Evaluation of Channel Estimation Algorithms Using Practically Measured Channels in FDD Massive MIMO

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Abstract. An important problem for massive multiple-input multiple-output (MIMO) systems operating with frequency-division duplexing (FDD) is to accurately estimate the channel response with low pilot signal overhead. Most existing algorithms for efficient channel estimation are based on compressive sensing (CS) and assume sparse structure of the channel vector. Relying on it, they try to minimize estimation error and reduce the number of required pilot signals. Utilizing real-world channel responses, we evaluate the performance of 11 state-of-the-art channel estimation algorithms for FDD massive MIMO systems. Results from simulation experiments with channel measurements for carrier frequency in the 2.4 GHz and 5 GHz bands for three environments and two levels of mobility are presented. Channel structures of theoretical and practically measured channels are compared and it is shown that the latter does not follow a specific sparse structure which leads to a significant increase in estimation errors according to our results. A comprehensive analysis of estimation quality and its dependence on signal-to-noise ratio (SNR) and number of pilot signals is provided. The results demonstrate that some algorithms perform well when applied to practical channels while others do not provide confident results. The effects of pilot matrix choice and angular domain channel representation are also studied and evaluated.

Keywords: Channel estimation · Massive Mimo · Practical channels · Frequency-division duplexing · Compressive sensing

1 Introduction

Up to the present moment, the amount of wireless communications has been growing at an exponential pace for many decades [2]. In order to satisfy the vast demands for mobile data rate and capacity, 5G techniques will be employed for future wireless networks. One potential technology to support this growth is the massive MIMO [9,16] which is a promising solution to handle several orders of magnitude increase in wireless data traffic than current technologies [2]. In order

V. Poulkov (Ed.): FABULOUS 2019, LNICST 283, pp. 228–242, 2019.
https://doi.org/10.1007/978-3-030-23976-3_21

to process the uplink (UL) and downlink (DL) signals and to fully exploit the potential benefits for efficient spectrum and energy utilization, accurate information about the channel responses is needed which presents one of the key challenges in practical application of massive MIMO [2]. The channel responses need to be estimated regularly and the current set of channel response realizations is called the channel state whereas the knowledge that the base station (BS) has of them is referred to as the channel state information (CSI) [2].

The main method for CSI acquisition is pilot signaling. To estimate the channel response from N transmitting antennas, N orthogonal pilot signals are required in order to ensure signal separation which introduces overhead and wastes resources [2]. In traditional systems, the BS sends pilots to user equipments (UEs) which feedback the DL channel estimation to the BS which does not scale well with the number of antennas at the BS [11]. In a system with K users utilizing time-division duplexing (TDD), the channels in the UL and DL are assumed to be reciprocal so the pilot overhead is proportional to K [2,17]. If FDD is used, the channels in the UL and DL are different [17] which leads to a pilot and feedback overhead of $N + K/2$ on average if the frequency resources are divided equally between UL and DL and the system operates in the preferable regime with $N/K \geq 4$ [2]. Such overhead is prohibitive for mobile scenarios, however designing and demonstrating an efficient FDD massive MIMO implementation is a great challenge which needs to be solved [2]. This is the reason why our work is focused on evaluating efficient channel estimation algorithms FDD systems.

1.1 Related Channel Estimation Techniques and Algorithms for FDD Massive MIMO Systems

One major approach to reduce the pilot and CSI feedback overheads in FDD massive MIMO systems is to exploit the hidden sparsity and low-rank properties of the massive MIMO channel via CS and sparse recovery methods [5,11,14]. According to CS, a signal which exhibits sparsity in some transformation domain can be recovered from far fewer samples than those required by the classical Shannon-Nyquist theorem [3]. Hence, channel estimation in massive MIMO systems can be realized by (i) transforming channel measurements into sparse matrices, (ii) compressing the sparse signals into signals with far lower dimensions than real channel estimates and (iii) recovering the original signals from the compressed signals. The goal is to estimate large-sized channels from small-sized measurements using few pilot signals by carefully designing the transformation matrix.

Examples for CS-based algorithms are the classical orthogonal matching pursuit (OMP) [6], least absolute shrinkage and selection operator (LASSO) [4,11], maximum likelihood (ML), expectation-maximization (EM), Turbo-CS [12] and others. The classical OMP algorithm [6] is a straightforward extension of the CS model to CSI estimation problems without assuming any common structure among channel responses of different users. The joint OMP (J-OMP) [14] exploits the hidden joint sparsity structure in the user channel matrices due to the shared local scatterers in the physical propagation environment. The select-discard simultaneous OMP (SD-SOMP) [10] is a universal robust recovery algorithm under different joint sparsity models. Compressive Sampling Matching Pursuit (CoSaMP) [6,13] is an iterative greedy algorithm which recovers the channels individually without taking into account any particular sparsity structure. The Distributed Sparsity Adaptive Matching Pursuit (DSAMP) [7] leverages the spatially common sparsity of massive MIMO channels to jointly estimate multiple channels associated with different subcarriers. The L1 LASSO [4,11] is a ℓ_1 minimization problem which aims to introduce a sparse structure in the recovered channel whereas the burst LASSO [11] assumes that the channel response has a burst sparse structure. Both LASSO algorithms recover the channel response individually while the joint burst LASSO algorithm [11] exploits the additional joint burst-sparse structure in MU massive MIMO channels. The EM Bernoulli-Gaussian (BG) approximate message passing (AMP), EM-BG-AMP [19], is a signal reconstruction algorithm which models the signal as i.i.d BG with unknown prior sparsity, mean and variance, while the noise is considered as zero-mean Gaussian with unknown variance. The signal is simultaneously reconstructed while learning the prior signal and noise parameters [19]. The Turbo-CS [12] algorithm is based on the turbo principle in iterative decoding. It consists of a minimum mean squared error (MSE)—MMSE, and a linear MMSE (LMMSE) estimators and assumes an i.i.d. prior distribution of the channel response. However, it cannot exploit the structured sparsity of massive MIMO channels and the structured Turbo-CS [5] algorithm was proposed to overcome this limitation by assuming a Markov prior. The conventional Least Squares (LS) method correlates the received signal with the known pilot sequence, but suffers from lack of orthogonality between desired and interfering pilots (pilot contamination). Hence, the estimation performance is limited by the signal-to-interference ratio at the BS [20]. The performance in terms of CSI recovery error of some of these CS methods is experimentally verified in this work and the results are presented in Sect. 4.

Other approaches to reduce the pilot and CSI feedback overheads in FDD massive MIMO systems are to use channel parametrizations [2], the opportunistic channel sounding policies [8] and methods exploiting machine learning and artificial neural networks.

1.2 Contributions

This work evaluates the practical performance of state-of-the-art channel estimation algorithms with real-world channel responses for application in FDD massive MIMO systems. Simulation experiments to demonstrate the dependence of CSI recovery error of the algorithms on SNR, number of pilot signals, pilot matrix choice and channel response representation have been carried out. The results are compared with a baseline for a realistic theoretical channel model. The channel structures of the theoretical model and practical channel responses are compared. To the best of the authors' knowledge, such study has not been considered in the literature before.

1.3 Structure of the Paper

The rest of the paper is organized as follows. The system model is presented in Sect. 2. A description of the measurement data and used methodology follow in Sect. 3. In Sect. 4, analysis and discussions of the simulation results are provided. Finally, Sect. 5 concludes the paper and highlights future research directions on the topic.

2 System Model

In the present work, we consider a flat block-fading MU massive MIMO system operating in FDD mode. There is one BS with N antennas serving K single-antenna user terminals. The BS transmits a sequence of M pilot signals $\mathbf{x}_t^{\mathsf{H}} \in \mathbb{C}^{1\times N}$, $t = 1,\ldots,M$ for estimating the downlink channel. User k receives the signal $\mathbf{y}_k \in \mathbb{C}^{M\times 1}$

$$\mathbf{y}_k = \mathbf{X}\mathbf{h}_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{X} = [\mathbf{x}_1,\ldots,\mathbf{x}_M]^{\mathsf{H}} \in \mathbb{C}^{M\times N}$ is a pilot matrix which is known in both the BS and UE, $\mathbf{h}_k \in \mathbb{C}^{N\times 1}$ is the channel response of user k and $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0},\sigma^2\mathbf{I}) \in \mathbb{C}^{M\times 1}$ is the additive complex Gaussian noise at user k with each element having zero mean and variance σ^2.

In many related works (e.g., [11,14]), the pilot signals matrix $\mathbf{X}$ is selected to have independent and identically distributed (i.i.d.) Gaussian elements. Nevertheless, as elaborated upon in [12], a partial orthogonal sensing matrix achieves better performance under the Turbo-CS algorithm than an i.i.d. Gaussian sensing matrix which is experimentally confirmed for various other algorithms in [5]. Therefore, the present work utilizes a partial discrete Fourier transform (DFT) random permutation (PDFT-RP) pilot matrix modeled as presented in [5]. Nevertheless, experiments have also been carried out with an i.i.d Gaussian sensing matrix to verify the performance gain.

Some works consider the channel transformed into the virtual angular domain $\mathbf{h}_k^{\omega} = \mathbf{F}\mathbf{h}_k$ where $\mathbf{F} \in \mathbb{C}^{N\times N}$ denotes the unitary matrices for the angular domain transformation at the BS [5,11,14]. Resulting from the limited scatterers at the BS, $\mathbf{h}_k^{\omega}$ usually exhibits individual burst sparsity due to local scattering at the BS and joint sparsity due to common scattering at the BS [11]. Assuming an angular domain transformation, the received signal 1 can be rewritten as

$$\mathbf{y}_k = \mathbf{X}\mathbf{F}^{\mathsf{H}}\mathbf{h}_k^{\omega} + \mathbf{n}_k = \mathbf{A}\mathbf{h}_k^{\omega} + \mathbf{n}_k, \tag{2}$$

which is a standard CS model with sensing matrix $\mathbf{A}$ and sparse channel $\mathbf{h}_k^{\omega}$.

3 Description of the Measurement Data and Methodology

Since the massive MIMO concept started to gain research interest around 2010, a number of testbeds to demonstrate the feasibility of massive MIMO systems have been developed by academia and industry. Some of the first publications describing practical design, realization, and evaluation of such systems are with regard to the Argos prototype by Rice University [17,18]. A detailed analysis of practically measured massive MIMO channels and their properties is presented in [16]. With the help of the Argos system, the authors have conducted a comprehensive many-antenna multi-user (MU) MIMO channel measurement campaign resulting in over 100 traces made publicly available for further research on [1]. The dataset spans 20 topologies providing over one billion channel measurements and approximately 1 terabyte of data covering measurements across the UHF (470–698 MHz), 2.4 GHz, and 5 GHz bands in diverse environments. At 2.4 GHz and 5 GHz, up to 104 BS antennas are deployed to serve 8 UEs.

Throughout the present work, this measurement dataset was utilized for evaluating various CS-based channel estimation methods. It was selected because it consists of a rich set of practically measured wireless channel responses in multiple environments with various levels of mobility and at three frequency bands. Moreover, this was the only publicly available massive MIMO measurement dataset at the time of writing this paper to the best of the authors' knowledge. Specifically, we use the "Asilomar2016" dataset described in [16].

Out of the dataset, 8 traces with carrier frequency in the 2.4 GHz and 5 GHz bands were selected for experimental analysis. They were conducted in three environments—indoor line of sight (LOS) and non-LOS (NLOS), as well as outdoor with two types of mobility—static and environmental [16]. The reason behind choosing only two types of marginal mobility is that our aim is to compare and evaluate the performance of various channel estimation algorithms and not to study the effects of mobility on channel correlation and aging. Nevertheless, aspects related to mobility can be further exploited for efficient channel estimation techniques, such as the opportunistic method outlined in [8]. From the selected 8 channel traces, only some subcarriers and frames were used in the experiments in order to reduce computation time amounting to a total of around 3840 simulated channel responses.

The authors of ArgosV2 provide a channel measurement and analysis software framework [1] which computes the actual frequency response of the wireless channel. Using this framework, the normalized magnitude of three wireless channels is depicted in Fig. 1—the theoretical 3rd Generation Partnership Project (3GPP) spatial channel model (SCM) [15] and two practically measured channels. The theoretical channel response (a) has a burst sparse nature and could also be jointly burst sparse among users depending on the multipath environment [11]. On the other hand, the practically measured channel responses (b) and (c) have many more significant elements and do not follow any certain sparsity structure, independent on the environment, scatterers located therein and carrier frequency.

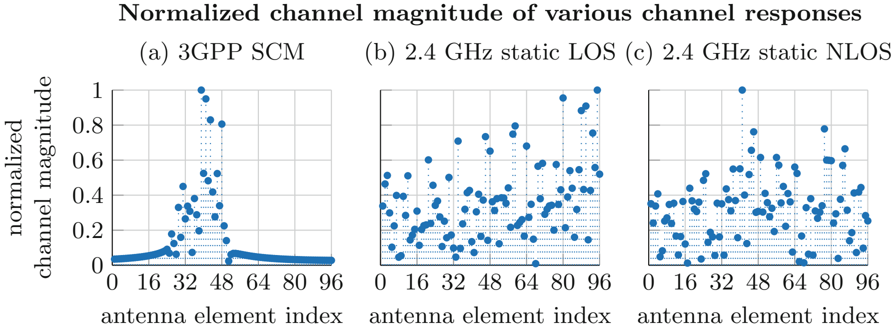

Fig. 1. Comparison of normalized channel magnitude of various channel responses—theoretical (3GPP SCM) and practically measured, for a BS antenna with 96 elements.

4 Simulation Results and Analysis

In this Section, the performance in terms of CSI recovery error of 11 CS-based channel estimation algorithms is evaluated and compared by utilizing practically measured channel responses as described in Sect. 3. Ranging from well-known estimators to algorithms tailored specifically to the massive MIMO channel response structure, the algorithms are listed with their specifics in Table 1. The algorithms were selected based on their applicability to FDD massive MIMO systems and reported low estimation error.

Table 2 presents an overview of the main simulation parameters. The Argos system operates in TDD and it is assumed that the UL and DL channel responses are perfectly reciprocal [2,16]. Hence, the channel response estimated by the Argos system can be used as the channel response to be estimated in the DL in Eqs. 1 and 2. The number of users K and of antenna elements in the BS array N match the Argos testbed measurements. The pilot signals M and SNR values are chosen in accordance with the widely used scenarios in recent works [5,11,14] and larger bounds for M are considered in order to highlight algorithm behavior in the borderline cases. The two selected carrier frequency bands are broadly used in modern wireless communications below 6 GHz. The environments and mobility levels selected for the simulation were outlined in Sect. 3. Although the practically measured channel response by the ArgosV2 testbed is mainly used, the CSI recovery error is compared with the results achieved with the theoretical 3GPP SCM channel model. Some of the experiments also consider the virtual angular domain channel representation $\mathbf{h}^{\omega}$ to illustrate how it affects estimation performance. Several experiments have also been carried out with an i.i.d. Gaussian pilot signals matrix to compare estimation quality with the PDFT-RP pilot signals matrix.

The normalized MSE (NMSE) of the estimated CSI was selected to serve as the performance metric as it is well-established in the literature for ranking algorithm performance [5,11,14,19,20]. The NMSE is defined as

Table 1. Simulated channel estimation algorithms and methods.

Algorithm	$\mathbf{h}_k$ recovery	Assumptions and comments	References
Classical OMP	*individual*	A naive extension of CS to CSI estimation	[6]
J-OMP	*joint*	Hidden joint sparsity is exploited	[14]
SD-SOMP	*joint*		[10]
CoSaMP	*individual*		[6,13]
DSAMP	*joint*		[7]
L1 LASSO	*individual*		[4,11]
Burst LASSO	*individual*	Burst sparsity in the structure of $\mathbf{h}_k$	[11]
EM-BG-AMP	*individual*	Apriori independent and Bernoulli-Gaussian distributed coefficients	[19]
Turbo-CS	*individual*	i.i.d. prior	[12]
Structured Turbo-CS	*individual*	Markov prior to model the structured sparsity of $\mathbf{h}_k$	[5]
Conventional LS	*individual*	$\hat{\mathbf{h}}_k = \mathbf{y}_k\mathbf{X}^\dagger$, $\mathbf{X}^\dagger$ – Moore-Penrose pseudoinverse	[14]

$$\text{NMSE} = \frac{1}{K}\sum_{k=1}^{K}\frac{\|\mathbf{h}_k - \hat{\mathbf{h}}_k\|^2}{\|\mathbf{h}_k\|^2}, \tag{3}$$

where $\hat{\mathbf{h}}_k \in \mathbb{C}^{N\times 1}$ is the estimated channel response vector. For clear representation and better readability of the results, the NMSE in decibels defined as $\text{NMSE(dB)} = 10\log_{10}\text{NMSE}$ is depicted. It is of high interest to draw the dependence of NMSE on M because the main goal of evaluated algorithms is to minimize M while maintaining a feasible error. On the other hand, noise can have detrimental effect on estimation errors, hence dependence of NMSE on SNR is also studied.

4.1 Average NMSE as a Function of SNR

Figure 2 compares the NMSE of the estimated CSI in decibels depending on the SNR of the received signal $\mathbf{y}_k$. The NMSE results have been averaged over all environments, frames, subcarriers and carrier frequency bands described in Sect. 3. An exception is made for the L1 and burst LASSO algorithms due to the high computational complexity of the burst LASSO, therefore simulations for only a single subcarrier and frame were carried out for these two algorithms.

The performance of analysed algorithms using the 3GPP SCM [15] is plotted with dotted lines to serve as a reference. Based on the difference between NMSE

Table 2. Simulation parameters.

Parameter	Notation	Modeling	Value	Dimension
Number of UEs	K	[16]	8	—
Number of BS antennas	N	[16]	96	—
Number of pilot signals	M	—	15–90	—
Carrier frequency bands	f	—	2.4; 5	GHz
SNR	SNR	—	0–40	dB
Channel response	$\mathbf{h}$	[15,16]	3GPP SCM; ArgosV2 measured	—
Pilot matrix	$\mathbf{X}$	[5,11,12]	PDFT-RP; i.i.d. CG	—
Noise	$\mathbf{n}$	[5,11]	Additive complex Gaussian	—
Environment	—	[16]	Indoor (LOS, NLOS); outdoor	—
Mobility	—	[16]	Static; environmental	—

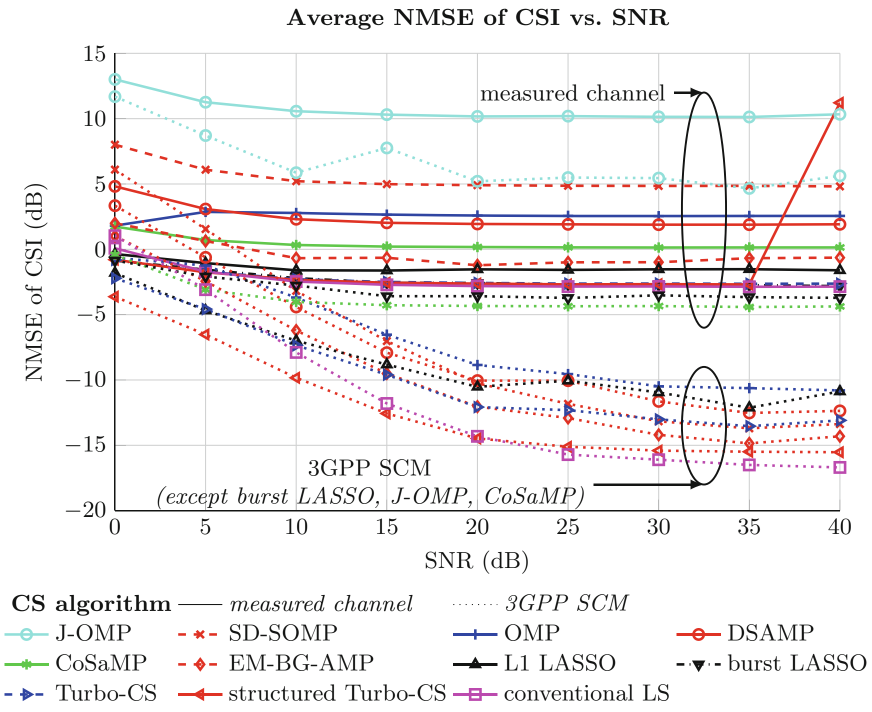

Fig. 2. Average NMSE of CSI in decibels versus SNR of analyzed algorithms for $M =$ 45 pilot signals. Results achieved with the 3GPP SCM [15] are depicted as a reference.

for practically measured channel responses and for channels generated with the 3GPP SCM, it can be concluded that all algorithms perform better when using the burst sparse channel response provided by the 3GPP SCM. With this model, all CS-based methods apart from the J-OMP provide a negative NMSE of down to -16 dB at SNR > 10 dB . Lowest NMSE is achieved with the structured Turbo-CS algorithm for SNR < 20 dB, while conventional LS provides lowest NMSE for higher SNR. EM-BG-AMP, SD-SOMP and Turbo-CS also perform moderately well at high SNR. Notably, highest error under this channel model is achieved by the CoSaMP, burst LASSO and J-OMP algorithms. The J-OMP algorithm exploits the hidden joint sparsity among user channel vectors in order to recover CSI with a smaller error [14]. As demonstrated in Fig. 1, such joint sparsity is not present in the measured channel responses. This could explain the low performance of J-OMP which would perhaps be further improved by fine-tuning algorithm parameters. This assumption is valid for all algorithms—the achieved results depend on the particular settings of algorithm-specific parameters. It is important to note that the structured Turbo-CS algorithm does not perform well at the highest simulated SNR $= 40$ dB setting. Such errors can be observed in other results described further in the work and could be based on slow convergence or ill-conditioning.

Figure 3(left) illustrates only the algorithms whose performance is feasible for practical implementation, i.e., which achieve NMSE < 0 dB. The conventional LS algorithm achieves best performance with a NMSE of down to around -2.8 dB followed by the burst LASSO and Turbo-CS (both in its canonical and structured variants) algorithms. However, at low SNR ≈ 0 dB, the LS algorithm recovers the channel vector with an unacceptable error. The L1 LASSO and EM-BG-AMP algorithms provide higher error while the dependence of EM-BG-AMP on SNR is inconsistent. The algorithms based on message passing, such as the EM-BG-AMP and structured Turbo-CS, learn the required channel statistical parameters automatically by the EM framework as pointed out in [5].

Both Figs. 2 and 3 show that the achieved error does not drop significantly when increasing the SNR after 20 dB. This is the reason why this setting was chosen for estimating the dependence of algorithm performance on the number of pilot signals M.

4.2 Average NMSE as a Function of the Number of Pilot Signals M

Figure 4 demonstrates the dependence of algorithm performance in terms of NMSE of CSI in decibels on the number of pilot signals M. The averaging explained in Subsect. 4.1 was applied. Achieved NMSE when using the 3GPP SCM [15] is plotted with dotted lines with NMSE down to almost -20 dB for $M = 90$ and the LS algorithm with all other algorithms achieving similar performance except the CoSaMP, J-OMP and burst LASSO. However, J-OMP performs well after $M = 70$ pilot signals. All simulated methods confirm the negative exponential dependence of NMSE on the number of pilot signals which leads to lower error as M grows. As pointed out in the previous discussions, the structured Turbo-CS algorithm does not perform well at $M = 50$ and $M > 65$ settings while the conventional LS algorithm leads to high error at the highest simulated $M = 90$ setting.

Figure 3 (right) illustrates only feasible algorithms with NMSE < 0 dB. The conventional LS algorithm performs best at $M \leq 85$ followed by the burst

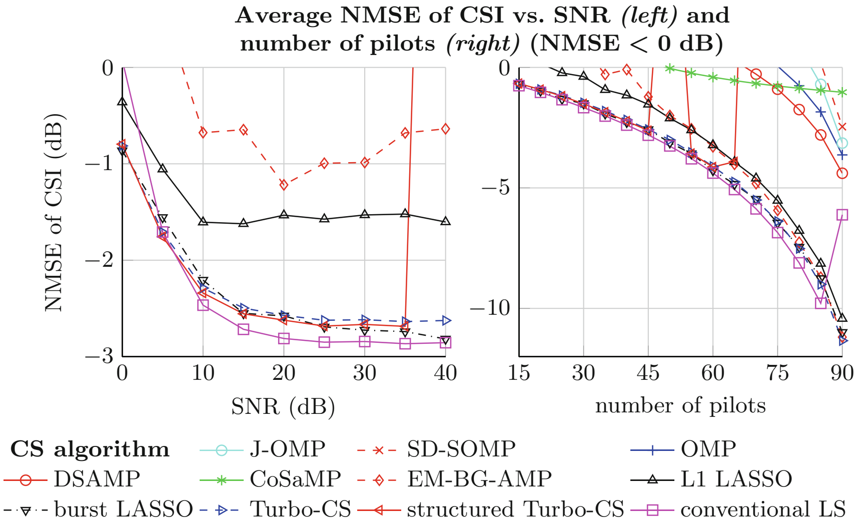

Fig. 3. Average NMSE of CSI in decibels versus SNR (left, $M = 45$ pilot signals) and number of pilot signals M (right, SNR $= 20$ dB). Only algorithms achieving NMSE < 0 dB are shown.

LASSO and Turbo-CS algorithms. The L1 LASSO and EM-BG-AMP algorithms also provide acceptable results for $M \geq 50$. It is noteworthy to mention that in order to reduce the pilot and feedback overhead, values of $M > 65$ do not make much sense in a practical FDD massive MIMO scenario due to the increased overhead.

4.3 Effect of Pilot Matrix Choice and Channel Representation on CSI Estimation Errors

As discussed in Sect. 2, a pilot matrix $\mathbf{X}$ comprised of PDFT-RP elements was reported to perform better than an i.i.d. Gaussian pilot matrix under various CS-based algorithms [5,12]. This was experimentally confirmed in our simulations and the results for indoor LOS environment at 2.4 GHz are presented in Fig. 5(top). Notably, the Turbo-CS algorithm, both in its canonical and structured version, performs much better with a PDFT-RP pilot matrix as the NMSE of both Turbo-CS algorithms with an i.i.d. Gaussian pilot matrix is around 40 dB. The performance difference for other algorithms is not so strongly expressed, however the EM-BG-AMP performs much better with an i.i.d. Gaussian pilot matrix which might be due to the i.i.d BG assumption on the signal model. This is in line with the described results in [5].

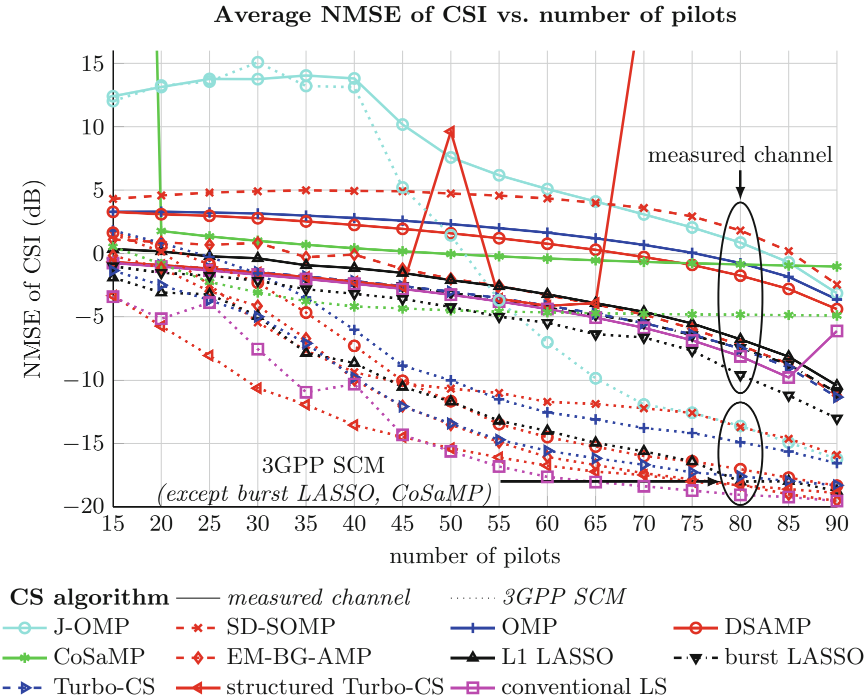

Fig. 4. Average NMSE of CSI in decibels versus number of pilot signals M of analyzed algorithms for SNR = 20 dB. Results achieved with the theoretical 3GPP SCM [15] are depicted as a reference.

All simulations described in the previous subsections suggest CSI estimation according to Eq. 1. However, considering Eq. 2, i.e. the transformed channel response into the virtual angular domain $\mathbf{h}_k^\omega$, might introduce additional sparsity in the channel vector as elaborated upon in Sect. 2. The usual definition of such transformation is $\mathbf{h}_k^\omega = \mathbf{F}\mathbf{h}_k$ where $\mathbf{F} \in \mathbb{C}^{N\times N}$ is a DFT matrix [5,11]. Simulation experiments with such transformation were carried out for indoor NLOS and static outdoor environments at 2.4 GHz and the results are depicted in Fig. 5 (bottom). For most algorithms except the J-OMP, the channel in angular domain leads to higher CSI recovery error, however the differences are minor.

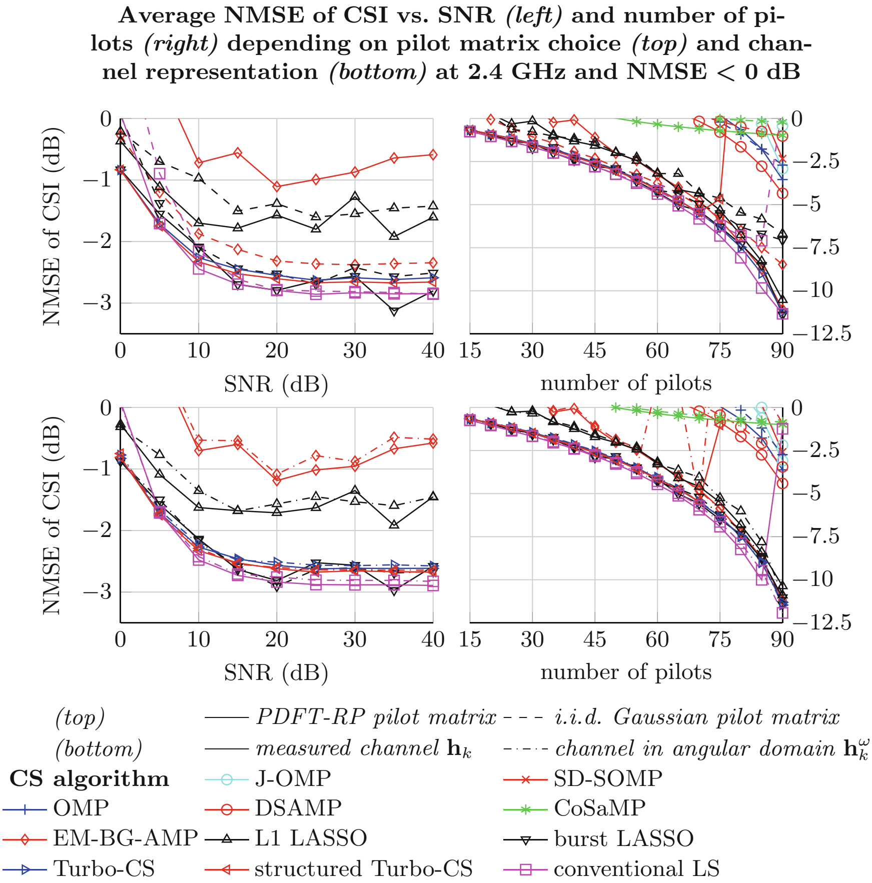

Fig. 5. NMSE of CSI in decibels versus SNR (left, $M = 45$) and number of pilot signals M (right, SNR = 20 dB) of analyzed algorithms depending on pilot matrix (top, averaged for indoor LOS) and channel representation (bottom, averaged for indoor NLOS and static outdoor environments) at 2.4 GHz.

5 Conclusions and Future Work

In this work, we evaluate the performance of 11 CS-based channel estimation algorithms utilizing real-world channel responses in the context of FDD massive MIMO systems. Most of the analysed algorithms assume some sort of sparse structure in the channel response vector and rely on it to reduce the number of required pilot signals and minimize estimation error. We show that the examined practically measured channels do not follow such structure. For the simulation experiments, channel measurements for carrier frequency in the 2.4 GHz and 5 GHz bands for three environments with two levels of mobility were selected from the publicly available measurement dataset recorded by the ArgosV2 system. Performance with the theoretical 3GPP SCM is used as a baseline and it clearly shows a reduction in estimation error due to the burst-sparse channel response structure. NMSE of the estimated CSI is the chosen performance metric and its dependence on SNR and number of pilot signals is studied.

The results show that the conventional LS algorithm achieves lowest NMSE followed by the burst LASSO and Turbo-CS (both in its canonical and structured variant) with the L1 LASSO and EM-BG-AMP algorithms also providing good results. The OMP, J-OMP, SD-SOMP, CoSaMP and DSAMP algorithms provide practically prohibitive results for most settings. Considering the good performance of burst LASSO, as part of a future research it would be interesting to evaluate the performance of its joint modification defined in [11]. Although the chosen pilot signals matrix is a PDFT-RP, its advantages to the i.i.d. Gaussian counterpart are quantitatively shown. Representation of the channel response in the angular domain is also evaluated and the results prove that using such transformation leads to minor performance differences, however the CSI recovery error is higher when exploiting the transformed channel.

For future wireless networks operating in mmWave bands, even larger antenna arrays with reduced number of RF chains will be used at both the BS and UEs which makes designing new and efficient methods for accurate channel estimation and feedback an open question. Performing verification of these methods with practical mmWave channels remains a topic of importance for future research. Other novel approaches for efficient CSI estimation and feedback, such as opportunistic channel estimation, should be further investigated and their feasibility for realistic channels needs to be proven in practice. In order to precisely estimate the performance of existing and future channel estimation techniques, it is vital to work with accurate and realistic channel models for massive MIMO propagation. Such models could benefit from the already available real-life channel measurement data which were gathered by various testbeds. It would be also interesting to practically determine the dependence of channel estimation quality on the number of BS and UE antennas, and particularly the effects of reducing the number of BS antennas. For this purpose, the dataset utilized in this work could be further examined.

Acknowledgement. This paper is published with the support of project No BG05M2OP001-2.009-0033 “Promotion of Contemporary Research Through Creation of Scientific and Innovative Environment to Encourage Young Researchers in Technical University - Sofia and The National Railway Infrastructure Company in The Field of Engineering Science and Technology Development” within the Intelligent Growth Science and Education Operational Programme co-funded by the European Structural and Investment Funds of the European Union.

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