Outage Analysis of MIMO-NOMA Relay System with User Clustering and Beamforming Under Imperfect CSI in Nakagami-m Fading Channels

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Abstract. In this paper, we propose and analyze a downlink multiple-user MIMO-NOMA relay system where all users are grouped into several clusters. To mitigate the inter-cluster interference, the superposition signals at the base station are beamed at the relay nodes. Then, the relays communicate with the users through superposition coding in power domain. We derive the exact closed-form expressions of the outage probability of each user in every cluster for the cases of perfect successive-interference cancellation (SIC) and imperfect SIC. The outage performance is analyzed the over Nakagami-m fading channels, taking into account the imperfection of channel state information (CSI). All analysis results are compared with Monte-Carlo simulation result to verify the correctness of the derived mathematical expressions. The results show that the channel fading severity, channel estimation error, and the quality of SIC structure have strong influences on the outage performance of the proposed MIMO-NOMA relay system.

Keywords: Multiple-input multiple-output · Non-orthogonal multiple-access · User clustering · Beamforming · Successive interference cancellation · Outage probability

1 Introduction

The fifth generation (5G) networks will be deployed in 2020 to provide the Internet of Things (IoT) service. IoT basically connects people, processes, data, and

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every possible things together. The key challenge in IoT is to maintain reliable communication in the condition of the limited spectrum and low cost [1]. Meanwhile, the non-orthogonal multiple-access (NOMA) is considered as a promising multiple-access technique for 5G and beyond mobile networks due to its superior spectral efficiency [2–4].

The main feature of NOMA is to use the power domain and code domain for multiple access, which adopts the superposition code at the transmitter and successive interference cancellation (SIC) method at the receiver to detect the desired signals [5]. NOMA can be applied for both uplink and downlink. In the NOMA systems, less transmission power is allocated to users which have better channel conditions while more transmission power is allocated to users which have worse channel conditions [6–8]. The purpose of this strategy is to achieve a balance between the system throughput and the fairness among users [9]. On the other hand, the power allocation in NOMA systems can be based on the priority of users, i.e. users with higher priority are allocated more power while users with lower priority are allocated less power [10,11]. The disadvantage of [10,11] is that the users with lower priority will have higher interference in the same the channel conditions.

In the literatures, there have been many research works on analyzing the performance of NOMA in various scenarios. The authors of [12] investigated a NOMA downlink system where all users locate randomly. They derived the closed-form expressions of the outage probability as well as the ergodic capacity. In [13], both downlink and uplink of NOMA systems were studied. The dynamic power allocation with undertaking QoS for different users was proposed to provide more performance fairness. Since the NOMA enhances the bandwidth efficiency and throughput, it is suitable for the multiple-user systems. The authors of [14] proposed and analyzed a multi-beam multiple-input single-output (MISO)-NOMA system. Unfortunately, the authors assumed that each cluster had only two users, namely near user and far user. A NOMA system under the condition of user quality fairness was studied in [15]. The authors concluded that it is important to allocate power according to the channel gains. In [16], the impact of user pairing on the performance of two NOMA systems, i.e. NOMA with fixed power allocation (F-NOMA) and cognitive-radio-inspired NOMA (CR-NOMA), was characterized. The power allocation coefficient was chosen to satisfy the predefined QoS requirements of users. In aforementioned works, the authors assumed that the transmitter was able to communicate with all users. However, in many cases it may not possible due to power limitation. Thus, the deployment of relay nodes is necessary. The authors of [17–19] proposed the multiple-user multiple-input-multiple-output (MIMO)-NOMA systems. The results showed that the performance is improved when multiple users are gathered into a cluster. However, the authors in these works studied the scenario where the base station (BS) directs beams towards users without having the CSI. In fact, the NOMA systems always require the accurate CSI to allocate power for users, but the variation in wireless environment may cause the imperfect CSI [20,21]. Moreover, when designing the transmission beamforming vectors, the BS needs to know the accurate CSI to precode the signals. Therefore, we are interested in examining the influence of the imperfect CSI on the performance of MIMO-NOMA downlink relay system.

Motivated by the above issues, in this paper we combine NOMA and beamforming into a downlink multiple-user MIMO relay system. This model is extremely useful in urban environment where several users may be blocked by high buildings or mountains. The contributions of our paper can be summarized as follows:

- We propose a downlink MIMO-NOMA relay system to improve the spectrum utilization efficiency and performance. We divide all users into subgroups to not only mitigate the extra-user interference but also reduce the complexity of the system. On the other hand, we use relays to forward signals to destination which helps to overcome the effect of channel fading and to significantly reduce the training sequences to estimate uplink CSI.
- We derive the exact closed-form expressions of the outage probability of each user in every clusters. For the practical purpose, we investigate the proposed MIMO-NOMA relay system over Nakagami fading channels with various fading levels.
- We analyze the system in the case of imperfect CSI which is caused by the channel estimation error. The results show that the outage performance is reduced remarkably under the influence of imperfect CSI. All analysis results are compared with simulation results to confirm the correctness of the derived mathematical expressions.

The rest of the paper is organized as follows. Section 2 presents the system model of the proposed MIMO-NOMA relay system with beamforming. Mathematical analysis of the outage probability of the proposed system is given in detail in Sect. 3. Numerical results are presented in Sect. 4. Finally, Sect. 5 concludes the paper.

2 System Model

We consider a downlink multiple-user relay system as shown in Fig. 1. In this system model, N users (D_n, $n = 1, 2, 3, ..., N$) are grouped into L clusters according to their spatial positions which are obtained by using the spatial direction methods such as Global Positioning System (GPS) technique or user location tracking algorithms. We denote $D_{l,n}$ as the nth user in the lth cluster. There are L relays (R) which are deployed according to the geographical location of the clusters to assist the BS in forwarding signals to users. The direct link from BS to each user in the groups is assumed not available because the distance from BS to users is larger than of of the covering area of BS or because of deep shadow fading. Moreover, the system operates in half-duplex mode, i.e. the communication from BS to users takes two time slots. To improve the spectral usage efficiency, the NOMA method is used for users in each cluster. Particularly, the signals from all users in a cluster is superposed at the BS, then they are decoded by SIC algorithms at the relays and users to reduce the interference from other signals to the desired signal. Moreover, the users in each cluster have different propagation distances, thus the channel gains between the relay and users are different. The channel state information (CSI) is imperfect. Based on these properties, it is possible to do the signal superposition in power domain.

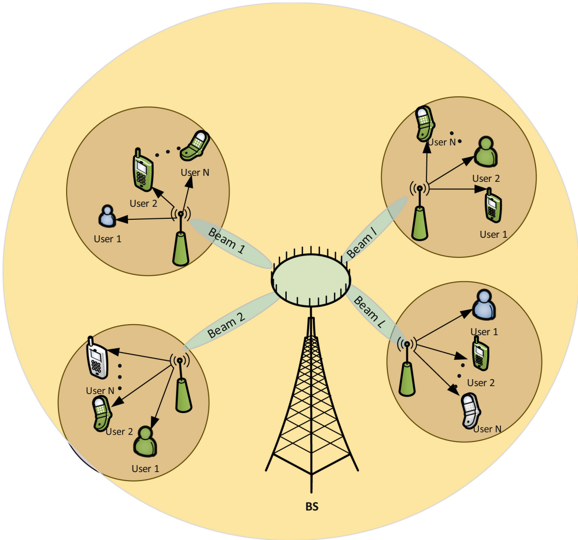

Fig. 1. System model of the proposed multiple-user MIMO-NOMA relay system with beamforming.

Regarding to the antenna configuration, the BS and each relay are equipped with N_t and N_r antennas, respectively. Meanwhile, the users only employ single antennas. This antenna configuration is absolutely reasonable because the BS and relay are big enough and have fixed locations to ensure the uncorrelated electromagnetic coupling between antennas. In this paper, we set the number of antennas of BS less than or equal to the number of antennas of relay because when BS has more antennas than relay, different approaches need to be used to implement MIMO-NOMA[1].

2.1 CSI Requests and Channel Model

To do beamforming and power allocation, the BS needs to know the statistical CSI. It is assumed that CSI is available at the BS because at the beginning of each time slot the relays send symbol pilots to the BS to estimate the uplink

[1] For example, one possible way is to allocate different beamforming vector to each user, then the precoding matrices at the base station can be optimized by taking user fairness into consideration.

channel. Since the BS and relay are imperfectly synchronized, the pilot signals of all relay transmissions to the BS are non-orthogonal and imperfect. Thus, perfect CSI at the BS is very difficult to be obtained due to two reasons, i.e. channel estimation error and feedback delay. It is assumed that the minimum mean squared error (MMSE) estimation is used [22]. Then, the relation between the estimated channel matrix $\hat{\mathbf{h}}_l$ and the actual channel matrix $\mathbf{h}_l$ can be given by

$$\mathbf{h}_l = \hat{\mathbf{h}}_l + \mathbf{e}_l, \tag{1}$$

where $\mathbf{e}_l \sim \mathcal{CN}(0, \sigma^2_{\mathrm{SR}_l}\mathbf{I}_l)$ is the channel estimation error vector with i.i.d. zero mean and unit variance complex Gaussian distributed elements.

To allocate power for users, the relays also estimate the CSIs of the links from relays to users. The relation between the estimated channel coefficient $\hat{g}_n$ and the actual channel coefficient g_n can be expressed as

$$g_n = \hat{g}_n + e_n, \tag{2}$$

where e_n denotes the error of the channel between R_l and D_n.

Moreover, $\hat{\varOmega}_{\mathcal{A}} = \varOmega_{\mathcal{A}} - \sigma^2_{e_{\mathcal{A}}}$ with $\mathcal{A} \in \{\mathrm{SR}_m, \mathrm{RD}_n\}$ are the normalized channel gains of $\hat{\mathbf{h}}_l$ and g_n. We should note that the elements of $\hat{\mathbf{h}}_l$, $\hat{g}_i$ are statistically independent of $\mathbf{e}_l$ and e_n. Denote $\rho_{\mathcal{A}}$, $0 \leq \rho_{\mathcal{A}} \leq 1$, as the estimation error coefficient which indicates the difference between the channel with estimation error and the perfect channel. Then, the normalized variance of the estimation error is $\sigma^2_{e_{\mathcal{A}}} = \rho_{\mathcal{A}}\varOmega_{\mathcal{A}}$ and the of estimation channel error is $\hat{\varOmega}_{\mathcal{A}} = (1-\rho_{\mathcal{A}})\varOmega_{\mathcal{A}}$.

To balance the implementation complexity and the system performance, we use zero-force beamforming (ZFBF) at the BS. We design a weight $\mathbf{w}_l \in \mathbb{C}^{N_t \times L}$ for the lth cluster to mitigate the interference from other clusters. $\mathbf{w}_l$ can be represented as the projection of $\mathbf{h}_l$ in the null space of the lth cluster interference channels so that the channel gain is maximized and the inter-cluster interference is canceled. Mathematically, $\mathbf{w}_l$ is computed as

$$\mathbf{w}_l = \frac{\varPi_l \mathbf{h}_l}{\|\varPi_l \mathbf{h}_l\|}, \tag{3}$$

where $\varPi_l = \mathbf{I}_N - \mathbf{H}_l(\mathbf{H}_l^H\mathbf{H}_l)^{-1}\mathbf{H}_l^H$ and $\mathbf{H}_l$ is

$$\mathbf{H}_l = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{l-1}, \mathbf{h}_{l+1}, \cdots, \mathbf{h}_L]^T, \tag{4}$$

$\mathbf{H}_l$ is an extended channel matrix which excludes only $\mathbf{h}_l^H$. $[\cdots]^T$ and $(\cdot)^H$ represent the transpose matrix and the conjugate transpose, respectively. The signals can be transmitted to lth relay if the null space of $\mathbf{H}_l$ has one greater-than-zero dimension. Thus, we have

$$\mathbf{h}_l^H \mathbf{w}_j = 0, \forall l \neq j. \tag{5}$$

In other words, the relay must completely cancel the interference of the first hop. Moreover, it is required that the size of the matrix for beamforming design is not too large and the computational complexity is bearable[2].

[2] This system can be extended to massive MIMO-NOMA.

The channel matrix between the BS and lth relay is denoted as $\mathbf{h}_l$, where $\mathbf{h}_l = [h_{l,1}, h_{l,2}, \cdots, h_{l,k}, \cdots, h_{l,N_r}] \in \mathbb{C}^{N_r \times 1}$ with $l \in \{1, \cdots, L\}$ and $k \in \{1, \cdots, N_r\}$, $h_{l,k} \sim \mathcal{G}(m_1, m_1/\Omega_{l,k})$ refers to the element complex channel coefficient of the channel from the BS to lth relay, and $\mathbb{E}\{|h_{l,k}|^2\} = \Omega_{l,k}$ is the variance of channel gain. All channel coefficients follow i.n.i.d Nakagami-m distribution where m denotes the fading parameter, which models the scale of line-of-sight (LoS) and multipath scatters. Moreover, it is well-known that the square of Nakagami distribution random variable is Gamma distribution [23], where $\frac{1}{2} \le m = \frac{\mathbb{E}^2\{Z\}}{var\{Z\}} \le \infty$ is the inverse of the normalized variance of Z with $Z \in \{|h_{l,k}|^2, |g_n|^2\}$.

2.2 Signal Model

A set of signals $\mathbf{x}_\mathrm{S} = [\mathbf{x}_{\mathrm{S},1}, \cdots, \mathbf{x}_{\mathrm{S},L}]^T \in \mathbb{C}^{L \times 1}$ is constructed at the BS and broadcasted to all relays. Then, it is multiplied by beamforming vector at the output antenna according to the principle of zero-forcing (ZF) method.

The superposition modulation in NOMA multiple antennas system consists of two steps, i.e. power allocation and beamforming. Every antenna of the BS transmits the superposition codebook, which includes N signals of the lth cluster and can be described as $\mathbf{w}_l \mathbf{x}_{\mathrm{S},l} = \mathbf{w}_l [x_{l,1}, \cdots, x_{l,N}]^T$. This superposition code book is taken from $\mathbf{x}_\mathrm{S}$, with $x_{l,n}$ is the signal of $D_{l,n}$, $\mathbb{E}\{|\mathbf{x}_{\mathrm{S},l}|^2\} = P_\mathrm{S}$.

Consequently, the signal which is transmitted to the lth cluster can be expressed as

$$\mathbf{x}_{\mathrm{S},l} = \mathbf{w}_l \sum\nolimits_{n=1}^{N} \sqrt{a_n P_\mathrm{S}} x_n, \tag{6}$$

where a_n and x_n denote the power allocation coefficient and signal of n-th user with $\sum_{n=1}^{N} a_n = 1$ and the normalization of $\mathbf{w}_l$, $\|\mathbf{w}_l\|^2 = 1$.

Then, the received signal at the relay R_l in the first time slot can be given by

$$\begin{aligned} y_{\mathrm{R}_l} &= \mathbf{h}_l^H \mathbf{w}_l \sum_{n=1}^{N} \sqrt{a_n P_\mathrm{S}} x_l + w_{l,n} \\ &= \underbrace{\mathbf{h}_l^H \mathbf{w}_l \sqrt{a_n P_\mathrm{S}} x_l}_{\text{desired signal of } (l,n)\text{-th user}} + \underbrace{\mathbf{h}_l^H \mathbf{w}_l \sum_{i=n+1}^{N} \sqrt{a_{l,i} P_\mathrm{S}} x_{l,i}}_{\text{interference of other users}} \\ &\quad + \underbrace{\mathbf{h}_l^H \mathbf{w}_l \sum\nolimits_{k=1}^{n-1} \sqrt{\xi_1 a_{l,k} P_\mathrm{S}} x_{l,k}}_{\text{interference of imperfect SIC}} + w_{l,n}, \end{aligned} \tag{7}$$

where $w_{l,n} \sim \mathcal{CN}(0, \sigma_l^2)$ is an i.i.d. additive white Gaussian noise (AWGN) at the lth relay. It should be noted that the term $\mathbf{h}_l^H \mathbf{w}_l \sum_{k=1}^{n-1} \sqrt{a_k P_\mathrm{S}} x_k$ in (7) equals to zero in the case of perfect SIC.

Denote the large-scale fading coefficient of n-th user by $\sqrt{d_n^{-\beta}}$, where d_n is the distance between relay and nth user and β is the path-loss factor. Moreover,

$\tilde{g}_n$ represents the small-scale fading coefficient of nth user, i.e. $g_n = \tilde{g}_n\sqrt{d_n^{-\beta}}$, then $g_n \sim \mathcal{G}(m_t, m_t/\Omega_{\mathrm{RD}_n})$ where $t \in \{2,3,4\}$, $\Omega_{\mathrm{RD}_n} = \mathbb{E}\{|g_n|^2\}$. The additive white Gaussian noise (AWGN) at D_n is $w_{\mathrm{D}_n} \sim \mathcal{CN}(0, N_0)$ with N_0 is the noise variance. All channel gains follow i.n.i.d. Rayleigh distribution. Without loss of generality, it is assumed that $d_1 > d_2, \cdots, > d_N$. Therefore, the channel gains are sorted according to an ascending order $|g_1|^2 <, \cdots, < |g_N|^2$.

During the second time slot, the relays re-encode and forward the messages to the users. Hence, the output signal at the SIC architecture of $\mathrm{D}_{l,n}$ is

$$y_{\mathrm{D}_{l,n}} = \underbrace{g_n\sqrt{b_n P_{\mathrm{R}}}x_l}_{\text{desired signal of } n\text{-th user}} + \underbrace{g_n \sum\nolimits_{i=n+1}^{N} \sqrt{b_i P_{\mathrm{R}}}x_i}_{\text{interference of other users}} + \underbrace{\sum\nolimits_{k=1}^{n-1}\sqrt{\xi_2 b_k P_{\mathrm{R}}}g_n x_k}_{\text{interference of imperfect SIC}} + w_{\mathrm{D}_n}. \tag{8}$$

At the relays, SIC is used to remove the interference from the signals of $(i = n+1)$th users, which have higher power. In the case of perfect SIC, the signal-to-interference-plus-noise ratio (SINR) of $\mathrm{D}_{l,n}$ at the relay is denoted by γ_{R_n}. From (7), the SINR is calculated as

$$\gamma_{\mathrm{R}_n} = \frac{P_{\mathrm{S}} a_n |\mathbf{h}_l^H \mathbf{w}_l|^2}{\sum_{i=n+1}^{N} P_{\mathrm{S}} a_i |\mathbf{h}_l^H \mathbf{w}_l|^2 + \sigma_{\mathrm{R}_n}^2}. \tag{9}$$

In the case of imperfect SIC, the term $\mathbf{h}_l^H \mathbf{w}_l \sum_{k=1}^{n-1}\sqrt{\xi_1 a_{l,k} P_{\mathrm{S}}}x_{l,k}$ in (7) is not equal to zero. It depends on the quality of SIC structure. As a result of this feature, the instantaneous SINR is

$$\gamma_{\mathrm{R}_n}^{\mathrm{ipSIC}} = \frac{P_{\mathrm{S}} a_n |\mathbf{h}_l^H \mathbf{w}_l|^2}{\sum\limits_{i=n+1}^{N} P_{\mathrm{S}} a_i |\mathbf{h}_l^H \mathbf{w}_l|^2 + |\mathbf{h}_l^H \mathbf{w}_l|^2 \xi_1 \sum\limits_{k=1}^{n-1} a_{l,k} P_{\mathrm{S}} + \sigma_{\mathrm{R}}^2}, \tag{10}$$

where ξ_1 represents the impact level of the residual interference at the relay.

From (8), the SINR of $\mathrm{D}_{l,n}$ when $b_i < b_n$ and under perfect SIC is given by

$$\gamma_{\mathrm{D}_n} = \frac{P_{\mathrm{R}} b_n |g_n|^2}{\sum_{i=n+1}^{N} b_i P_{\mathrm{R}} |g_n|^2 + \sigma_{\mathrm{D}_n}^2}. \tag{11}$$

When the signal power of $\mathrm{D}_{l,i}$ is larger than $\mathrm{D}_{l,n}$, i.e. $b_i > b_n$, the SINR for $\mathrm{D}_{l,n}$ to detect the signals of $\mathrm{D}_{l,i}$ can be expressed as

$$\gamma_{\mathrm{D}_{l,i}} = \begin{cases} \dfrac{P_{\mathrm{R}} b_i |g_n|^2}{\sum_{k=i+1}^{N} b_k P_{\mathrm{R}} |g_n|^2 + \sigma_{\mathrm{D}_n}^2}, \text{ if } i < N, & \text{(12a)} \\ \dfrac{P_{\mathrm{R}} b_i |g_n|^2}{\sigma_{\mathrm{D}_n}^2} \qquad \text{if } i = N. & \text{(12b)} \end{cases}$$

After x_i is decoded successfully, i.e. $\gamma_{\mathrm{D}_{l,i}} \geq \gamma_{\mathrm{th}i}$, it will be canceled out at the SIC structure of $\mathrm{D}_{l,n}$, where $\gamma_{\mathrm{th}i}$ is the targeted SINR of $\mathrm{D}_{l,i}$. SIC is carried out continuously until all signals of $\mathrm{D}_{l,n}$ are decoded successfully.

In the case of imperfect SIC, the SINR is

$$\gamma_{\mathrm{D}_n}^{\mathrm{ipSIC}} = \frac{P_{\mathrm{R}} b_n |g_n|^2}{\sum_{i=n+1}^{N} b_i P_{\mathrm{R}} |g_n|^2 + \sum_{k=1}^{n-1} \xi_2 b_k P_{\mathrm{R}} |g_n|^2 + \sigma_{\mathrm{D}_n}^2}, \tag{13}$$

where ξ_2 represents the impact level of the residual interference at the D_n.

We should note that for the DF protocol, the end-to-end SINR of the system is the minimum of the SINRs of BS – R and R – D_n links, i.e.,

$$\gamma_{e2e} = \min(\gamma_{\mathrm{R}_n}, \gamma_{\mathrm{D}_n}). \tag{14}$$

3 Performance Analysis

3.1 Outage Probability

In this section, the outage probability (OP) of $D_{l,n}$ in the proposed system is derived in two cases, perfect SIC and imperfect SIC. The OP is defined as the probability that the transmission rate of the system falls below the minimum required data rate. Let r_1, r_n (bit/s/Hz) be the minimum required data rate from BS to R and from R to D_n, respectively. For simply, we set $r_1 = r_n = r$, then the OP of the system is calculated as

$$\begin{aligned} \mathrm{OP}_{\mathrm{D}_n} &= \Pr\Big[\min(\gamma_{\mathrm{R}_n}, \gamma_{\mathrm{D}_n}) < \gamma_{\mathrm{th}}\Big] \\ &= F_{\gamma_{\mathrm{R}_n}}(\gamma) + F_{\gamma_{\mathrm{D}_n}}(\gamma) - F_{\gamma_{\mathrm{R}_n}}(\gamma) F_{\gamma_{\mathrm{D}_n}}(\gamma), \end{aligned} \tag{15}$$

where the threshold $\gamma_{\mathrm{th}} = 2^{2r} - 1$ is used as the protected value of the SINR to ensure the quality of service of the network and satisfy the target data rate r of R_l or $D_{l,n}$.

From (9) and (11), we have the $F_{\gamma_{\mathrm{R}_n}}(\gamma)$ and $F_{\gamma_{\mathrm{D}_n}}(\gamma)$ in the case of perfect SIC as

$$F_{\gamma_{\mathrm{D}_n}}^{\mathrm{ipSIC}}(\gamma) = \Pr\left(\frac{P_{\mathrm{S}} a_n |\mathbf{h}_l^H \mathbf{w}_l|^2}{\sum_{i=n+1}^{N} P_{\mathrm{S}} a_i |\mathbf{h}_l^H \mathbf{w}_l|^2 + \sigma_{\mathrm{R}}^2} < \gamma_{\mathrm{th}}\right), \tag{16}$$

$$F_{\gamma_{\mathrm{D}_n}}^{\mathrm{ipSIC}}(\gamma) = \Pr\left(\frac{P_{\mathrm{R}} b_n |g_n|^2}{\sum_{i=n+1}^{N} b_i P_{\mathrm{R}} |g_n|^2 + \sigma_{\mathrm{D}_n}^2} < \gamma_{\mathrm{th}}\right). \tag{17}$$

Then, based on (10) and (13), we can rewrite (16) and (17) as

$$F_{\gamma_{\mathrm{R}_n}}^{\mathrm{ipSIC}}(\gamma) = \Pr\left(\frac{P_{\mathrm{S}} a_n |\mathbf{h}_l^H \mathbf{w}_l|^2}{\sum_{i=n+1}^{N} P_{\mathrm{S}} a_i |\mathbf{h}_l^H \mathbf{w}_l|^2 + |\mathbf{h}_l^H \mathbf{w}_l|^2 \xi_1 \sum_{k=1}^{n-1} a_k P_{\mathrm{S}} + \sigma_{\mathrm{R}}^2} < \gamma_{\mathrm{th}}\right). \tag{18}$$

$$F_{\gamma_{\mathrm{D}_n}}^{\mathrm{ipSIC}}(\gamma) = \Pr\left(\frac{P_{\mathrm{R}} b_n |g_n|^2}{\sum_{i=n+1}^{N} b_i P_{\mathrm{R}} |g_n|^2 + \sum_{k=1}^{n-1} \xi_2 b_k P_{\mathrm{R}} |g_n|^2 + \sigma_{\mathrm{D}_n}^2} < \gamma_{\mathrm{th}}\right). \tag{19}$$

Let us denote $X = |\mathbf{h}_l^H \mathbf{w}_l|^2$ and $Y = |g_n|^2$ for the notation convenience. Since the normalized $\mathbf{w}_l$ is independent of $\mathbf{h}_l^H$, $|\mathbf{h}_{l,k}^H \mathbf{w}_l|^2 = \|\mathbf{h}_{l,k}\|^2$ is Chi-square distributed[3] with the degree of freedom is $2K$, where $k \in \{1, \cdots, K\}$ and $K = N_r(N_t - (L-1))$ [25], $|g_n|^2$ is Chi-square distributed with the degree of freedom is 2. From (16), the CDF of γ_{R_n} can be calculated as

$$\begin{aligned} F_{\gamma_{\mathrm{R}_n}}(\gamma) =& \Pr\left(X < \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}(a_n - \gamma_{\mathrm{th}}\tilde{a})} \right) \\ =& \frac{1}{\Gamma(m_1 K)} \gamma\left(m_1 K, \frac{m_1 \gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}(a_n - \gamma_{\mathrm{th}}\tilde{a})\hat{\Omega}_{\mathrm{SR}}} \right). \end{aligned} \tag{20}$$

where $\tilde{a} = \sum_{i=n+1}^{N} a_i$.

From (17), the CDF of the SINR at $D_{l,n}$ is given by

$$\begin{aligned} F_{\gamma_{\mathrm{D}_n}}(\gamma) &= \Pr\left(\max_{n,\cdots,N} Y_n < \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})} \right) \\ &= 1 - \prod_{n}^{N} \Pr\left(Y_n \geq \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})} \right), \end{aligned} \tag{21}$$

where $\tilde{b} = \sum_{i=n+1}^{N} b_i$.

It is required that $a_n > \gamma_{\mathrm{th}}\tilde{a}$ and $b_n > \gamma_{\mathrm{th}}\tilde{b}$. Otherwise, the outage always occurs because we only consider $X, Y \in (0, \infty]$. Hence, the allocated power for D_n should be more than the total power of other users. Finally, the outage probability in the case of perfect SIC is.

$$\begin{aligned} \mathrm{OP}_{\mathrm{D}_n} =& \frac{1}{\Gamma(m_1 K)} \gamma\left(m_1 K, \frac{m_1 \gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}(a_n - \gamma_{\mathrm{th}}\tilde{a})\hat{\Omega}_{\mathrm{SR}}} \right) \\ &+ \sum_{j=0}^{N} \binom{N}{j} (-1)^j e^{-\frac{j m_t \gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})\hat{\Omega}_{\mathrm{RD}_j}}} \sum_{\ell=0}^{j(m_t-1)} c_\ell^j \left(\frac{m_t \gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})\hat{\Omega}_{\mathrm{RD}_j}} \right)^\ell \\ &- \frac{1}{\Gamma(m_1 K)} \gamma\left(m_1 K, \frac{m_1 \gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}(a_n - \gamma_{\mathrm{th}}\tilde{a})\hat{\Omega}_{\mathrm{SR}}} \right) \\ &\times \sum_{j=0}^{N} \binom{N}{j} (-1)^j e^{-\frac{j m_t \gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})\hat{\Omega}_{\mathrm{RD}_j}}} \sum_{\ell=0}^{j(m_t-1)} c_\ell^j \left(\frac{m_t \gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}(b_n - \gamma_{\mathrm{th}}\tilde{b})\hat{\Omega}_{\mathrm{RD}_j}} \right)^\ell. \end{aligned} \tag{22}$$

[3] If a random variable has Chi-square distribution with the degree of freedom is K, it can be presented as the summation of K Rayleigh distribution random variables [24, p. 16].

Next, we consider the case of imperfect SIC. From (18) and (19), we can rewrite the CDF of γ_{R_n} and γ_{D_n} as

$$\begin{aligned} F_{\gamma_{\mathrm{R}_n}}^{\mathrm{ipSIC}}(\gamma) &= \Pr\left(X < \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}[a_n - \gamma_{\mathrm{th}}(\beta_1+\beta_2)]}\right) \\ &= \frac{1}{\Gamma(m_1 K)}\gamma\left(m_1 K, \frac{m_1\gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}[a_n - \gamma_{\mathrm{th}}(\beta_1+\beta_2)]\hat{\Omega}_{\mathrm{SR}}}\right), \end{aligned} \tag{23}$$

$$\begin{aligned} F_{\gamma_{\mathrm{D}_n}}^{\mathrm{ipSIC}}(\gamma) &= \Pr\left(\max_{n,\cdots,N} Y_n < \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}[b_n - \gamma_{\mathrm{th}}(\psi_1+\psi_2)]}\right) \\ &= 1 - \prod_{n}^{N}\Pr\left(Y_n \geq \frac{\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}[b_n - \gamma_{\mathrm{th}}(\psi_1+\psi_2)]}\right), \end{aligned} \tag{24}$$

where $\gamma(,\cdot,)$ is the lower incomplete Gamma function [26], $\beta_1 = \sum_{i=n+1}^{N} a_i$, $\beta_2 = \xi_1\sum_{k=1}^{n-1} a_k$, and $\psi_1 = \sum_{i=n+1}^{N} b_i$, $\psi_2 = \xi_2\sum_{k=1}^{n-1} b_k$. It is also required that $a_n > \gamma_{\mathrm{th}}(\beta_1+\beta_2)$ and $b_n > \gamma_{\mathrm{th}}(\psi_1+\psi_2)$ in (23) and (24), respectively. Otherwise, the outage always occurs.

Similar for the case of perfect SIC, we obtain the outage probability in the case of imperfect SIC as

$$\begin{aligned} \mathrm{OP}_{\mathrm{D}_n}^{\mathrm{ipSIC}} &= \frac{1}{\Gamma(m_1 K)}\gamma\left(m_1 K, \frac{m_1\gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}\Delta(n)\hat{\Omega}_{\mathrm{SR}}}\right) \\ &+ \sum_{j=0}^{N}\binom{N}{j}(-1)^j \exp\left(-\frac{j m_t\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}\Delta_1(n)\hat{\Omega}_{\mathrm{RD}_j}}\right)\sum_{\ell=0}^{j(m_t-1)} c_\ell^j\left(-\frac{m_t\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}\Delta_1(n)\hat{\Omega}_{\mathrm{RD}_j}}\right)^{\ell} \\ &- \frac{1}{\Gamma(m_1 K)}\gamma\left(m_1 K, \frac{m_1\gamma_{\mathrm{th}}\sigma_{\mathrm{R}}^2}{P_{\mathrm{S}}\Delta(n)\hat{\Omega}_{\mathrm{SR}}}\right) \\ &\times \sum_{j=0}^{N}\binom{N}{j}(-1)^j \exp\left(-\frac{j m_t\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}\Delta_1(n)\hat{\Omega}_{\mathrm{RD}_j}}\right)\sum_{\ell=0}^{j(m_t-1)} c_\ell^j\left(-\frac{m_t\gamma_{\mathrm{th}}\sigma_{\mathrm{D},n}^2}{P_{\mathrm{R}}\Delta_1(n)\hat{\Omega}_{\mathrm{RD}_j}}\right)^{\ell}, \end{aligned} \tag{25}$$

where $\Delta(n) = [a_n - \gamma_{\mathrm{th}}(\beta_1+\beta_2)]$ and $\Delta_1(n) = [b_n - \gamma_{\mathrm{th}}(\psi_1+\psi_2)]$.

4 Numerical Results

We provide some typical numerical results to evaluate the performance in terms of the outage probability and ergodic rate of the proposed MIMO-NOMA relay system. The system parameters are set as follows. The number of relay nodes, transmission antennas, and reception antennas are $L = 3$, $N_t = N_r = 3$. There are three users in each cluster with the threshold data rates $r_1 = r_2 = r_3 = 1$ b/s/Hz, respectively. The number of users in each cluster is set to three because

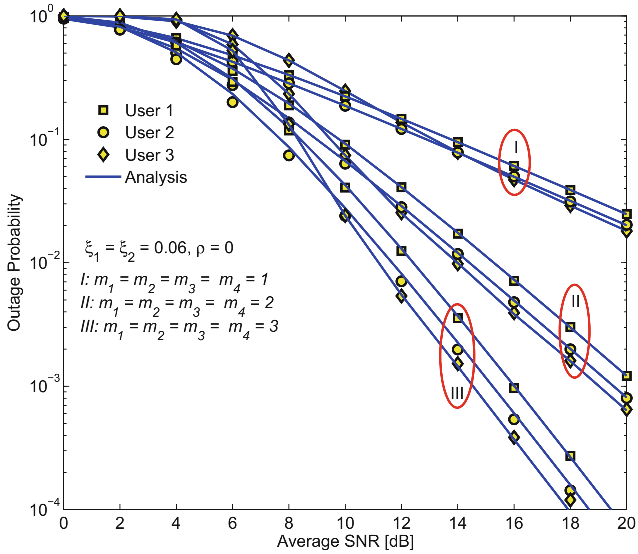

Fig. 2. Outage probability versus SNRs for different fading levels. $a_1 = 0.6, a_2 = 0.3, a_3 = 0.1$, $\xi_1 = \xi_2 = 0.06$.

the complexity and performance degradation of each user is proportional to the number of users [27]. All channels between the BS and each user are assumed to be Nakagami-m distributions. More specifically, the fading parameter m of each channel is denoted as follows: m_1 for BS – R_l link, m_t for R_l – $D_{l,1}$ link, and m_3, m_4 for R_l – $D_{l,2}$ and R_l – $D_{l,3}$ links, respectively. Without loss of generality, we assume the first user is farthest from the BS while the third user is nearest to the BS. Consequently, we can choose the average channel gains as $\Omega_{\mathrm{BS-R}_l} = \Omega_{l,1} = 2$, $\Omega_{l,2} = 3$, and $\Omega_{l,3} = 6$. Then, the power allocation coefficients are in an increasing order $a_1 > a_2 > a_3$. In general, the power allocation coefficient is given by $a_n = (N - n + 1)/\mu$, where μ is chosen such that $\sum_{n=1}^{N} \sqrt{a_n} = 1$. To simplify the system design and settings, we use similar power allocation coefficients for the BS and the relay, i.e. $a_n = b_n$.

Figure 2, presents the outage probability (OP) of each user versus the SNR in dB for different fading levels. We use three cases of fading, i.e. case I: $m_1 = m_2 = m_3 = m_4 = 1$, case II: $m_1 = m_2 = m_3 = m_4 = 2$, case III: $m_1 = m_2 = m_3 = m_4 = 3$, and fixed power allocation. From Fig. 2, we can see that the outage performance of the user 3 is always the best among three users although the allocated power for user 3 is the lowest. The reason is user 3 is the closest to the relay, then the channel gain from the relay to $D_{l,3}$ is the largest[4]. On the other hand, we can also see that the diversity gain of case III is the largest while case I is the lowest. Particularly, the diversity gain of case I is one while

[4] The decay of the magnitude power signal is proportional to the squared distance of the multipath fading rays.

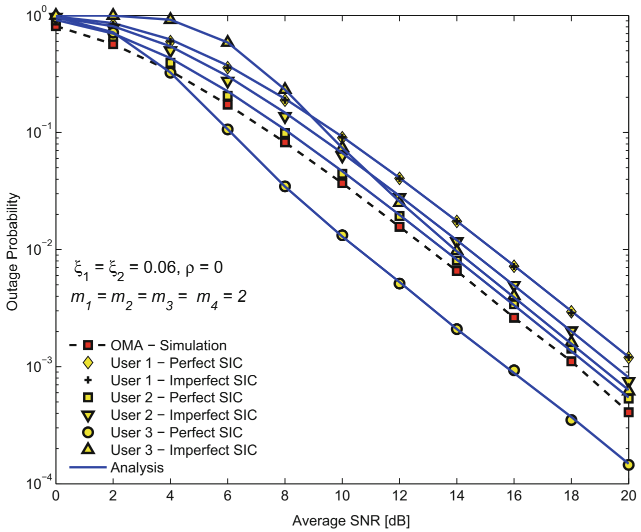

Fig. 3. Outage probability of NOMA-MIMO and OMA-MIMO relay systems versus the SNR for perfect/imperfect SIC. $a_1 = 0.6, a_2 = 0.3, a_3 = 0.1$, $\xi_1 = \xi_2 = 0.06$.

the diversity gains of case II and the case III are two and three, respectively. Therefore, we can conclude that the OP is improved as m increases because m represents the strength of the LoS.

Figure 3 shows the outage probability of each user versus the average SNR in dB for both perfect SIC and imperfect SIC. The fading parameters are set as $m_1 = m_t = m_3 = 2$. For simplicity, we assume that the coefficients of imperfect SIC at the relay and users are similar. We also compare the OPs of the proposed NOMA-MIMO relay systems and OMA-MIMO relay systems. To ensure the fairness between the NOMA and OMA relay systems, we use the same threshold γ_{th} for these two systems. As observed from Fig. 3, the OP of user 1 does not change for both cases of perfect SIC and imperfect SIC because user 1 does not use SIC. Additionally, it is the worst outage performance compared with the OPs of other users in high average SINR regime. We should notice that the OP of user 3 in the case of imperfect SIC is the worst when the average SNR is less than 9 dB. On the other hand, for user 2 and user 3, the OP increases significantly when SIC is imperfect. This is because the residual intra-cluster interference which appears after SIC will be increased if the number of users in a cluster is large. Due to the existence of SIC error (or can be called as error propagation [28, p. 242]) in the case of imperfect SIC, the remaining power of signal pre-decoding process impacts the demodulation of next signals. Thus, when designing the NOMA systems high quality SIC structure is needed. Moreover, the OP of user 3 in NOMA relay system is lower than the OMA relay system, but the OPs of user 1 and user 2 are higher.

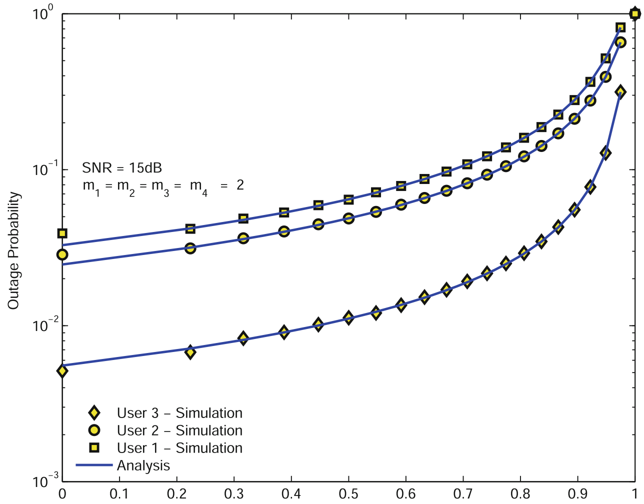

Fig. 4. Outage probability of each users versus estimation error coefficient. $a_1 = 0.6, a_2 = 0.3, a_3 = 0.1$, $\xi_1 = \xi_2 = 0.05$.

Figure 4 plots the outage probability of each user versus the estimation error coefficient in the case of imperfect SIC. We aim to investigate how the imperfect CSI impacts the outage performance of the proposed MIMO-NOMA relay system. As shown in Fig. 4, the outage probability increases when the estimation error coefficient is higher. In the worst case $\rho = 1$, the outage probability always occur. When $\rho < 0.6$, the OP reduces slowly but when $\rho > 0.6$ it drops rapidly. Another feature is that the OP of user 3 is the lowest compared with the OPs of user 1 and user 2.

5 Conclusions

In this paper, we proposed and investigated a downlink multiple-user MIMO-NOMA relay system under perfect/imperfect CSI over Nakagami-m fading channels. We provided the exact closed-form expression of the outage probability of each user in every cluster of the proposed system. All analysis results closely match with the Monte-Carlo simulation results, confirming the accuracy of the derived mathematical expressions. It is indicated that the channel fading severity, channel estimation error, and the quality of SIC structure greatly impact the outage performance of the proposed MIMO-NOMA relay system.

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