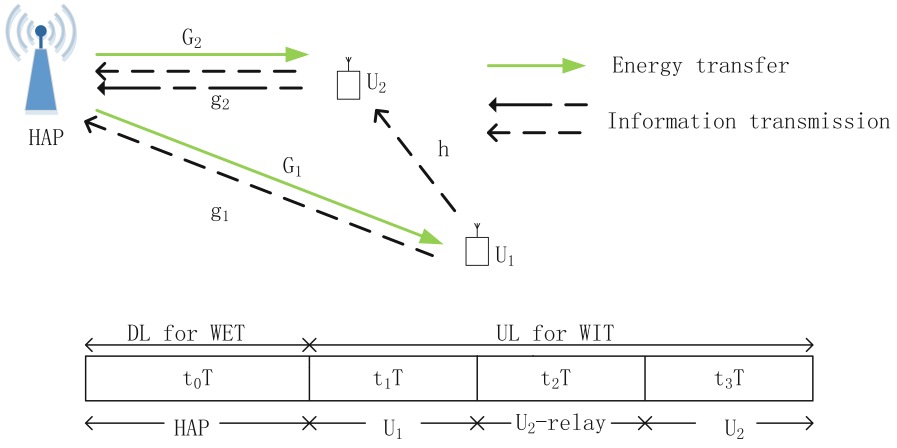

Fig. 1. System model of UCR-WPCN.

it is assumed that HAP and all users have a single antenna in the UCR-WPCN. Meanwhile, Time Division Multiple Access (TDMA) technology is also used in the same frequency band. Similar to [17], the "harvest and then transmit" protocol is adopted in UCR-WPCN. That is to say, the entire communication process is divided into two parts: wireless energy transmission (WET) and wireless information transmission (WIT). In the WET stage, all users collect energy from the radio frequency signals broadcast by HAP, and convert the collected electromagnetic energy into electricity and storage using the conversion circuit. In the WIT stage, all users use the energy collected during the WET phase to transmit its information to HAP. This phase is divided into three separate parts. First, U_1 broadcasts information to HAP and U_2. Secondly, U_2 decodes U_1 information, amplifies and forwards the HAP. Finally, U_2 transmits his own information to HAP. Thus, the total time can be limited as

$$\sum_{k=0}^{3} t_k \le T, \quad k = 0, 1, 2, 3. \tag{1}$$

In the WET and WIT phases, we assume all channels are quasi-static block fading channels. In WET, the channel power gain between the users of U1 and U2 and HAP is expressed as G_1 and G_2, respectively. In WIT, the channel gain factor between the users of U_1 and U_2 and HAP is expressed as g_1 and g_2 respectively. And the channel gain factor of U_1 and U_2 can be expressed as h. All the channel gain factors represent the characteristics of channel path loss, shadow effect and multipath fading. It is assumed that Channel State Information (CSI) is known by UCR-WPCN and the total time T is limited to 1 s.

In the WET stage, HAP broadcast radio frequency signals for a time duration t_0 at transmit power P_{HAP}. Due to the noise power is far less than the power of received signal, and the transmission power of the users is less than the transmission power of HAP, therefore, we assume that the energy can be collected by users from the channel noise and each other can be negligible [19].

Therefore, the energy collected by the users in the WET stage can be expressed as

$$E_k^h = \eta_k t_0 T G_k^2 P_{HAP}, \quad k = 1, 2. \tag{2}$$

Where $\eta_k \in (0, 1]$ is the energy conversion efficiency of U_i. Without loss of generality, we assume that $\eta_1 = \eta_2 = \eta$.

At WIT stage, each user independently transmits information for a time duration t_k at transmit power P_k with the TDMA technology.

The information received by HAP from U_1 is expressed as

$$y_1 = \sqrt{P_1} g_1 x_1 + n_1 \tag{3}$$

The information received by HAP from U_1 is expressed as

$$y_2 = \sqrt{P_1} h x_1 + n_2 \tag{4}$$

The information received by HAP from U_2 forwarding U_1 is expressed as

$$y_3 = \sqrt{P_2^1} g_2 \overline{x_1} + n_3 \tag{5}$$

The information received by HAP from U_2 is expressed as

$$y_4 = \sqrt{P_2^2} g_2 x_2 + n_4 \tag{6}$$

Wherein, $x_1,\ \overline{x_1},\ x_2$ are the information signal sent by U_1, the signal of U_2 relay U_1 and the signal sent by U_2 to transmit its own information. P_1, P_2^1 and P_2^2 are the transmission power of U_1, the signal power of U_2 forwarding U_1, and the power of U_2 to transmit its own information signal. $n_1 \sim n_4$ represent the Additive white Gaussian noise at the end of HAP and U_2, without loss of generality, here we assume $n_i \sim CN(0, N_0), i = 1, \ldots, 4$

Therefore, the achievable rate of U1 end-to-end is expressed as [20]

$$R_1 = \min\left\{R_{direct},\ R_{realy}\right\} \tag{7}$$

R_{direct} represents the total information rate of U_1 to HAP and U_2 forwarding U_1 information to HAP, which can be expressed as

$$R_{direct} = t_1 \log_2(1 + \frac{P_1 \|g_1\|^2}{\sigma_1^2}) + t_2 \log_2(1 + \frac{P_2^1 \|g_2\|^2}{\sigma_3^2}) \tag{8}$$

R_{relay} represents the information rate from U_1 to U_2

$$R_{relay} = t_1 \log_2(1 + \frac{P_1 \|h\|^2}{\sigma_2^2}) \tag{9}$$

The achievable rate of U_2 end-to-end is expressed as

$$R_2 = t_3 \log_2(1 + \frac{P_2^2 \|g_2\|^2}{\sigma_4^2}) \tag{10}$$

Therefore, the total throughput of the UCR-WPCN is expressed as

$$R = R_1 + R_2 \tag{11}$$

2.2 Power Consumption Model of Users

The main concern about wireless sensor networks and the Internet is low power devices. Therefore, this section focuses on the total energy consumption of users in the UCR-WPCN.

The user's energy consumption is divided into two parts, transmission power consumption and circuit consumption during hardware processing.

At WIT stage, each user independently transmits information for a time duration t_k at transmit power P_k. Therefore, the energy consumption of U_1 and U_2 can be respective modeled as

$$E_1 = (P_1 + P_{1c})t_1 \tag{12}$$

$$E_2 = (P_2^1 + P_{2c})t_2 + (P_2^2 + P_{2c})t_3 \tag{13}$$

Wherein, P_{1c} and P_{2c} are the circuit consumption of U_1 and U_2, respectively. According to the law of conservation of energy, we can obtain

$$E_k \leq E_k^h, \quad k = 1, 2. \tag{14}$$

Thus, the total energy consumption of the whole users system can be model as

$$E = E_1 + E_2 \tag{15}$$

3 The EE Model of the UCR-WPCN

3.1 The Optimization Model

The goal of this section is to maximize the EE of users. EE can be defined as the ratio of the total throughput to the total energy consumption of UCR-WPCN, i.e.EE = R/E. In order to maximize the EE, the time distribution and power control are jointly optimized. The EE model is defined as

$$\begin{aligned}
&\max_{t,P} : \frac{R}{E} \\
&\text{S.t. c1: } t_0 + t_1 + t_2 + t_3 \leq T \\
&\quad \text{c2: } (P_1 + P_{1c})t_1 \leq E_1 \\
&\quad \text{c3: } (P_2^1 + P_{2c})t_2 + (P_2^2 + P_{2c})t_3 \leq E_2 \\
&\quad \text{c4:} \{R_1, R_2\} \geq R_{min} \\
&\quad \text{c5: } t_0,\ t_1,\ t_2, t_3, P_1, P_2^1, P_2^2 \geq 0
\end{aligned} \tag{16}$$

Wherein, $t = [t_0,\ t_1,\ t_2,\ t_3], P = [P_1, P_2^1, P_2^2]$.

In problem (16), c1 means the transmission time limit. c2, c3 ensures that energy consumption of users in the WIT phase does not exceed the harvested energy in the

WET phase respectively. c4 meets the minimum rate constraint of users. And c5 represents the non-negative constraints of time allocation and power control.

Note that problem (16) is neither convex nor quasi-convex, because the objective function is the fractional form and the coupled optimization variables is contained in c2, c3 and c4. In order to solve this problem effectively and quickly, in the following section, the principle of fractional programming and variable substitution is used to transform it into a standard convex problem.

3.2 The Transformation of the Objective Function

According to the principle of nonlinear fractional programming [21], the objective function of the problem (16) can be expressed as

$$q^{*} = \max \frac{R}{E} \tag{17}$$

The equivalent form of the objective function is as follows

$$F(q^{*}) = \max\{R - q^{*}E\} = 0 \tag{18}$$

Compared with the problem (16), the equivalent form of the objective function (18) is more handleable. But, it is still a non-convex problem, because there still contain the coupled optimization variables.

In order to solve the optimization problem (16), we introduce the auxiliary variables, which represents users' power consumption, i.e. $\gamma_1 = P_1 t_1$, $\gamma_2 = P_2^1 t_2, \gamma_3 = P_2^2 t_2$.

Therefore, (8) is further translated into

$$R_{driect}^{'} = t_1 \log_2(1 + \frac{\gamma_1 \|g_1\|^2}{t_1 \sigma_1^2}) + t_2 \log_2(1 + \frac{\gamma_2 \|g_2\|^2}{t_2 \sigma_3^2}) \tag{19}$$

(9) can further translate into

$$R_{relay}^{'} = t_1 \log_2(1 + \frac{\gamma_1 \|h\|^2}{t_1 \sigma_2^2}) \tag{20}$$

(7) can be rewritten as

$$R_1^{'} = \min\left\{R_{direct}^{'}, R_{realy}^{'}\right\} \tag{21}$$

(10) changes into

$$R_2^{'} = t_3 \log_2(1 + \frac{\gamma_3 \|g_2\|^2}{t_3 \sigma_4^2}) \tag{22}$$

(11) transforms into

$$R^{'} = R_1^{'} + R_2^{'} \tag{23}$$

Therefore, problem (16) can be reformulated as

$$\begin{aligned}
&\max_{t,\gamma} : \ R^{'} - q(\gamma_1 + P_{1c}t_1 + \gamma_2 + P_{2c}t_2 + \gamma_3 + P_{2c}t_3) \\
&\text{S.t. c1: } t_0 + t_1 + t_2 + t_3 \le T \\
&\quad \text{c2: } \gamma_1 + P_{1c}t_1 \le E_1 \\
&\quad \text{c3: } \gamma_2 + P_{2c}t_2 + \gamma_3 + P_{2c}t_3 \le E_2 \\
&\quad \text{c4:} \left\{R_1^{'}, R_2^{'}\right\} \ge R_{\min} \\
&\quad \text{c5: } t_0,\ t_1,\ t_2, t_3, \gamma_1, \gamma_2, \gamma_3 \ge 0
\end{aligned} \tag{24}$$

It is not difficult to find the problem (24) is a standard convex optimization problem.

Proof: Firstly, we can define a function like $f(x) = \log_2(1+x), x \ge 0$

We can find $g(x,y) = yf(x/y), y > 0$ is the perspective functions of the concave function. According to the convexity of the convex function, the optimization objective function in question (24) is also a concave function.

Furthermore, c1, c2, c3 are linear function. Therefore, problem (24) is a standard convex optimization problem [22].

3.3 Optimization Iterative Algorithm

In order to solve the problem (24) quickly and effectively, we propose an optimization iterative algorithm in this section. The algorithm is summarized in Table 1.

Table 1. Optimization iterative algorithm for UCR-WPCN.

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Algorithm
1: initialize $q$ , $t$ , $\gamma$ and the maximum tolerance $\varepsilon$ ;
2: obtain $t^*$ , $\gamma^*$ from (24)
3: obtain $F(q^*)$ from (18)
4: compare $F(q^*)$ with $\varepsilon$
   if $F(q^*) \le \varepsilon$
       obtain $q^*$ from (17)
       jump to 6;
   elseif
       jump to 5;
   end
5: obtain and update $q$ from (17), then, jump to 2;
6: break and output $q^*$ ;
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4 Simulation Results

In this section, we present simulation results by a series of numerical experiments to validate our theoretical findings, and to demonstrate the system EE of UCR-WPCN. The distance between U_1, U_2 and HAP is 15 m and 10 m respectively, and the distance between U_1 and U_2 is 6 m. All channels obey the Rayleigh distribution $CN(0, 10^{-2}d_{x,y}^{-\alpha})$, α is the channel fading coefficient and is set to 3, $d_{x,y}$ represents the distance between nodes x, y. According to the mutuality of the channels, $G_1 = g_1$ and $G_2 = g_2$ can be obtained. The HAP transmission power is set to 40 W. The conversion efficiency is set to 0.9, the maximum tolerance is set to 0.01. The circuit energy consumption of HAP, U_1 and U_2 are set to 0.05 W, 0.5 mW and 0.5 mW respectively, and the minimum rate per unit bandwidth is 0.2bits/Hz.

Figure 2 reflects the relationship between the number of iteration steps of the proposed Algorithm 1 and the total EE of the user in the case of satisfying different minimum rate constraints of the user in the UCR-WPCN. It can be observed from the figure, on average at most three iterations are needed to reach the optimal solution, which reflects the fast convergence of the proposed algorithm.

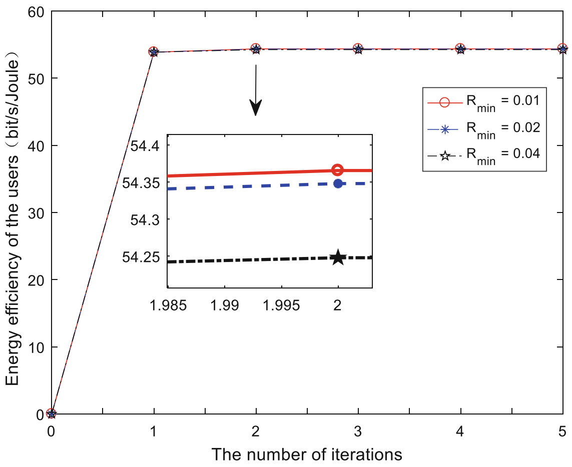

Fig. 2. EE of the users versus the number of iteration

It can be clearly seen from Fig. 3 that the energy harvesting time t_0 required by the users is decreasing with the HAP transmission power increases. Therefore, users can enjoy more information transmission time, which proves the correctness of the mode built in this paper.

As is shown in Fig. 4, under the same rate constraint conditions, the total rates in the relay system is greater than in the non-relay system. When the HAP is set to 40 W, the total rate of users in the relay system is approximately 3.5 times bigger than the total rate of the non-relay system users. Under different rate constraints, as the

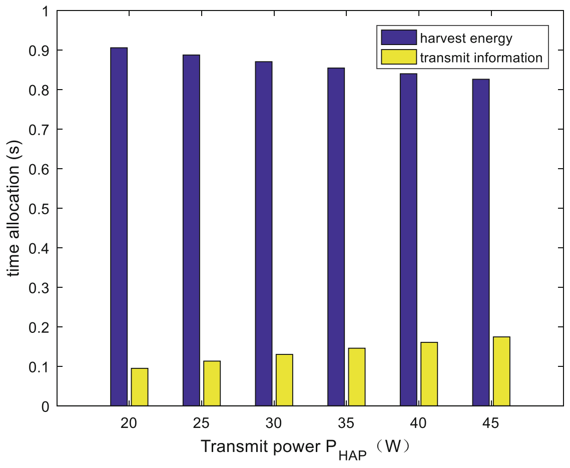

Fig. 3. The relationship between HAP and time allocation

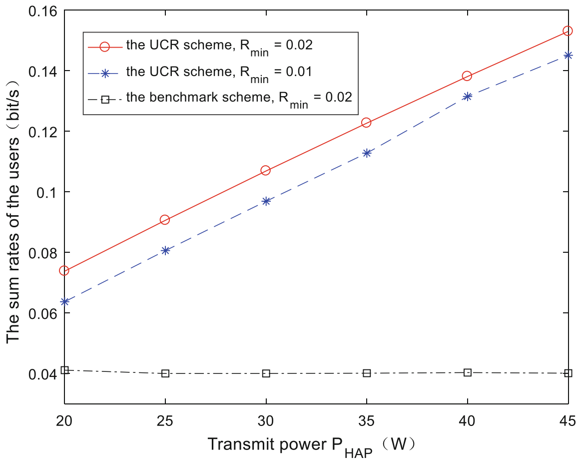

Fig. 4. The relationship between the sum rates of the users and HAP

minimum rate requirement of the users increases, the total rate of users in the relay system also increases accordingly. When the HAP is set to 40 W and the minimum rate doubled, the total rate of users under the relay system becomes about 1.07 times than the original.

It can be observed from Fig. 5 that the total energy efficiency of the users increase with the increase of the HAP transmission power. Because the HAP transmission power increase, the time required for the users to collect energy will decrease and the time for users to transmit information will increase, so that the information rate of the users will increase. In addition, under the same HAP transmission power, the EE of the users under the relay wireless energy communication network is always more than

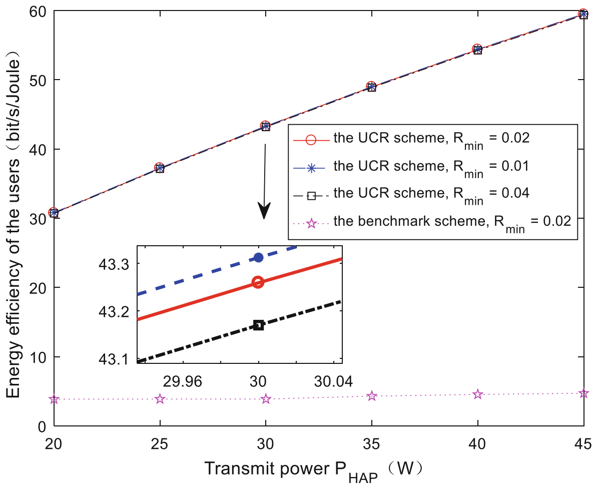

Fig. 5. The relationship between EE of the users and HAP

the non-relay system. When the HAP is set to 40 W and the minimum rate is set to 0.02 bit/s, the EE of users in the relay system is about 10.08 times bigger than without the relay system.

In combination with Figs. 4 and 5, it can be obtained that the form of user cooperative relay in the wireless energy communication network can not only significantly improve the total information rate of the users, but also the total energy efficiency of the users.

Figure 6 shows the relationship between EE of the users and channel fading coefficients in relay system. It can be observed from the figure that the energy efficiency

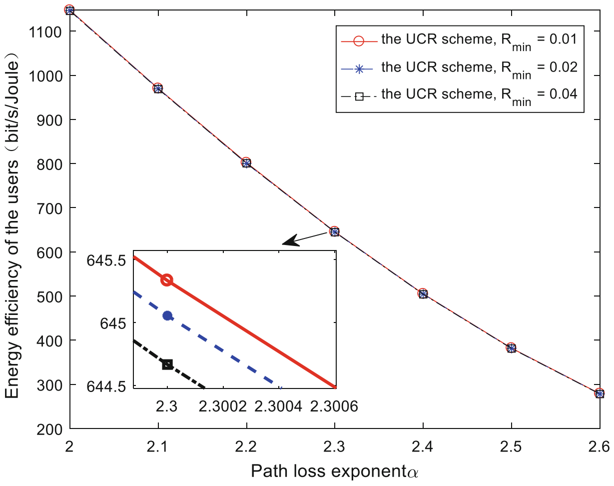

Fig. 6. The relationship between EE of the users and path loss exponent

of the users decreases as the channel fading coefficient increases. Because as the channel fading coefficient increases, more energy is consumed in the signal propagation, and the energy harvesting by the users will be reduced accordingly. In order to provide sufficient energy for the user to supply, the energy harvesting time in the WET stage will increase accordingly, resulting in a reduction in the time for users to use information communication, thereby reducing the information rate. In addition, it can also be observed from the figure that is in the case of the same channel fading coefficient, the EE also exhibits a decreasing condition as the minimum rate requirement increases.

5 Conclusion

Aiming at the "doubly-near-far" problem in WPCN, we propose an efficient user cooperative relay transmission scheme called UCR transmission scheme. In the WIT phase, the scheme can ensure the user closer to the HAP as the relay to cooperate and forward the information of the relatively distant user to the HAP in order to improve the communication quality of the remote users. Based on this scheme, the user energy efficiency maximization problem with joint optimization time allocation and power control resources is modeled under the condition of satisfying the minimum rate constraint of the user. And an effective optimization iterative algorithm is proposed to solve this problem. The simulation results show that the UCR scheme proposed in this paper improves not only the total information rate of users in the system, but also the total energy efficiency of users, compared with the non-cooperative transmission scheme. This enjoys important application value and significance in future communication systems, especially Internet of Things and wireless sensor networks.

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