



# Stabilized Distributed Layered Grant-Free Narrow-Band NOMA for mMTC

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**Abstract.** The main challenge of supporting Internet of Things (IoT) in 5G network is to provide massive connectivity to machine-type communication devices (MTCs), with sporadic small-size data transmission. Narrowband technology is energyefficient with extended coverage, on a narrow bandwidth, for low-rate and low-cost MTCs. Grant-free transmission is expected to support random uplink communication, however, this distributed manner leads to high collision probability. Nonorthogonal multiple access (NOMA) can be used in grantfree transmission, which multiplies connection opportunities by exploiting power domain. However, coordinated NOMA schemes where base station performs coordination is not suitable for grant-free transmissions. In this paper, based on a detailed analysis of the novel distributed grant-free NOMA scheme proposed in our previous work, a stabilized distributed narrow-band NOMA scheme is proposed to reduce collision probability, which derives the optimal (re)transmission probability for each MTC. With the stabilized scheme, the system can be always stable and its throughput can be guaranteed whatever the new arrival rate is. Simulation results reveal that, when the system is overloaded, for uplink throughput, our proposed scheme outperforms by 45.2% and 87.5%, respectively, compared with the distributed NOMA scheme without transmission probability control and the coordinate OMA scheme considering transmission control.

**Keywords:** Stability · Grant-free · Distributed NOMA · Narrowband · Massive MTC · IoT

## 1 Introduction

Narrowband IoT (NB-IoT) can enable low-rate energyefficient communication with extended-coverage, which is standardized by Third Generation Partnership Project (3GPP) [1, 2]. More specifically, it can support throughput of 20–50 kbps and extend the network coverage by up to 20 dB [3], which means a 100 times

bigger coverage. The coverage enhancement margin makes NB-IoT communications immune against propagation and indoor penetration losses. In addition, devices can reduce their uplink transmission power to prolong the battery lifetime, which fits massive MTC (mMTC) well.

Unlike human-type communications, which involve a small amount of high-rate devices with large-sized data [4], mMTC is generally characterized with massive low-computational-capability devices and sporadic transmissions. Thus uplink access is a serious challenge for mMTC [5]. Devices access the network via a four-step random access (RA) procedure in conventional grant-based schemes, however, it is inefficient to establish dedicated bearers for small data transmission in the scenario of mMTC, since the consequential signaling overheads are proportional to the number of devices. Therefore, grant-free schemes are more promising to mMTC.

Grant-free is gaining attention recently, which allows devices transmitting without base stations (BSs)' radio resources granting [6,7], which is perfect for mMTC due to their low signaling overhead. Conventionally, slotted ALOHA [8] based on orthogonal multiple access (OMA) is used for uplink communications. However, it seriously suffers from the nuisance of collision resulting from contention based access by multiple devices. Fortunately, by exploiting power domain, nonorthogonal multiple access (NOMA) enables multiple devices to share one time-frequency resource. Therefore, NOMA based grant-free can support significantly increased connections [9,10]. However, most of existing studies on NOMA focus on coordination with known channel state information (CSI) at both transmitter and receiver sides, to optimize subchannel and power allocation [11,12], which is not suitable for grant-free transmissions.

To address these challenges, we adopt a distributed NOMA, power division multiple access [13], in narrowband system, and propose a low-complexity distributed layered grant-free narrowband NOMA scheme to realize a hybrid transmission. With this scheme, the inherent drawback of grant-free random access, high collision probability due to its distributed manner, can be greatly alleviated. The key of the proposed scheme is, based on predetermined inter-layer received power difference, firstly dividing the extended-coverage cell into several regions. Secondly, power domain NOMA can be used to drastically reduce the number of MTCs that compete for grant-free transmission in each region. However, grantfree transmission probably lead to unstable system without effective control scheme. To further guarantee the stability of the system, no matter what the new arrival rate is, we apply an optimal (re)transmission probability self-control scheme. With the stabilized scheme, the system can be always stable and its throughput can be guaranteed whatever the new arrival rate is. Simulation results reveal that, when the system is overloaded, for uplink throughput, our proposed scheme outperforms by 45.2% and 87.5%, respectively, compared with the distributed NOMA scheme without transmission control and the coordinate OMA scheme considering transmission control.

The main contributions of this paper are as follows.

- A detailed analysis of the novel distributed grant-free NOMA scheme proposed in our previous work is made, which reveals that grant-free transmission probably lead to unstable system without effective control scheme.
- We propose a distributed layered grant-free NOMA based hybrid transmission scheme for narrowband system. Moreover, considering the stability of the system, we derive the optimal self-control (re)transmission probability for each MTC D.
- The system is modeled as a Markov chain, from which both the average throughput and MTC D delay can be effectively calculated. Simulation results demonstrate that the analysis matches well with the simulation, and the proposed self-control (re)transmission probability scheme works well whatever the new arrival rate of the system is.

The rest of the paper is organized as follow. In Sect. 2, we introduce the system model. In Sect. 3, stability of the proposed grant-free NOMA scheme in our previous work is analyzed. In Sect. 4, we propose a stabilized scheme with (re)transmission probability control, and the performance evaluation is listed. In Sect. 5, we present simulation results and the paper is concluded in Sect. 6.

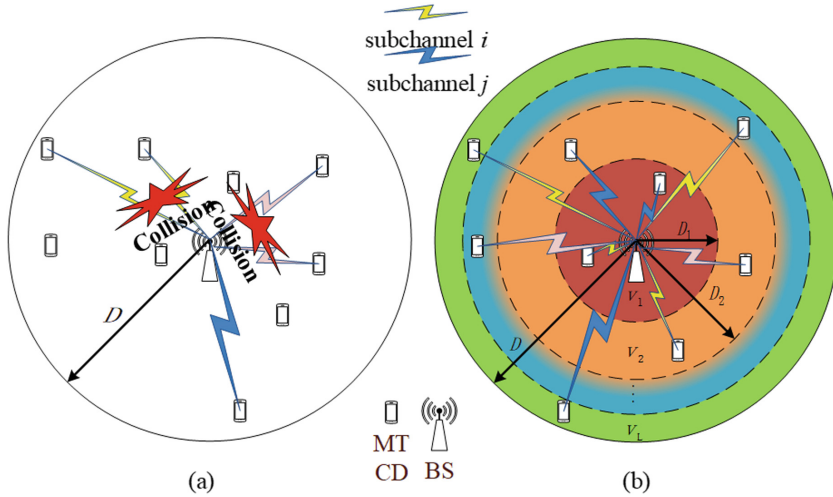
## 2 System Model

Consider a time slotted narrowband cellular network as shown in Fig. 1, with a single BS located in the origin, serving  $Q$  MTC Ds in the area. We assume that the MTC Ds are uniformly distributed in a circle of radius  $D$ , which is much longer than that of other systems. All MTC Ds share a narrow bandwidth of  $B_T$  for uplink data transmissions. The available system bandwidth is divided into frequency resource blocks (subchannels), each of bandwidth  $B$ . Thus, the total number of frequency resource blocks is given as  $M = B_T/B$ . In such a system, each MTC D always starts its transmission at the beginning of a time slot. At the end of the time slot, BS broadcasts the feedback message for all MTC Ds.

In this paper, we use *Connection Opportunity* (CO) to represent a connection resource. In OMA systems, the number of COs in a time slot is determined only by the number of available subchannels. Due to the limited frequency spectrum, the number of COs is inadequate for massive grantfree transmission. As seen in Fig. 1(a), if two users in a cell simultaneously access the BS with the same subchannel, collision happens.

In our previous work [14], a distributed NOMA concept is applied. Suppose that there are  $L$  predetermined aiming received power levels that are denoted as  $v_1 > v_2 > \dots > v_L > 0$ , where  $v_l = \Gamma(\Gamma + 1)^{L-l}$  ( $l = 1, 2, \dots, L$ ), and  $\Gamma$  is the target signal to interference-plus-noise ratio (SINR) at the BS for all MTC Ds, which guarantees the throughput performance of each MTC D. Therefore, according to the predetermined received power levels, the single-BS cell can be divided into  $L$  concentric layers, which is reasonable to the narrowband system for its broad coverage. MTC Ds in different layers have different aiming received power, as shown in Fig. 1(b). The outsider layer denotes the smaller received power. MTC Ds decide their transmission power according to locations

and CSI. For example, for an MTC  $k$ ,  $d_k$  is the distance to the BS, if it belongs to set  $K_l = \{k | D_{l-1} < d_k \leq D_l\}$ , where  $D_0 = 0$ ,  $D_l = D\sqrt{l/L}$ , then its aiming received power level is  $v_l$ . Based on its knowledge of channel gain of different subchannel  $i$ , referred as  $g_{i,k}$  ( $i = 1, \dots, M$ ), its transmission power is decided as  $P_k = \frac{v_l}{\max_i g_{i,k}}$ .



**Fig. 1.** (a) Grant-free transmission; (b) principle diagram of a distributed layered NOMA, where different colored circle and rings indicate the layers of different aiming received powers, and darker color represents bigger received power [14]. (Color figure online)

The key of the proposed scheme is dividing the extended-coverage cell into different regions (layers) based on predetermined inter-layer received power difference, thus, power domain NOMA can be used to drastically reduce the number of MTCs that compete for grant-free transmission in each region. To be specific, given  $M$  subchannels and  $Q_T$  MTCs, under the distributed layered grant-free NOMA scheme, it is equal to the situation that there are  $L$  layers, each layer has  $M$  subchannels. Since the MTCs are uniformly distributed, to simplify mathematics, we assume that the number of active MTCs in each layer is the same (i.e.  $Q = \frac{Q_T}{L}$ ). The assumption can be easily extended to the general scenario with different number of MTCs for each layer. For a subchannel, if there are multiple MTCs who choose the same power level, the signals cannot be decoded, which is called power collision. Otherwise, due to the clear power gap between the predetermined received power levels, the BS can decode any signal which is the only one who chose a subchannel with some predetermined received power level. If the transmitted packet experiences collision, it will retransmit through a subchannel in the next time slot immediately, which

is called fast retransmission [15], that is, fast retransmission and infinite retransmissions are assumed.

In this paper, an MTCD with a stored packet to transmit is called *activer*, while one without packet to transmit is called *inactiver*. *Activer* and *inactiver* are convertible, that is, an *inactiver* becomes *activer* once it has the arrival of a new packet, and an *activer* becomes an *inactiver* after a successful transmission. To simplify the system, there are some assumptions as follows:

- (1) Each MTCD has only one buffer to store a packet;
- (2) When a new packet arrives at an *inactiver*, it changes to an *activer* in the next time slot, and is treated equally as any other *inactives*;
- (3) Each *activer* can decide whether to transmit in the next time slot or not with transmission probability  $p_t$ ;

### 3 Stability Analysis of the Proposed Grant-Free NOMA Scheme

In this section, using the Foster-Lyapunov criterion [17], the stability of the proposed grant-free NOMA scheme with fast retransmission and infinite retransmissions is analyzed. According to the model description and analysis in the previous section, we can analyze each layer region, respectively and identically. Thus, in the following sections, we take one layer region as an example.

#### 3.1 Drift of the Proposed Grant-Free NOMA

For a grant-free system with  $M$  subchannels, since an MTCD in each layer selects a subchannel at random, each subchannel is selected with equal probability  $\frac{1}{M}$ . Let  $N_k$  denote the number of *actives* in one layer at time slot  $k$ , and  $A_{k+1}$  denote the number of *inactives* having new arrival at time slot  $k$ . According to the principle of fast retransmission,  $N_{k+1}$  can be given by

$$N_{k+1} = [N_k]_M + A_{k+1}, \quad (1)$$

which is actually a Markov chain, where  $[n]_M$  denotes the number of the backlogged (due to stopping transmitting) and collided (due to transmitting collision) packets among  $n$  transmitted packets, when the number of subchannels is  $M$ . If fast retransmission is applied, these  $[n]_M$  packets are all to be retransmitted in the next slot.

Since  $N_k$  is non-negative, we can analyze it with Lyapunov function. Given  $N_k = n$ , the drift of  $N_k$  can be expressed as [18]

$$d_n = \mathbb{E}[N_{k+1}|N_k] - N_k = \mathbb{E}[N_{k+1}|N_k = n] - n. \quad (2)$$

From Eq. (1), we can find that

$$\mathbb{E}[N_{k+1}|N_k = n] = \mathbb{E}[[n]_M] + \mathbb{E}[A_{k+1}|N_k = n] = \mathbb{E}[[n]_M] + \bar{A}_n, \quad (3)$$

where  $\bar{A}_n = E[A_{k+1}|N_k = n]$ , which is an important function impacting the stability of the system with fast-retrial and infinite retransmissions.

With  $M$  subchannels, let  $X_m$  denote the number of *activers* that choose some subchannel  $m$  ( $m \in [1, \dots, M]$ ) at the same time slot in the layer region. Since a subchannel is chosen by each *activer* uniformly at random, provided that there are  $N_k = n$  *activers* to transmit their packets, the probability that any subchannel  $m$  is perfectly chosen by one MTCD is

$$\Pr(X_m = 1) = \binom{n}{1} \frac{1}{M} \left(1 - \frac{1}{M}\right)^{n-1} = \text{bin}\left(1, n, \frac{1}{M}\right) = \frac{n}{M} q^{n-1}, \tag{4}$$

where  $q = 1 - \frac{1}{M}$  and  $\text{bin}(\ast)$  denotes a binomial probability. Thus, the average number of packets that are collided among  $n$  *activers* is given by

$$E[[n]_M] = n - \sum_{m=1}^M \Pr(X_m = 1) = n(1 - q^{n-1}). \tag{5}$$

According to Eqs. (2), (3) and (5), the drift of  $N_k$  can be expressed as

$$d_n = \bar{A}_n - nq^{n-1}. \tag{6}$$

From Eq. (6) we can find that, since  $nq^{n-1} \rightarrow 0$  as  $n \rightarrow \infty$ , the drift of  $N_k$  cannot be negative if the average number of *inactivers* having new arrival packets is a constant value, i.e.,  $\bar{A}_n = \lambda$ . In other words, the proposed grant-free NOMA transmission with fast retrial cannot be stable with a constant arrival rate  $\bar{A}_n = \lambda > 0$ . In other words, the system needs to control the new arrival rate in order to stay stable.

### 3.2 New Arrival Rate Control Scheme for Stability

Control criteria of the new arrival rate can relate to the number of *activers* at the present time slot,  $N_k$ , which is usually not directly obtained but can be estimated in many ways. In our previous work about stochastic online learning [19], a new kind of maximum likelihood technique, the number of active users,  $N_k$ , can be reliably estimated, based on the maximum likelihood estimation at the BS, by taking advantage of stochastic online learning technique.

Suppose there exists the *activer* number threshold, which denotes the sign of an overloaded system with fast-retrial. That is, if  $N_k = n > \bar{N}$ , the new arrival rate control should be implemented. For simplicity and rationality, we assume that  $\bar{N} = M$  in a grant-free system with  $M$  subchannels. Then, according to Eq. (6), we consider the following average new arrival rate control function

$$\bar{A}_n = \begin{cases} \lambda, & \text{if } n < M \\ \lambda q^{n-M}, & \text{if } n \geq M \end{cases}. \tag{7}$$

Let set  $S = \{0, \dots, M - 1\}$ , if the number of transmitting packets  $n$  is bigger than the threshold, i.e.,  $n \in \mathbb{Z} - S = \{M, M + 1, \dots\}$ , with Eq. (7), it can be found

that  $d_n = \lambda q^{n-M} - nq^{n-1} = (\lambda q^{-M+1} - n)q^{n-1}$ . So, if  $\lambda q^{-M+1} \leq M < n \in Z-S$ , that is,  $\lambda \leq Mq^{M-1}$ , is a sufficient condition for

$$d_n < 0, \quad n \in Z - S. \quad (8)$$

For  $n \in S$ , we can find that

$$d_n = \lambda - nq^{n-1} \leq \lambda, \quad n \in S. \quad (9)$$

Since set  $S$  is finite, we can see that Eqs. (8) and (9) satisfy the Foster-Lyapunov criterion [17], which denotes a stable system. When  $M$  increases, which is a usual situation with narrow-band mMTC system, we can have  $\lim_{M \rightarrow \infty} q^{M-1} = \lim_{M \rightarrow \infty} (1 - \frac{1}{M})^{M-1} = \frac{1}{e}$ , so  $Mq^{M-1}$  is approaching  $\frac{M}{e}$ , which means that the average new packet arrival rate should be less than  $\frac{M}{e}$  when only new arrival rate control scheme is applied.

However, average new arrival rate control scheme is not easy to implement in practice, since the probability to generate a new packet cannot be controlled. Thus, we take another feasible control strategy, based on (re)transmission probability control scheme.

## 4 Stabilized Scheme with Transmission Probability Control

### 4.1 Transmission Probability Control

In this scheme, when an activer has a packet to (re)transmit, it transmits with a controlled transmission probability  $p_g$  instead of 100% fast retransmission. For a grant-free system, since an MTCD in each layer select a subchannel from all  $M$  subchannels at random, once the MTCD decides to transmit, each subchannel is selected with equal probability  $\frac{1}{M}$ . We assume the number of packet transmissions in one time slot is a Poisson random variable. The average packet attempting rate is  $N$ . Since the probability for an attempting MTCD to choose a subchannel is equal, the number of packet arrivals at a subchannel is also Poisson with rate  $\frac{N}{M}$ . So the system throughput for a subchannel  $m$  can be expressed as

$$T_m = \frac{N}{M} e^{-\frac{N}{M}}, \quad (10)$$

and the overall throughput at a time slot is  $T = \sum_{m=1}^M T_m = N e^{-\frac{N}{M}}$ .

From Eq. (10), it is easy to find that the maximal throughput for any subchannel  $m$  is achieved when  $\frac{N}{M} = 1$ , i.e.,  $N = M$ , and the maximal value is  $e^{-1} \approx 0.368$ . Thus the maximal throughput of all  $M$  subchannels is  $Me^{-1}$ , which can be regarded as the system capacity upper limit. Obviously, for a stable system, average new packet arrival rate  $\lambda$  should satisfy  $\lambda \leq Me^{-1}$ , which is also consistent with the conclusion of the previous section.

Thus, for each layer, if the overall MTCd transmission rate is  $M$ , the maximal throughput is achieved theoretically. Therefore, if the number of *activers* in the layer is known as  $N$ , with new packet arrival rate  $\lambda \leq Me^{-1}$ , the maximum throughput of the system can be achieved stably, if the transmission probability for each *activer* in the layer is controlled adaptively as  $p_t = \min\{1, \frac{M}{N}\}$ . Thus, the controlled transmission probability for the MTCdS in the layer at time slot  $k$  is

$$p_t(k) = \min\{1, \frac{M}{N_k}\}. \tag{11}$$

### 4.2 State Transition Probability

As indicated in the previous section, an *inactiver* having a new packet becomes an *activer* at the start of the next time slot.  $N_k$  denotes the number of *activer* at the start of time slot  $k$ . Assuming that there are  $Q$  MTCdS, and an *inactiver* has a probability of  $p_g$  to generate a new packet, which meets the stability requirement, that is,  $p_g \leq \frac{M}{(Q-N_k)e} \leq \frac{M}{Qe}$ . In the following, for simplicity purpose, we assume that  $p_g = \frac{M}{Qe}$ , which denotes a maximal new packet generating probability. With a given controlled (re)transmission probability  $p_t(k)$ ,  $N_{k+1}$  is only depends on  $N_k$ . Thus, the number of *activers* in each time slot  $k$ , denotes as  $N_k$  ( $k = 1, 2, \dots$ ), is a Markov chain with  $p_t(k)$ , which is given according to Eq. (11), with state space  $\{0, 1, 2, \dots, Q\}$ .

Let  $p_{i,j}$  denote the state transition probability of  $N_k$  from state  $i$  to state  $j$ , i.e.,  $p_{i,j} = \Pr(N_{k+1} = j | N_k = i), 0 \leq i, j \leq Q$ . Let  $D_k$  denote the number of successful departure packets (or the number of *activers* transmitted and decoded successfully) at time slot  $k$ , and satisfy  $D_k \in [0, \min\{M, N_k\}]$ . Let  $A_{k+1}, 0 \leq A_{k+1} \leq Q - N_k$  denote the number of *inactivers* having new packet arrivals in time slot  $k$ , which means these  $A_{k+1}$  MTCdS will become *activers* at time slot  $k + 1$ . The state transition of  $N_k$  ( $k = 1, 2, \dots$ ) satisfies

$$N_{k+1} = N_k - D_k + A_{k+1}. \tag{12}$$

For simplicity, with  $D_k = d, N_k = i, N_{k+1} = j, A_{k+1}$  can be written as

$$A_{k+1} = j - i + d. \tag{13}$$

Next we will discuss how to obtain the state transition matrix  $P$ , which consists of state transition probability  $p_{i,j}$ . For  $i = 0$ , that is when there is no *activers* at this time slot, there is no departure packet, that is,  $d = 0$ . So according to Eq. (13),  $N_{k+1} = A_{k+1}$ . For  $i = 1$ , that is, when there is only one *activer* at this time slot, we have  $p_t = 1$  according to our controlled (re)transmission probability scheme denoted as Eq. (11). Thus in this situation, the only *activer* will always transmit and be successful, that is,  $d = 1$ . So we have  $N_{k+1} = A_{k+1}$ . Therefore, it is easy to get the transition probabilities when  $i = 0$  and  $i = 1$  as

$$p_{0,j} = p_{1,j} = \binom{Q}{j} p_g^j (1 - p_g)^{Q-j} = \text{bin}(j, Q, p_g). \tag{14}$$



With new arrival rate control scheme,  $p_g = \frac{M}{Qe}$ .

Apart from  $i = 0$  and  $i = 1$ , when  $i \geq 2$ , the state transition probability  $p_{i,j}$  can be expressed as

$$p_{i,j} = \sum_{d=0}^{\min\{M,i\}} \Pr\{N_{k+1} = j | N_k = i, D_k = d\} \cdot \Pr\{D_k = d | N_k = i\}, \quad (15)$$

which is the conditional probability of the number of successfully transmitted *activers* at time slot  $k$ ,  $D_k$ .

For the first multiplier factor of Eq. (15),  $\Pr\{N_{k+1} = j | N_k = i, D_k = d\} = \Pr\{A_{k+1} = j - i + d | N_k = i, D_k = d\}$  when  $i - d \leq j \leq Q - d$ , otherwise  $\Pr\{N_{k+1} = j | N_k = i, D_k = d\} = 0$ . Since *inactivers* have the new packet arrival probability as  $p_g$ , then  $A_{k+1}$  is binomial as

$$\Pr\{A_{k+1} = a | N_k = i, D_k = d\} = \Pr\{A_{k+1} = a | N_k = i\} = \text{bin}(a, Q - i, p_g) \quad (16)$$

For the second multiplier factor of Eq. (15), we assume that there are  $T_k$  transmitted MTCs when there are  $N_k$  *activers* at time slot  $k$ , so  $\Pr\{D_k = d | N_k = i\}$  can be expressed as the conditional probability of  $T_k$ , that is,

$$\begin{aligned} \Pr\{D_k = d | N_k = i\} &= \sum_{t=d}^i \Pr\{D_k = d | N_k = i, T_k = t\} \cdot \Pr\{T_k = t | N_k = i\}. \\ &= \sum_{t=d}^i \Pr\{D_k = d | T_k = t\} \cdot \Pr\{T_k = t | N_k = i\} \end{aligned} \quad (17)$$

For the first multiplier factor of Eq. (17),  $\Pr\{D_k = d | T_k = t\}$  can be derived from combinatorial problem where  $T_k$  balls is randomly distributed to  $M$  boxes, resulting in exactly  $D_k$  boxes with perfect one ball, the probability can be easily obtained as [16]

$$\Pr\{D_k = d | T_k = t\} = \frac{(-1)^d M! t!}{M^t d!} \cdot \sum_{x=d}^{\min\{M,t\}} \frac{(-1)^x (M-t)^{t-x}}{(x-d)!(M-x)!(t-x)!}. \quad (18)$$

For the second multiplier factor of Eq. (17), since with given (re)transmission probability  $p_t(k)$ , the number of transmitted MTCs  $T_k$  is binomial with given number of *activers*  $N_k$ , that is,

$$\Pr\{T_k = t | N_k = i\} = \text{bin}(t, i, p_t(k)), \quad (19)$$

where  $p_t(k) = \min\{1, \frac{M}{i}\}$  according to (re)transmission probability control scheme denoted by Eq. (11).

With Eqs. (18) and (19),  $\Pr\{D_k = d | N_k = i\}$  can be calculated according to (17). So the whole transition probability can finally be obtained from (14) and (15), which can be obtained from Eqs. (16), (17), (18) and (19).

### 4.3 Performance Evaluation Criteria

After the derivation and analysis above, with known state transition matrix  $P$ , for a Markov chain, the steady state probability  $\pi = [\pi_0, \pi_1, \pi_2, \dots, \pi_Q]$  can be obtained through solving the following equations

$$\begin{cases} \pi = \pi \cdot P \\ \sum_{q=0}^Q \pi_q = 1 \end{cases}. \quad (20)$$

The elements of  $P$  are calculated from Eqs. (14) and (15).

Since the average number of *activers* can be given as  $\bar{N} = \sum_{q=0}^Q q \cdot \pi_q$ , we can obtain the first system performance evaluation criteria, the average throughput, which can be calculated as

$$\bar{D} = \sum_{d=0}^M d \cdot \Pr(D_k = d), \quad (21)$$

where  $\Pr(D_k = d)$  can be easily calculated by the formula of full probability, which is

$$\begin{aligned} \Pr(D_k = d) &= \sum_{i=d}^Q \Pr(D_k = d | N_k = i) \cdot \Pr(N_k = i) \\ &= \sum_{i=d}^Q \Pr(D_k = d | N_k = i) \cdot \pi_i. \end{aligned} \quad (22)$$

For the second system performance evaluation criteria, the average number of backlogged packets, since at the end of a time slot, there are  $D_k$  MTCs who successfully transmitted their packets, which can be obtained with Eq. (22), we have  $B_k = N_k - D_k$  MTCs become backlogged in the time slot  $k$ , who will continue to be a part of *activers* in the next slot. So the average number of backlogged MTCs can be calculated as

$$\bar{B} = \bar{N} - \bar{D}. \quad (23)$$

For a time-tolerate mMTC system, the average number of backlogged MTCs is not at the highest priority. But the system should maintain stability, which requires that the number of backlogged packets plus new-generated ones not to be out-of-control, that is, the sum should be less than the system capacity upper limit.

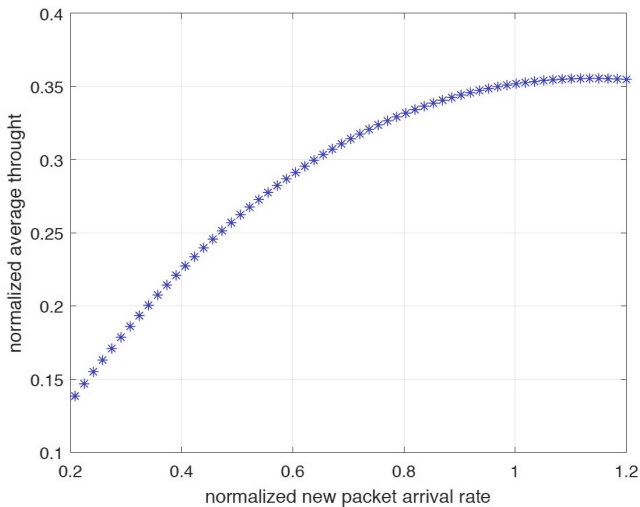
## 5 Simulation Results

In this section, we present simulation results to evaluate the performance of proposed stabilized scheme. The list of key mathematical symbols used in this paper are summarized in Table 1. The new packet arrival rate  $\lambda$  is normalized by  $Me^{-1}$ , which is the system capacity limit of one layer. The performance of the algorithm is characterized by the normalized average throughput and average backlog. The normalized average throughput of one layer is the average number of successful packets in a time slot normalized by  $M$ .

From Fig. 2, we notice the normalized system throughput is in proportion to the normalized new arrival rate  $\bar{\lambda}$  when  $0 \leq \bar{\lambda} \leq 1$ , which is expected. The system throughput can be viewed as the packet departure rate. It is easy to know, for a stabilized system, the departure rate almost equals to the arrival rate. Thus Fig. 2 shows the system is stable for all new packet arrival rates of  $0 \leq \bar{\lambda} \leq 1$ , which has achieved the equal effect with new packet arrival rate control scheme. However, in Fig. 2, we also plot the normal average throughput for  $\bar{\lambda} > 1$ , which is the advancing effect of the proposed stabilized scheme. It is obvious when  $\bar{\lambda} > 1$ , the system exists increasing backlogs phenomenon,

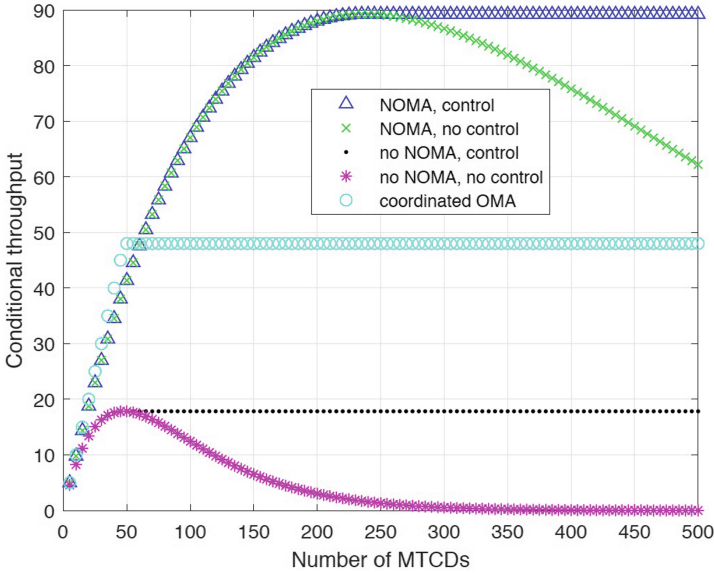
**Table 1.** Notation summary

Notation	Description	Value
$B_T$	Total bandwidth of the system in Hz	180
$B$	The bandwidth of subcarrier in Hz	3.75
$M$	The number of subchannels	48
$D$	The radius of the cell in km	10
$L$	The number of NOMA power level	5
$\lambda$	The mean new packets arrival rate	5
$Q_T$	The number of MTCs in one slot	500
$\Gamma$	Target SINR in dB	6

**Fig. 2.** Normalized average throughput of the stabilized distributed grant-free NOMA scheme.

according to the theoretical analysis above. However, the normalized average throughput can be guaranteed by our stabilized algorithm. And we can notice the throughput is stabilized around the maximal possible rate of  $e^{-1}$ .

In Fig. 3, when the system is overloaded, our proposed transmission scheme with transmission probability control outperforms pure grant-free NOMA without transmission probability control, non-NOMA schemes with/without transmission probability control, high-complexity&overhead coordinated OMA scheme, for different values of  $Q$  in terms of conditional throughput. For example, when the system is overloaded, uplink throughput is expected to increase by 45.2% and 87.5%, respectively, compared with the distributed NOMA scheme without transmission control and the coordinate OMA scheme considering transmission control. System throughput can be guaranteed by our stabilized algorithm.



**Fig. 3.** Conditional throughput comparison between proposed grant-free NOMA with/without transmission probability control, non-NOMA schemes with/without transmission probability control, coordinated OMA scheme, for different values of  $Q$ .

## 6 Conclusion

In this work, to support more connectivity in uplink grantfree mMTC, we proposed a novel distributed layered grant-free NOMA framework based on distributed NOMA. The proposed hybrid transmission scheme can significantly reduce signaling overhead comparing to coordinated schemes. We prove that the scheme with fast-retrial and without transmission probability control is unstable. For the stability analysis, the Foster Lyapunov criterion is considered. For the proposed grant-free NOMA system, a stabilization algorithm was proposed. We give a theoretical analysis on the stabilized algorithm performance. The simulation results show that the performance of the stabilized algorithm is much better than the non-stabilized algorithm. With the stabilized algorithm, the system is always stable when the new packet arrival rate is less than system capacity. Even when the arrival rate is higher than capacity, system throughput can still be guaranteed.

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