# A Queue in Overall Telecommunication System with Quality of Service Guarantees

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**Abstract.** For the first time a queue, related to the shortage of network resources, is included in a model of overall telecommunication system with finite number of users and facilities which makes the model closer to the real system. The service in the queue depends on feedbacks of call attempts and of the state and duration of services in the overall system. The server of the queuing system has more than one exits. The results presented are a base for future development of tools for management, design, dimensioning and redimensioning of the system.

**Keywords:** Overall telecommunication system · Conceptual model · Queuing system with feedbacks · Quality of Service

## 1 Introduction

The classical conceptual model of overall telecommunication system is described in [4] and developed in more details in [5]. We briefly mention the most important features of the model and some basic notation.

The classical conceptual model considers user's behaviour, finite number of homogenous users and terminals, losses due to abandoned and interrupted dialing, blocked and interrupted switching, unavailable intent terminal, blocked and abandoned ringing and abandoned communication. The traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately but in their interrelation. Two types of virtual devices are included in the model: base and comprising base devices.

At the bottom of the structural model presentation, we consider basic virtual devices that do not contain any other virtual devices. A basic virtual device has a general graphical representation as shown in Fig. 1.

The parameters of the basic virtual device x are the following (see [2] for terms definition):


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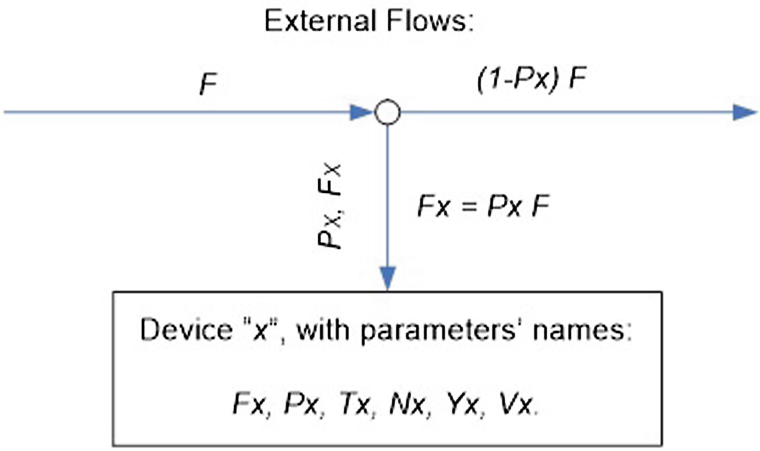


**Fig. 1.** A graphical representation of a basic virtual device x.

- $Fx$ - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;
- $Px$ - probability of directing the requests towards device x;
- $Tx$ - service time (duration of servicing of a request) in device x;
- $Yx$ - traffic intensity [Erlang];
- $Vx$ - traffic volume [Erlang - time unit];
- $Nx$ - number of lines (service resources, positions, capacity) of device x.

The graphic representations of the base virtual devices together with their names and types are shown in Fig. 2 (see [4]). The type of each of the basic virtual devices is also shown in Fig. 2. Each basic virtual device belongs to one of the following types: Generator, Terminator, Modifier, Server, Enter Switch, Switch and Graphic connector. With the exception of the Switch, which has one or two entrances and one or two exits, every other virtual device has one entrance and/or one exit.

The names of the virtual devices are concatenations of the first letters of the branch exit, branch and stage in that order (see Fig. 2). For example **ad** stands for the virtual device "abandoned dialling" while **rad** – for "repeated abandoned dialling".

For the better understanding of the model and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [2]) is used. For example $dem.F$ for demand flow; $inc.Y$ stands for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic.

The following comprising virtual devices denoted by **a, b, s** (see Fig. 2) and **ab** (not shown in Fig. 2) are considered in the model.

- **a** comprises all calling terminals (A-terminals) in the system. It is shown with continuous line box in Fig. 2;

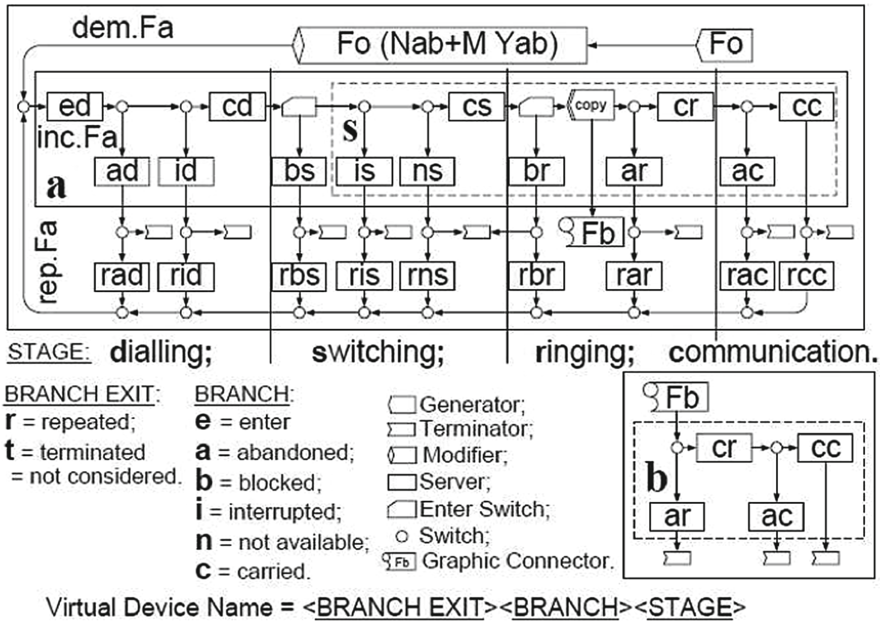


**Fig. 2.** Classical conceptual model of an overall telecommunication system (see [4]).

- **b** comprises all called terminals (B-terminals) in the system. It is shown in box with dashed line in the down right corner in Fig. 2;
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Fig. 2;
- **s** virtual device corresponding to the switching system. It is shown with dashed line box into the a-device in Fig. 2.

## 2 Representation of the Queuing System Within the Switching Stage of an Overall Telecommunication System

In this section, we propose a representation of a Queuing system in the Switching system of an overall telecommunication system. In the classical conceptual model (see [4]), once the Switching system reaches its capacity, the incoming call attempts are blocked and they are redirected to the "blocked switch" branch which begins at the virtual device denoted by **bs** on Fig. 2.

With the inclusion of queue in the Switching stage of the model when the Switching system has reached its capacity the incoming call attempts wait in a buffer until a service line in the Switching system becomes available. We consider the buffer size of the queuing system to be of finite length and the number of servers (service lines) also to be finite. In such queuing system, the call attempts will be blocked only when both the Switching system and the buffer have reached their capacity. The conceptual model of the Switching system with a queue in terms of Service Systems Theory is shown in Fig. 3. In comparison to the classical conceptual model in Fig. 2, the branch **bs** is removed because the blocked call attempts from the Enter Switch remain in the queue and they are not redirected to other virtual devices. The Switching system with a queue consists of a device of type Queue denoted by **q**, the Enter Switch before it and all devices of the **bq** branch. The switching system is denoted by **s** in Fig. 3 as well as in Fig. 2. The Enter Switch device before the **q** device redirects the call attempts when the queue is full. The base device **q** has the same parameters as the other base devices: $Fq, Yq, Tq, Pq, Nq$. The capacity of the buffer is $Nq$. The queue discipline considered in the model is FIFO. The Enter switch device between the **q** device and the **s** device has one important parameter – the probability of blocked switching ($Pbs$) with which the call attempts remain in the **q** device.

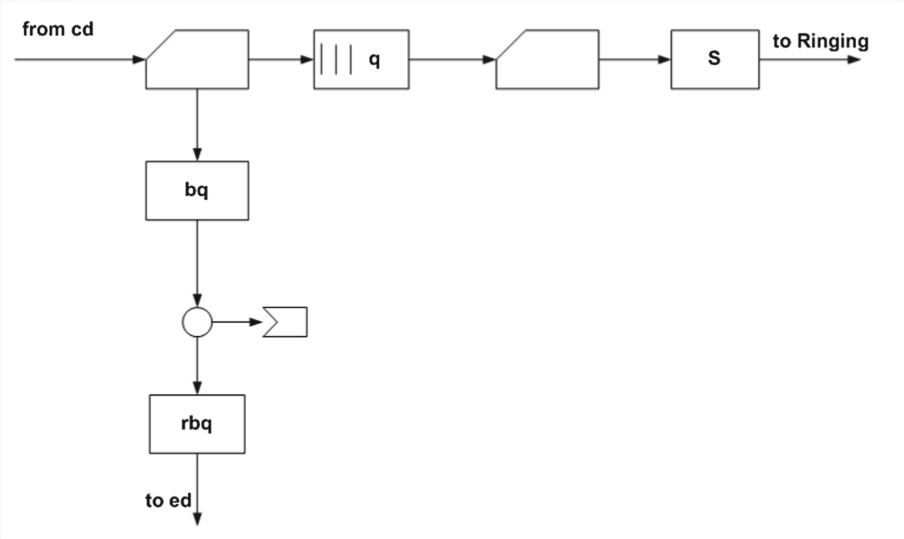


**Fig. 3.** Conceptual model of a part of the Switching stage of an overall telecommunication system with a queue. **cd** stands for "carried dialing", **q** for the Queue device, **s** for "Switching system", **bq** for "blocked queuing", **rbq** for "repeated blocked queuing", **ed** for "enter dialing".

In [6] four conceptual models of a queuing system are compared. One of the models (see Fig. 4) illustrates the important concept of "zero queuing". The internal structure of the queue is presented, including two virtual devices: "carried queue" (cq) and "zero queuing" (zq). Requests pass the Queue without delay, with probability $Pzq = 1 - Pbs$, in case there are free places available in the Server, in the moment of their arrival. The duration of the zero queuing ($Tzq$) may be zero, or close to it. The total queuing time ($Tq$) is given by

$$Tq = Pbs(1 - Pbq)Tcq + (1 - Pbs)Tzq. \quad (1)$$

This approach is a detailization of the classical approach (Fig. 3). It represents explicitly the important concept of zero queuing and the probability of blocked server ($Pbs$). It is more complex, but allows more clear and full presentation of the processes in the queuing system.

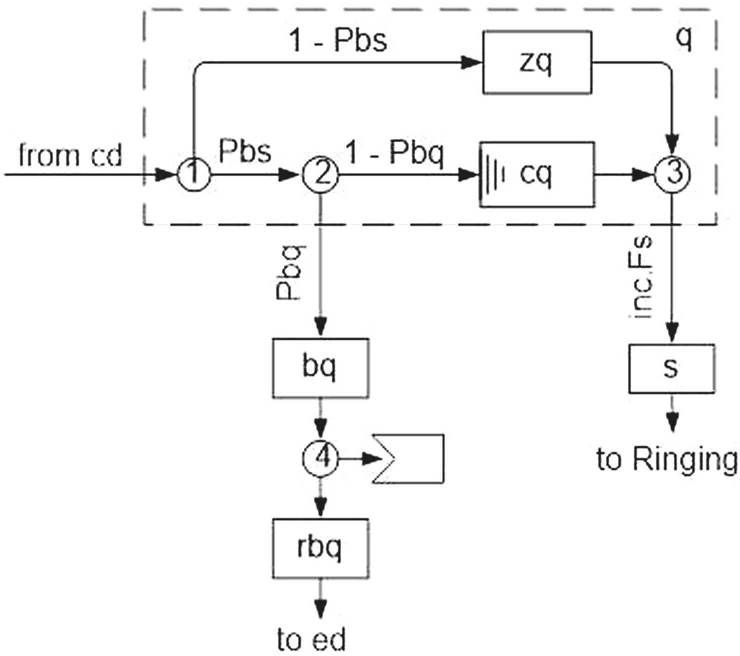


**Fig. 4.** More detailed representation of a queuing system.

The parameters of the queuing system in the case of service of the call attempts without waiting can be obtained using Eqs. (42) and (43) from Sect. 4.

In order to compactly describe single queuing stations in an unambiguous way, the so called Kendall notation is often used (see [1]). A queuing system is described by 6 identifiers separated by vertical bars in the following way:

$$Arrivals \,|\, Services \,|\, Servers \,|\, Buffersize \,|\, Population \,|\, Scheduling$$

where "Arrivals" characterises the arrival process (arrival distribution), "Service" characterizes the service process (service distribution), "Servers" – the number of servers, "Buffersize" – the total capacity, which includes the customers possibly in the server (infinite if not specified), "Population" – the size of the customer population (infinite if not specified), and finally, "Scheduling" – the employed service discipline.

In our model, the queuing system in the Switching stage of the telecommunication network in Kendall notation is represented as $M|M|Ns|Ns+Nq|Nab|$

$FIFO$, where $M$ stands for exponential distribution, $Ns$ is the capacity of the Switching system (number of equivalent internal switching lines) and $Nab$ is the total number of active terminals which can be calling and called. This is related to the derivation of the analytical model of the system.

The important parameters of the devices in Fig. 3 can be divided into two groups. The first group consists of parameters whose values can be obtained from the environment of the Queuing system in the way described in [4,5]. These parameters are $Ts, Ns, Pbs, Ys, ofr.Fq$. The second group consists of the unknown parameters of the queuing process $Fq, Pbq, Tq, Yq$. In order to describe the queuing process in details we consider the following cases separately depending on the value of $Ys$ – the traffic of the Switching system.

Case 1. If the Switching system has reached its capacity, i.e. $Ys = Ns$, and there are call attempts waiting to be serviced in the Queue device, i.e. $Yq > 0$, then $Pbs > 0$. In this case the intensity of the flow carried by the Queue device is equal to the intensity of the flow leaving the Switching system, i.e. $crr.Fq = out.Fs$ where the qualifier "out" is abbreviation of outgoing. Generally, for the outgoing flow from the Switching system we have

$$out.Fs = \frac{Ys}{Ts} \tag{2}$$

which is a restatement of the Little's formula. Since $Ys = Ns$ we also have

$$out.Fs = crr.Fq. \tag{3}$$

Case 2. If the Switching system has not reached its capacity but there are call attempts being serviced, i.e. $0 < Ys < Ns$, then $Yq < Nq$ and $Pbs < 1$. The equality $Fq = Fs$ holds.

Case 3. Finally, if there are no call attempts being serviced by the Switching system, i.e. $Ys = 0$, then $Fq = Fs = 0$ and $Yq = 0$.

## 3 Conceptual Model of Overall Telecommunication System with Queue in the Switching Stage

By combining the representation of the queue in terms of Service System Theory shown in the previous section and the classical model from [4] we obtain conceptual model of the overall telecommunication system with queue in the Switching stage. Its graphical representation is shown in Fig. 5.

In the conceptual model in Fig. 5 there are at least 37 important virtual devices. Of them 33 are basic virtual devices and 4 (**a, b, s, ab**) are comprising. They are of interest because the values of their parameters characterize the state of the overall telecommunication system. Every device has five parameters: $P, F, T, Y$ and $N$. Therefore the total number of parameters is 185.

The meaning of two of the parameters – $Fo$ and $M'$ – should be explained separately. $Fo$ is the intent intensity of calls of one idle terminal. $M'$ is a constant, characterizing the Bernoulli-Poisson-Pascal (BPP) flow of demand calls

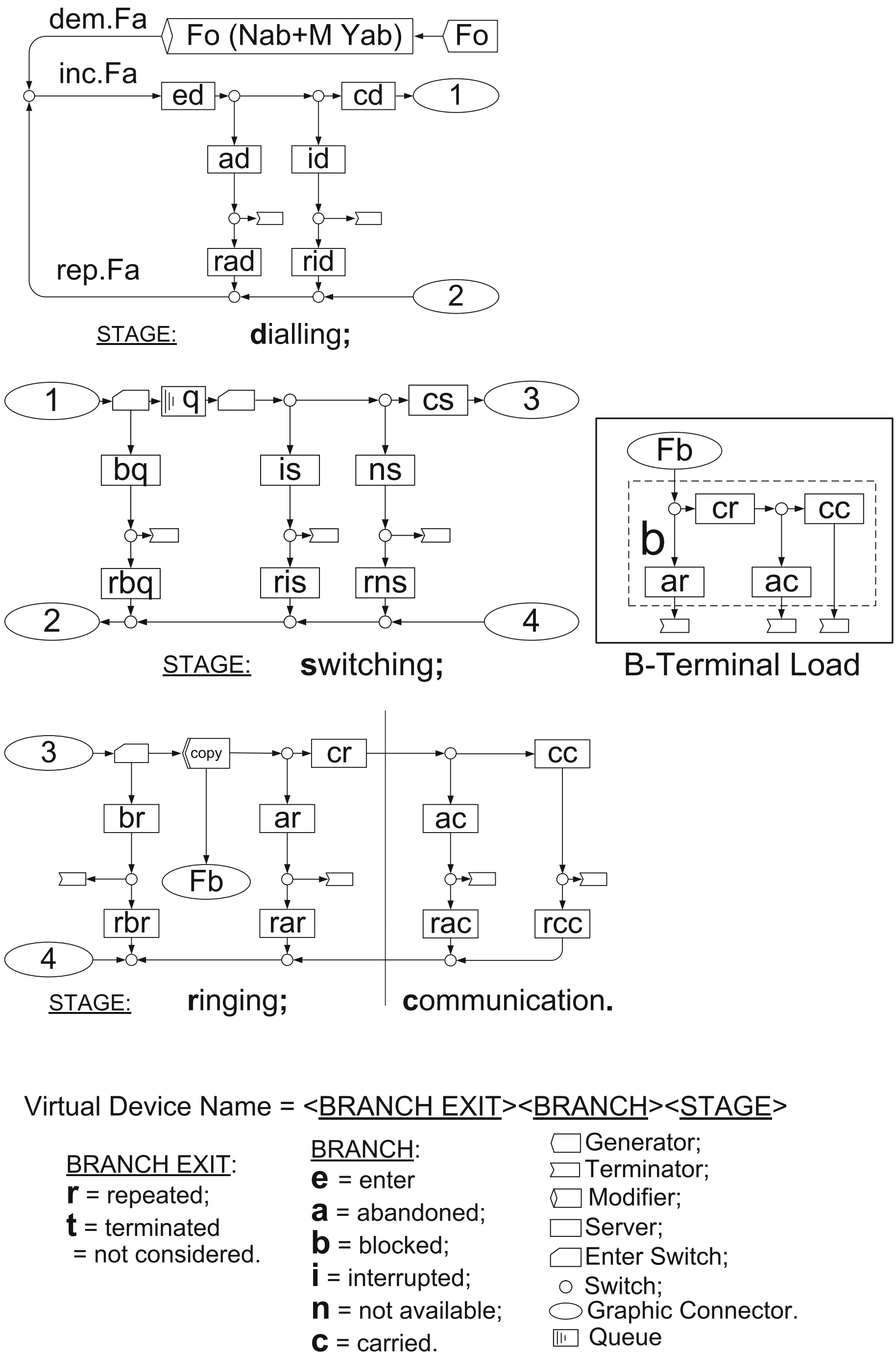


**Fig. 5.** Conceptual model of an overall telecommunication system with a queue in the Switching stage.

($dem.Fa$). The intensity of the flow of demand calls is given by the equation $dem.Fa = Fo(Nab + M'Yab)$. If $M' = 1$, the intensity of demand flow corresponds to Bernoulli (Engset) distribution. If $M' = 0$, the intensity of demand calls corresponds to the Poisson (Erlang) distribution. If $M' = 1$, the intensity of demand calls corresponds to the Pascal (Negative Binomial) distribution. In our analytical model every value of $M'$ in the interval $[-1, +1]$ is allowed. The BPP-traffic model is a very suitable one (see [3]).

To simplify the characterization of the parameters of the network we need to introduce, following [4], the terms **system tuple** and **base tuple**. A **system tuple** is a finite set of parameters' values which satisfy the following conditions:

1. All parameters in the system tuple belong to one particular system;
2. All values of the parameters are obtained (measured) during one and the same period of time;
3. The beginning and the length of the time period to which the values of the paramers correspond are elements of the system tuple.

The definition of system tuple is similar to the tuple definitions in Computer Science and Mathematics and allows for real time measurements, modeling and simulation to be performed. In practice, the duration of the time interval varies between 15 min and one hour. Since we study the system in stationary state the beginning and the duration of the time interval are not important. Every subset of the system tuple is referred to as **sub-tuple**.

**A base tuple** is a subset of the system tuple (sub-tuple) with the property that if we know the values of these parameters we may calculate the values of all other parameters of the same system tuple.

The parameters of the base tuple may be divided into two groups as follows:

– Static parameters: $M', Nab, Ns, Ted, Pad, Tad, Prad, Pid, Tid, Prid, Ted, Pis, Tis, Pris, Pns, Tns, Tes, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc, Nq, Tbq, Trbq, Prbq$. Their values are considered independent of the system state $Yab$ (see [5]) but may depend on other factors. For the model time interval they are considered constants.
– Dynamic parameters: $Fo, Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fq, crr.Fs, Tq, Pbq$. Their values are mutually dependent. Equations expressing their dependencies can be derived with the help of the graphical representation of the conceptual model in Fig. 5.

The parameters can be also classified on the basis of the origin of their values.

– Parameters related to the technical characteristics of the system: $Pid, Pis, Tcs, Ns, Nq$;
– Parameters describing the human behaviour: $Fo, Nab, Prad, Tid, Prid, Pris, Tis, Pns, Tns, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Prac, Tcc, Prcc, Tbq, Trbq, Prbq$;
– Mix factors' parameters: $Ted, Pad, Tad, Tcd, Pac, Tac$. They are dependent on the first two groups;

– Parameters whose value is determined by the modellers: $M'$. It characterizes a Bernoulli-Poisson-Pascal (BPP) flow;
– Parameters derived from the previous groups: $Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fq, crr.Fs, Tq, Pbq$.

The parameters characterizing the Quality of Service (QoS) are $Pbr, Pbs, Tq$.

This classification of the parameters allows for different types of teletrafic tasks to be formulated and solved. These tasks are divided into two groups: Stationary teletraffic tasks and Dynamic teletraffic tasks. There are three main Stationary teletraffic tasks.

– **QoS prediction task** is the task of finding values of the parameters determining the QoS ($Pbs, Pbr, Pbq$) if the parameters related to the technical characteristics and the human behavior are known. This allows for the values of the indicators for QoS to be obtained (see [7]);
– **Technical characteristics task** is the task of finding the values of those parameters related to the technical characteristics (the first group) which guarantee a given QoS if the values of the parameters describing the human behavior and the desired QoS are known. The rest of the base parameters in the same base tuple are known. The important Network Dimensioning and Redimensioning tasks belong to this type.
– **Human behavior task** is the task of finding the values of a set of parameters characterizing the behavior of the users who would generate call attempts serviced with the desired QoS if the parameters related to the technical characteristics and the QoS are given. The users' behavior can be influenced through changes in the tariff policies and the technical limitations. For instance, the allowed duration of listening to busy and dialing tone.

In the Dynamic teletraffic tasks the system's dynamic is represented as a series of tuples. There are long and short term dynamics. In the long term dynamics, all parameters of the system may have variable values while in short analysis, some of the parameters are assumed to have constant values. In the present paper systems in stationary state for a short time interval are considered.

## 4 Main Assupmtions and Derivation of Some Analytical Expressions for Parameters of the Queuing System

We consider the conceptual model of overall telecommunication system with queue shown in Fig. 5 and described in the previous section. Parameters with known values are all probabilites for directing the call to a device (the P–parameters), the holding time parameters of the base virtual devices (T –parameters) and the values of the intensity of the incoming calls flow – $F_a$. The unknown parameters are the parameters of the comprising virtual devices except $Fa$ and $Nab$. We want to express analytically the unknown parameters' values of the Queue: $Tq, Pbq, Yq$.

### 4.1 Main Assumptions

To obtain simple analytical models of the system in the process of solving different teletraffic tasks, as in [4], we need to state the following assumptions.

1. The telecommunication system considered is represented graphically and functionally in Fig. 5 and it is closed.
2. All base virtual devices except the Queue device have unlimited capacity. The Queue has capacity $Nq$ which is the buffer size. The comprising virtual devices have limited capacity: the **ab** device contains all active terminals $Nab \in [2, \infty)$. The switching system (**s**) has capacity $Ns$. One internal switching line can carry only one call for both incoming and outgoing calls.
3. Every call from the incoming flow to the system ($inc.Fa$) occupies only a free terminal which becomes a busy A-terminal.
4. The system is in a stationary state and the Little's theorem can be applied for every device.
5. Every call occupies one place in a base virtual device independently from the other devices.
6. Any calls in the telecommunication network's environment (outside the **a** and **b** devices) do not occupy any of the telecommunication system's devices.
7. The probabilities of directing the calls to the base virtual devices and the holding time in the devices are independent from each other and from the intensity of the incoming flow $inc.Fa$. Their values are determined by the users' behavior and the technical characteristics of the telecommunication system. Exception to this assumption are the devices of type Enter Switch corresponding to $Pbq$ and $Pbs$, and $Pbr$ (see Fig. 5).
8. For the base virtual devices **ar, cr, ac** and **cc** the probabilities of directing the calls to them and the duration of occupation of the device are the same for the A and B calls.
9. The variables in the model are random with fixed distributions. The Little's theorem allows us to use their mean values.
10. Every call occupies simultaneously all base virtual devices through which it has passed, including the device where it is at the current moment of observation. When a call leaves the comprising devices **a** or **b** the occupied places by it in all base virtual devices are released.

### 4.2 Expressing Analytically the Parameters of the Queue

In most publications on Queuing Theory and its applications in Telecommunication Systems the queuing systems studied have either one service line (server) or infinite servers and the buffer size of the queue is also infinite. In the few sources where queuing systems of type M|M|n|m|FIFO are studied such as [8,9], analytical expressions for their parameters such as mean duration of service, queue length, probability of blocking due to full buffer, probability of waiting in the queue etc, are only partially found. Here we start, following [8], with the simplest queuing systems to determine the queue parameters. Then, using the same approach, we determine the parameters of the queuing system M|M|Ns|Ns + Nq|FIFO, where as usual, M stands for exponential distribution, $Ns$ is the number of switching lines in Switching system (finite), $Nq$ is the length of the buffer (also finite) and FIFO stands for First In First Out discipline of service.

**Finding the Parameters of the Queuing System M|M|1|FIFO**. The density functions of the arrival and service times are respectively

$$a(t) = \lambda e^{-\lambda t}, \tag{4}$$

$$b(t) = \mu e^{-\mu t}, \tag{5}$$

where $1/\lambda$ is the mean value of time between two arrivals (interrarival time) and $1/\mu$ is the mean time of service. For our model $\lambda = ofr.Fq$ and $\mu = (crr.Fs + Fis+Fns+Fbr+Far+Fac)/Ys = crr.Fs+Fis+Fns+Fbr+Far+Fac$ because in this case there is only one service line in the Switching system. They are assumed to be statistically independent which results in a birth-death process. Let us denote with $p_n$ the probability that the queuing system is in state $n$ that is

$$p_n = Pr\{\text{there are n call attempts in the queuing system}\}.$$

There are different ways to solve the birth-death equations. The solution is well-known and can be found for example in [8]:

$$p_n = p_0(\lambda/\mu)^n, \ (n \geq 1) \tag{6}$$

$$p_0 = \frac{1}{\sum\limits_{n=0}^{\infty} (\frac{\lambda}{\mu})^n}. \tag{7}$$

Since the fraction $\lambda/\mu$ is found often below, in order to simplify the expressions we introduce the notation $\rho = \lambda/\mu$. Then the expression for $p_0$ for $\rho < 1$ becomes

$$p_0 = \frac{1}{\sum\limits_{n=0}^{\infty} \rho^n} = 1 - \rho. \tag{8}$$

Let $X_{s+q}$ be the random variable "number of call attempts in the queuing system" and $Y_{s+q}$ be its expected value. Then we obtain

$$Y_{s+q} = E[X_{s+q}] = \sum_{n=0}^{\infty} np_n = (1-\rho)\sum_{n=0}^{\infty} n\rho^n. \tag{9}$$

The last sum can be written as

$$\sum_{n=0}^{\infty} n\rho^n = \rho + 2\rho^2 + 3\rho^3 + ... = \rho\sum_{n=1}^{\infty} n\rho^{n-1} = \rho\frac{d[1/(1-\rho)]}{d\rho} = \frac{\rho}{(1-\rho)^2}. \tag{10}$$

Therefore

$$Y_{s+q} = \frac{(1-\rho)\rho}{(1-\rho)^2} = \frac{\lambda}{\mu - \lambda}. \tag{11}$$

Let $Yq$ be the expected number of call attempts in the buffer. Then

$$Yq = \sum_{n=1}^{\infty}(n-1)p_n = \sum_{n=1}^{\infty} np_n - \sum_{n=1}^{\infty} p_n = Y_{s+q} - (1-p_0) = \frac{\rho}{1-\rho} - \rho$$
$$= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}\,. \tag{12}$$

Now, using Little's theorem we can find the mean waiting time of a call attempt in the queue ($T_q$) and the mean time spent in the queuing system of a call attempt ($T_{s+q}$):

$$T_{s+q} = \frac{Y_{s+q}}{\lambda} = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu-\lambda}, \tag{13}$$

$$Tq = \frac{Yq}{\lambda} = \frac{\rho^2}{\lambda(1-\rho)} = \frac{\rho}{\mu-\lambda}. \tag{14}$$

After reverse substitution of $\lambda$, $\mu$ and $\rho$ in (11), (12), (13) and (14) we obtain

$$Y_{s+q} = \frac{ofr.Fq}{crr.Fs + Fis + Fns + Fbr + Far + Fac - ofr.Fq}. \tag{15}$$

$$Yq = \frac{(ofr.Fq)^2}{crr.Fs + Fis + Fns + Fbr + Far + Fac}$$
$$\cdot \frac{1}{crr.Fs + Fis + Fns + Fbr + Far + Fac - ofr.Fq}. \tag{16}$$

$$T_{s+q} = \frac{1}{crr.Fs + Fis + Fns + Fbr + Far + Fac - ofr.Fq}. \tag{17}$$

$$Tq = \frac{ofr.Fq}{crr.Fs + Fis + Fns + Fbr + Far + Fac}$$
$$\cdot \frac{1}{crr.Fs + Fis + Fns + Fbr + Far + Fac - ofr.Fq}. \tag{18}$$

**Finding Parameters of the Queuing System M|M|Ns|FIFO**. In this queuing system, there are $Ns$ serving lines in the Switching system and the buffer has infinite length. The serving lines are assumed to have independent and identically exponentially distributed service times and the arrival process is Poisson. Again, we have a birth-death process and $\lambda_n = \lambda$ for all $n$. If there are more than $Ns$ call attempts in the Queuing system, i.e. $Ys + Yq > Ns$, all $Ns$ lines in the Switching system are occupied and each of them is serving the call attempts with mean rate $\mu = (crr.Fs + Fis + Fns + Fbr + Far + Fac)/Ys$ and the output rate is $Ns\,\mu$. If there are $n$ call attempts in the queuing system and $n < Ns$, only $n$ of the Switching lines are occupied and the system output rate is $\mu_n$ where

$$\mu_n = \begin{cases} n\mu & \text{for } 1 \leq n < Ns. \\ Ns\ \mu & \text{for } n \geq Ns. \end{cases} \tag{19}$$

Through the same procedure as in the case with one line in the Switching system we obtain the probability of the system to be in state $n$:

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & \text{for } 1 \leq n < Ns. \\ \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} p_0 & \text{for } n \geq Ns. \end{cases} \tag{20}$$

Similarly, since the sum of the probabilities must be equal to 1, we obtain

$$p_0 = \left( \sum_{n=0}^{Ns-1} \frac{\lambda^n}{n!\mu^n} + \sum_{n=Ns}^{\infty} \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} \right)^{-1} . \tag{21}$$

To simplify the expressions we introduce the notation $r = \lambda/\mu$ and $\rho = r/Ns = \lambda/(Ns\mu)$. Now the expression for $p_0$ becomes

$$p_0 = \left( \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \sum_{n=Ns}^{\infty} \frac{r^n}{Ns^{n-Ns} Ns!} \right)^{-1} . \tag{22}$$

Since $r/Ns = \rho < 1$, the second sum above can be further simplified in the following way:

$$\begin{aligned} \sum_{n=Ns}^{\infty} \frac{r^n}{Ns^{n-Ns} Ns!} &= \frac{r^{Ns}}{Ns!} \sum_{n=Ns}^{\infty} \left( \frac{r}{Ns} \right)^{n-Ns} = \frac{r^{Ns}}{Ns!} \sum_{m=0}^{\infty} \left( \frac{r}{Ns} \right)^{m} \\ &= \frac{r^{Ns}}{Ns!} \frac{1}{1-\rho}. \end{aligned} \tag{23}$$

After substitution of (23) in (22) we obtain

$$p_0^{-1} = \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} \frac{1}{1-\rho}. \tag{24}$$

For the expected length of the queue $Yq$ we have

$$\begin{aligned} Yq = \sum_{n=Ns+1}^{\infty} (n-Ns) p_n &= \sum_{n=Ns+1}^{\infty} (n-Ns) \frac{r^n}{Ns^{n-Ns} Ns!} p_0 = \frac{r^{Ns} p_0}{Ns!} \sum_{i=1}^{\infty} i\rho^i \\ &= \frac{r^{Ns} \rho\, p_0}{Ns!} \sum_{i=1}^{\infty} i\rho^{i-1} = \frac{r^{Ns} \rho\, p_0}{Ns!} \frac{d}{d\rho} \frac{1}{1-\rho} = \frac{r^{Ns} \rho\, p_0}{Ns!(1-\rho)^2} \,. \end{aligned} \tag{25}$$

Again, using Little's formula we obtain the mean waiting time of a call in the queue:

$$Tq = \frac{Yq}{\lambda} = \frac{r^{Ns}}{Ns!\ Ns\ \mu(1-\rho)^2} p_0. \tag{26}$$

Now, we can find the expected number of call attempts in the queuing system ($Y_{s+q}$). First we notice that

$$T_{s+q} = Ts + Tq = \frac{r^{Ns}}{Ns!\ Ns\ \mu(1-\rho)^2} p_0 + \frac{1}{\mu}. \tag{27}$$

From the Little's formula we have $Y_{s+q} = \lambda T_{s+q}$. Therefore

$$Y_{s+q} = r + \frac{r^{Ns}\rho}{Ns!(1-\rho)^2} p_0. \tag{28}$$

Recall that

$$r = \frac{\lambda}{\mu} = \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} \tag{29}$$

and

$$\rho = \frac{r}{Ns} = \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}. \tag{30}$$

After substitution of (29) and (30) in (24), (25), (26), (27) and (28) we obtain the following expressions for the parameters of the queue:

$$\begin{aligned} Yq = &\left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \\ &\cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \cdot p_0 \\ &\cdot \frac{1}{Ns!\left(1 - \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^2}, \end{aligned} \tag{31}$$

where

$$\begin{aligned} p_0^{-1} = &\sum_{n=0}^{Ns-1} \left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^n \frac{1}{n!} \\ &+ \left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \frac{1}{Ns!} \\ &\cdot \frac{1}{1 - \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}}. \end{aligned} \tag{32}$$

$$Tq = \left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \cdot \frac{Ys}{Ns!Ns(crr.Fs + Fis + Fns + Fbr + Far + Fac)} \cdot \frac{1}{\left(1 - \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^2} p_0. \quad (33)$$

$$T_{s+q} = \left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \cdot \frac{Ys}{Ns!Ns(crr.Fs + Fis + Fns + Fbr + Far + Fac)} \cdot \frac{1}{\left(1 - \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^2} p_0 + \frac{Ys}{crr.Fs + Fis + Fns + Fbr + Far + Fac}. \quad (34)$$

$$Y_{s+q} = \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} + \left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \cdot \frac{1}{Ns!\left(1 - \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^2} p_0. \quad (35)$$

**Finding Parameters of the Queuing System M|M|Ns|Ns + Nq|FIFO.** Finally, we consider the queuing system which is used in the conceptual model of overall telecommunication system with queue in the switching stage. The difference between this queuing system and the one from the previous section is that the buffer has finite length denoted by $Nq$. This sets a limit on the total number of call attempts in the queuing system – they cannot be more than $Nq + Ns$. Although often used, we could not find in literature expressions for all parameters of the queuing system in one source. There are partial results in [8,9]. Here, by analogy with the previous types of queuing systems we find analytical expressions for the parameters of the queue. First, we notice that the arrival rate $\lambda_n$ is equal to 0 when $n \geq Ns + Nq$. The probability for the system to be in state $n$ is now given by

$$p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & \text{for } 1 \leq n < Ns. \\ \frac{\lambda^n}{Ns^{n-Ns} Ns!\mu^n} p_0 & \text{for } Ns \leq n \leq Ns + Nq. \end{cases} \quad (36)$$

Again, the condition that the sum of the probabilities $p_n$ should be equal to 1, gives us the following expression for $p_0$:

$$p_0 = \left( \sum_{n=0}^{Ns-1} \frac{\lambda^n}{n!\mu^n} + \sum_{n=Ns}^{Ns+Nq} \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} \right)^{-1} . \tag{37}$$

In order to simplify the expression we set $r = \lambda/\mu$ and $\rho = r/Ns$. For the second sum in (37) we have

$$\sum_{n=Ns}^{Ns+Nq} \frac{\lambda^n}{Ns^{n-Ns} Ns! \mu^n} = \frac{r^{Ns}}{Ns!} \sum_{n=Ns}^{Ns+Nq} \rho^{n-Ns}$$
$$= \begin{cases} \frac{r^{Ns}}{Ns!} \frac{1-\rho^{Nq+1}}{1-\rho} & \text{for } \rho \neq 1. \\ \frac{r^{Ns}}{Ns!} (Nq+1) & \text{for } \rho = 1\,. \end{cases} \tag{38}$$

After substitution in (37) we obtain

$$p_0^{-1} = \begin{cases} \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} \frac{1-\rho^{Nq+1}}{1-\rho} & \text{for } \rho \neq 1\,. \\ \sum_{n=0}^{Ns-1} \frac{r^n}{n!} + \frac{r^{Ns}}{Ns!} (Nq+1) & \text{for } \rho = 1. \end{cases} \tag{39}$$

For the expected length of the queue in this case we have

$$Yq = \sum_{n=Ns+1}^{Ns+Nq} (n - Ns) p_n = \frac{p_0 r^{Ns}}{Ns!} \sum_{n=Ns+1}^{Ns+Nq} \frac{(n-Ns) r^{n-Ns}}{Ns^{n-Ns}}$$
$$= \frac{p_0 r^{Ns} \rho}{Ns!} \sum_{n=Ns+1}^{Ns+Nq} (n - Ns) \rho^{n-Ns-1} = \frac{p_0 r^{Ns} \rho}{Ns!} \sum_{i=1}^{Nq} i \rho^{i-1}$$
$$= \frac{p_0 r^{Ns} \rho}{Ns!} \frac{d}{d\rho} \left( \frac{\rho - \rho^{Nq+1}}{1-\rho} \right) = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(1 - \rho^{Nq}(Nq+1))(1-\rho) + \rho - \rho^{Nq+1}]$$
$$= \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(\rho - 1)\rho^{Nq}(Nq+1) + 1 - \rho^{Nq+1}]\,. \tag{40}$$

The above holds for $\rho \neq 1$.

To obtain the number of call attempts in the system $Y_{s+q}$ we notice that a part $p_{Ns+Nq}$ of the incoming flow of call attempts are blocked because the buffer has finite length $Nq + Ns$. This probability in the conceptual model is equal to $Pbq$. Therefore, the incoming rate becomes $\lambda(1 - Pbq)$ and as in (28) we have

$$Y_{s+q} = Yq + Ys = \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(\rho - 1)\rho^{Nq}(Nq+1) + 1 - \rho^{Nq+1}] + \frac{\lambda(1 - Pbq)}{\mu}$$
$$= \frac{p_0 r^{Ns} \rho}{Ns!(1-\rho)^2} [(\rho - 1)\rho^{Nq}(Nq+1) + 1 - \rho^{Nq+1}] + r(1 - Pbq). \tag{41}$$

With the Little's formula we obtain

$$T_{s+q} = \frac{Y_{s+q}}{\lambda(1 - Pbq)}. \tag{42}$$

and

$$Tq = T_{s+q} - \frac{1}{\mu} = \frac{Yq}{\lambda(1-Pbq)}$$
$$= \frac{p_0 r^{Ns}\rho}{Ns!(1-\rho)^2}\frac{[(\rho-1)\rho^{Nq}(Nq+1)+1-\rho^{Nq+1}]}{\lambda(1-Pbq)}. \quad (43)$$

Finally, the probability of blocked queue ($Pbq$) is equal to the probability that the system is in state $Ns + Nq$ and from (36) we have

$$Pbq = \frac{\lambda^{Ns+Nq}}{Ns^{Nq}Ns!\mu^{Ns+Nq}}p_0. \quad (44)$$

Recall that

$$r = \frac{\lambda}{\mu} = \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} \quad (45)$$

and

$$\rho = \frac{r}{Ns} = \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}. \quad (46)$$

After substitution in (40)–(44) we obtain

$$\begin{aligned} Yq = p_0 &\left(\frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac}\right)^{Ns} \\ &\cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \cdot \frac{1}{Ns!} \\ &\cdot \frac{1}{[1 - \frac{ofr.Fq.Ys}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}]^2} \\ &\cdot \left[\left(\frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} - 1\right)\right. \\ &\cdot \left(\frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^{Nq}(Nq+1) + 1 \\ &\left. - \left(\frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}\right)^{Nq+1}\right], \end{aligned} \quad (47)$$

where

$$\begin{aligned} p_0^{-1} = &\sum_{n=0}^{Ns-1}\frac{(ofr.Fq.Ys)^n}{n!(crr.Fs + Fis + Fns + Fbr + Far + Fac)^n} \\ &+ \sum_{n=Ns}^{Ns+Nq}\frac{(ofr.FqYs)^n}{Ns^{n-Ns}Ns!(crr.Fs + Fis + Fns + Fbr + Far + Fac)^n}. \end{aligned} \quad (48)$$

$$Y_{s+q} = p_0 \left( \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} \right)^{Ns}$$
$$\cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \cdot \frac{1}{Ns!}$$
$$\cdot \frac{1}{[1 - \frac{ofr.Fq.Ys}{(crr.Fs\ +\ Fis\ +\ Fns\ +\ Fbr\ +\ Far\ +\ Fac)Ns}]^2}$$
$$\cdot \left[ \left( \frac{ofr.FqYs}{(crr.Fs\ +\ Fis\ +\ Fns\ +\ Fbr\ +\ Far\ +\ Fac)Ns} - 1 \right) \right.$$
$$\cdot \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq} (Nq + 1) + 1$$
$$\left. - \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq+1} \right]$$
$$+ \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} (1 - Pbq). \quad (49)$$

$$T_{s+q} = \left[ p_0 \left( \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} \right)^{Ns} \right.$$
$$\cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \cdot \frac{1}{Ns!}$$
$$\cdot \frac{1}{[1 - \frac{ofr.Fq.Ys}{(crr.Fs\ +\ Fis\ +\ Fns\ +\ Fbr\ +\ Far\ +\ Fac)Ns}]^2}$$
$$\cdot \left[ \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} - 1 \right) \right.$$
$$\cdot \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq} (Nq + 1) + 1$$
$$\left. - \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq+1} \right]$$
$$\left. + \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} (1 - Pbq) \right] \cdot \frac{1}{ofr.Fq(1 - Pbq)}. \quad (50)$$

$$Tq = p_0 \left( \frac{ofr.FqYs}{crr.Fs + Fis + Fns + Fbr + Far + Fac} \right)^{Ns}$$
$$\cdot \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns}$$
$$\cdot \frac{1}{Ns! \left( 1 - \frac{ofr.FqYs}{(crr.Fs\ +\ Fis\ +\ Fns\ +\ Fbr\ +\ Far\ +\ Fac)Ns} \right)^2}$$

$$\cdot \left[ \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} - 1 \right) \right.$$

$$\cdot \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq} (Nq + 1)$$

$$\left. + 1 - \left( \frac{ofr.FqYs}{(crr.Fs + Fis + Fns + Fbr + Far + Fac)Ns} \right)^{Nq+1} \right]$$

$$\cdot \frac{1}{ofr.Fq(1 - Pbq)}. \tag{51}$$

$$Pbq = \frac{(ofr.FqYs)^{Ns+Nq}}{Ns^{Nq}\, Ns!(crr.Fs + Fis + Fns + Fbr + Far + Fac)^{Ns+Nq}} .p_0. \tag{52}$$

## 5 Conclusion

The conceptual model of overall telecommunication system with queue described here in details is a base for the development of analytical model of the network. The analytical model can be used to solve important teletraffic tasks such as dimensioning and redimensioning of the network and predicting the QoS. The analytical expressions obtained in Sect. 4 for the parameters of the queuing system are the first step in the development of the analytical model.

In our future work, we shall use the conceptual model described in the present paper and the expressions for the parameters of the queuing system to obtain a system of equations for the dynamic parameters. We shall propose numerical methods for solving the resulting non-linear system.

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