Dual Optimal Robust Power Control Algorithm Based on Channel Uncertainty

Guanglong Yang(✉), Xuezhi Tan, and Xiao Wang

School of Electronics Information Engineering, Harbin Institute of Technology, Harbin 150001, People's Republic of China
yang3616@126.com, {Tanxz1957,hitwx}@hit.edu.cn

Abstract. In order to improve the fault-tolerant ability under parameter perturbation, the robust power control problem based on channel uncertainty is studied. In this paper, the robust optimization theory and stochastic optimization theory commonly used to deal with uncertain parameters are deeply analyzed, and the mathematical significance, application scenarios, advantages and disadvantages of the two optimization theories are summarized. A comprehensive solution for bounded uncertainty and probabilistic constraints is proposed. On the one hand, the scheme guarantees the rights and interests of the primary user under the worst error, on the other hand, the secondary user is satisfied with a certain interrupt probability under the condition of system robustness. In this paper, the main user interference temperature and the secondary user probability SINR are taken as the constraint conditions, the maximum throughput of the system is transformed into a convex optimization form, and the Lagrange dual (LD, Lagrange Duality) principle is used to solve the problem. The results show that the double optimization solution is a compromise between probabilistic constraint algorithm, Worst-case algorithm and non-robust algorithm, which not only fully protects the rights and interests of the primary user, but also takes into account the robustness. At the same time, the conservatism of the bounded uncertain design method is avoided.

Keywords: Cognitive radio · Underlay spectrum sharing · Distributed power control

1 Introduction

The uncertainty of channel parameters can be described by a variety of uncertain models. For example, the additive uncertainty model describes the uncertainty of channel parameters as the sum of estimated values and estimated errors, that is, the channel parameters vary in a bounded range [1]. In dealing with uncertain parameters, robust optimization theory [2] and stochastic optimization theory [3] are usually used. The robust optimization theory describes the parameter uncertainty as an appropriate closed-loop set of uncertainties, which contains all uncertain estimates, and transforms the robust uncertainty constraints by the worst-case principle (Worst-case). Becomes a deterministic solvable problem. The stochastic optimization theory considers that it is difficult to obtain the upper bound of the error of uncertain parameters, but its statistical

J. Jin et al. (Eds.): GreeNets 2019, LNICST 282, pp. 36–46, 2019.
https://doi.org/10.1007/978-3-030-21730-3_4

model can be obtained by training or other means, and the optimization mathematical model can be described by means of probabilistic constraint. The non-deterministic optimization problem is transformed into a deterministic solvable form by certain mathematical means.

In reference [4], the methods of robust optimization are described in detail, and the different methods of robust optimization in practical application are discussed, including mathematical programming, deterministic nonlinear optimization, direct search, etc. The advantages and disadvantages of different methods are discussed, which provides the basis for adopting the most suitable robust method. In reference [5], the channel gain is defined by the bounded distance between the estimated value and the exact value. Considering the conservatism of worst-case interference, a compromise between robust worst-case interference control and secondary user throughput is proposed. The interference probability of the secondary user to the primary user is controlled under the given threshold. A distributed uplink robust power control algorithm was proposed in reference [6]. Each uncertain parameter is modeled by the bounded distance between the estimated value and the exact value of each uncertain parameter, and the robust power allocation problem is described by the constraint protection value. Lagrange duality decomposition is used to solve the problem by distributed algorithm. SU needs to estimate the worst-case total interference at the PU receiver to ensure that PU is not interfered with.

To sum up, the power control problem of cognitive radio in Underlay mode is studied in the perfect channel and imperfect channel respectively.

2 System Model

In this paper, the underlying spectrum access mode is considered, so it is unnecessary to consider the communication situation of the primary user, and the time of sensing and judging the primary user's activity is reduced indirectly. This paper considers the underlying multi-user distributed cognitive radio scene, as shown in Fig. 1. Primary and secondary users coexist in the network, including M for secondary users, N for primary users. The secondary user is represented by the set $\mathrm{A} = \{1, 2, \cdots, M\}$ and the primary user by the set $\mathrm{B} = \{1, 2, \cdots, N\}$.

Order $\forall i, j \in \mathrm{A}$, $\forall k \in \mathrm{B}$.

In this paper, the interference temperature model in reference [7] is used to describe the interference size of the primary user receiver. In order to achieve the maximum throughput of secondary users, achieve optimal power control, and ensure the stability of primary user communication, the optimization problems considered in this chapter need to satisfy the following three conditions simultaneously:

$$\max \sum\nolimits_i \log(1 + \gamma_i)$$
$$\text{s.t.} \begin{cases} c_1 : 0 \le p_i \le p_i^{\max} \\ c_2 : \sum\nolimits_i p_i h_{i \leftrightarrow k} \le I_k^{th} \\ c_3 : \gamma_i \ge \gamma_i^d \end{cases} \tag{1}$$

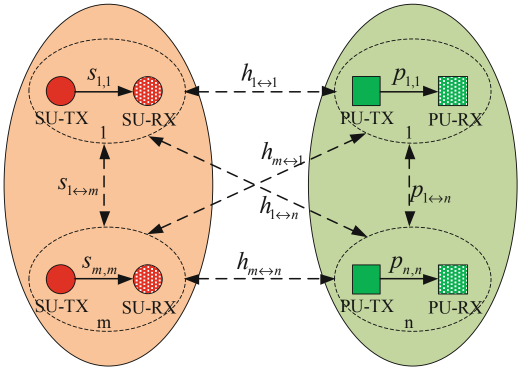

Fig. 1. Multi-user cognitive radio system

Formula (1) is a non-robust optimization problem. There are many uncertain parameters in the formula. The real value is difficult to obtain and can only be obtained by the method of estimation. At the same time, there is a certain error between the formula and the real value.

It can be seen from the constraint c_2 that there is uncertainty in channel gain $h_{i\leftrightarrow k}$, which may lead to the primary user receiving more interference power, thus leading to the deterioration of the primary user's communication quality and even the interruption of the communication. Constrained c_3 shows that there are uncertainties between channel gain $s_{i\leftrightarrow j}$ and primary interference I_{ip} between secondary users. This uncertainty may result in the secondary user communication γ_i not reaching the target value, thus the secondary user QoS.

3 Mathematical Model

For the mathematical model of (1), it is necessary to use appropriate methods to describe the uncertain parameters and perform robust control. The methods used to deal with parameter uncertainty are robust optimization and stochastic optimization. In the cognitive radio communication scenario, the communication quality requirement of the primary user is the highest, and any secondary user activities that affect the communication quality of the primary user are not allowed, so the most important prerequisite of this chapter is to ensure the communication quality of the primary user. Because there is no information feedback between primary and secondary users, the estimation error probability distribution of channel gain uncertainty between primary and secondary users is not easy to obtain. The robust optimization will adopt Worst-case method according to the uncertain set. The secondary user transmit power is sacrificed to guarantee the communication quality of the primary user. For cognitive users, a variety of means can be used to obtain a certain amount of information effectively. At the same time, the communication of cognitive users also allows the existence of a certain interruption probability of user equipment. Therefore, stochastic optimization based on probabilistic constraints among cognitive users can keep the overall performance of the system at a stable average.

3.1 Robust Optimization Mathematical Model

The channel gain uncertainty between secondary and primary users can be described using additive uncertainty [8], as follows:

$$\wp = \left\{ h_{i \leftrightarrow k} \middle| \bar{h}_{i \leftrightarrow k} + \Delta h_{i \leftrightarrow k} : |\Delta h_{i \leftrightarrow k}| \leq \kappa_{ik} \bar{h}_{i \leftrightarrow k} \right\} \tag{2}$$

Which, $|\bullet|$ represents an absolute operator. (2) the formula can be described as $h_{i \leftrightarrow k} = \bar{h}_{i \leftrightarrow k} + \Delta h_{i \leftrightarrow k}$, where $\bar{h}_{i \leftrightarrow k}$ represents the estimated value of channel gain, $\Delta h_{i \leftrightarrow k}$ denotes the corresponding estimation error, and $\kappa_{ik} \in [0, 1)$ denotes the uncertainty factor, which can describe the magnitude of uncertainty as well as the accuracy of parameter estimation. When the κ_{ik} is small, the estimation error is small, and the estimated channel gain is close to the real physical channel, that is, $\bar{h}_{i \leftrightarrow k} \rightarrow h_{i \leftrightarrow k}$. In special cases, when $\kappa_{ik} = 0$, the equivalent exact channel parameter optimization model based on (2) description of robust optimization problem is presented. When the κ_{ik} is very large, the representative parameter estimation is very large, and the estimation accuracy is not high. Therefore, the magnitude of κ_{ik} value is closely related to the robustness and optimality of the system. In general, the value of κ_{ik} can be determined by estimating the accuracy of the algorithm and the channel state model.

Similarly, considering the uncertainty of secondary users being interfered by primary users, the problem of robust resource allocation can be written as

$$\begin{gathered} \max \sum\nolimits_i \log(1 + \gamma_i) \\ \text{s.t.} \begin{cases} c_1 : 0 \leq p_i \leq p_i^{\max} \\ c_2'' : \sum_i p_i \bar{h}_{i \leftrightarrow k}(1 + \kappa_{ik}) \leq I_k^{th} \\ c_3 : \gamma_i \geq \gamma_i^d \end{cases} \end{gathered} \tag{3}$$

From the constraint c_2'', we can see that the upper limit of secondary user transmit power is smaller than that of accurate channel parameter model (such as $\kappa_{ik} = 0$) because of the introduction of κ_{ik}. Therefore, theoretically speaking, the throughput of the robust optimization problem of cognitive radio system is smaller than that of the non-robust optimization problem. In the uncertain mode, in order to prevent the secondary user's transmit power from affecting the primary user, the secondary user increases the protection of the primary user by transmitting lower power. The purpose of constrained c_3 is to prevent the parameter estimation error from causing the sub-user receiver to actually receive less γ_i than the target value. Next we discuss the uncertainty of c_3 by means of stochastic optimization.

3.2 Stochastic Optimization Mathematical Model

In the previous section, the uncertainty of channel gain between secondary user and primary user is solved by using robust optimization theory. In this section, the stochastic optimization theory is used to solve the uncertainty of channel gain between secondary user and primary user. In order to ensure that the constraint c_3 is true in most communication scenarios in the case of parameter perturbation, the service probability of the secondary user is as follows:

$$\max \sum\nolimits_i \log(1+\gamma_i)$$
$$\text{s.t.} \begin{cases} c_1 : 0 \le p_i \le p_i^{\max} \\ c_2'' : \sum_i p_i \bar{h}_{ik}(1+\kappa_{ik}) \le I_k^{th} \\ c_3' : \Pr\{\gamma_i \ge \gamma_i^d\} \ge \beta_i \end{cases} \tag{4}$$

Which, $\beta_i \in [0,1]$ is the satisfaction probability of the secondary user set in advance, representing the degree of the actual signal-to-noise ratio higher than the target γ_i^d. The larger the value of β_i is, the less interference between secondary users is required, that is, the lower the transmitting power is, the lower the mutual interference between primary users is to avoid interruptions.

Similar to the traditional stochastic optimization problem [11], problem (4) is not easy to solve, and it needs to be transformed into a deterministic form to obtain an analytical solution.

The stochastic optimization problem is formulated as:

$$\min \sum\nolimits_i p_i$$
$$\text{s.t.} \begin{cases} c_1 : 0 \le p_i \le p_i^{\max} \\ c_2'' : \sum_i p_i \bar{h}_{ik}(1+\kappa_{ik}) \le I_k^{th} \\ c_3'' : \gamma_i^d / \bar{\gamma}_i \le \hat{\beta}_i \end{cases} \tag{5}$$

which $\hat{\beta}_i = \ln \frac{1}{\beta_i}$.

4 Double Optimal Robust Power Control

Since problem (5) is a convex optimization problem, the following Lagrangian function can be constructed to solve the problem.

$$\begin{aligned} J(\{p_i\},\{\varpi_{i\leftrightarrow k}\},\{\psi_i\}) = & \sum\nolimits_i p_i \\ & + \sum\nolimits_k \varpi_{i\leftrightarrow k}\left(\sum\nolimits_i p_i \bar{h}_{i\leftrightarrow k}(1+\kappa_{ik}) - I_k^{th}\right) \\ & + \sum\nolimits_i \psi_i\left(\frac{\gamma_i^d}{\bar{\gamma}_i} - \hat{\beta}_i\right) \end{aligned} \tag{6}$$

Where $\varpi_{i\leftrightarrow k} \geq 0$ and $\psi_i \geq 0$ are the Lagrangian multipliers corresponding to the constraint conditions of problem (5) c_2'' and c_3''. Dual variables can be obtained by updating the following algorithm:

$$\varpi_{i\leftrightarrow k}(t+1) = \left[\varpi_{i\leftrightarrow k}(t) + \lambda\left(p_i\bar{h}_{i\leftrightarrow k}(1+\kappa_{ik}) - I_k^{th}\right)\right]^+ \tag{7}$$

$$\psi_i(t+1) = \left[\psi_i(t) + \theta\left(\gamma_i^d/\bar{\gamma}_i(t) - \hat{\beta}_i\right)\right]^+ \tag{8}$$

Where λ and θ are non-negative step-size factors.

5 Simulation Results and Analysis

In this section, the effectiveness of the algorithm is verified by simulation results. Firstly, the influence of channel uncertainty between primary and secondary users and the probability of secondary user satisfaction on system performance is analyzed. Then compared with Worst-case robust algorithm [9], probabilistic constraint algorithm [10] and non-robust algorithm, the superiority of the proposed algorithm is illustrated. Suppose there are three secondary users and two primary users in the network model. Channel interference gain $\bar{h}_{i\leftrightarrow k} \in [0, 0.1]$. between secondary and primary users Satisfaction probability $\beta = 0.9$ of secondary users Background noise $\sigma_i \in [0, 0.1]$. Assume that the primary user interferes with the secondary user sum is $I_{ip} = 2\sigma_i$. Assume the minimum SINR $\gamma_i^d \in [2\,\text{dB}, 6\,\text{dB}]$, maximum transmit power $p_i^{\max} = 1\,\text{mW}$ for each secondary user in the network model.

Figure 2 illustrates the relationship between channel gain uncertainty and total throughput between primary and secondary users at different interference temperatures.

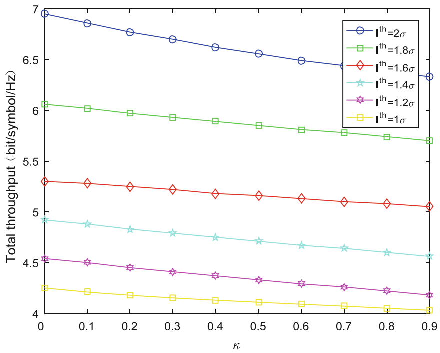

Fig. 2. Effect curve with the change of uncertainty κ in interference channel for the total throughput.

When the disturbance uncertainty is $\kappa = 0$, the uncertainty estimation error of the system is 0. The larger the uncertain parameter κ is, the greater the estimation error is, and the bigger the deviation between the estimated channel gain and the real value is. From Fig. 2, assume that each secondary user has the same SINR, that is, $\gamma_1^d = 2\,\text{dB}$. It can be seen that the total throughput of the secondary users decreases with the increase of the channel gain uncertainty parameter κ between the primary and secondary users for the fixed interference temperature I^{th}. Due to the uncertainty of the estimation error increasing, the secondary user can only reduce the transmit power in order to satisfy the constraint condition of the primary user interference. In the case of the same parameter κ, the larger the interference temperature I^{th} of the primary user is, the larger the total throughput of the secondary user is. This is because as the primary user I^{th} increases, the secondary user transmit power range is more relaxed, secondary users can use higher transmission power. However, due to $p_i^{\max}$ constraints, secondary users have the maximum transmit power limit.

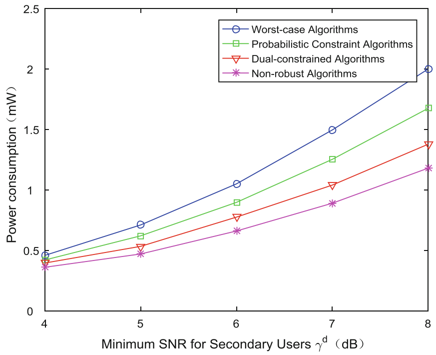

Fig. 3. Contrast curve of power dissipation for different algorithms

Figures 3 and 4 show the superiority of the algorithm by comparing it with Worst-case algorithm, probabilistic constraint algorithm and non-robust algorithm respectively.

Figure 3 describes the relationship between the total power consumption of secondary users and the target SINR. Assume that the interference temperature threshold in the network model is $I_{th} = 0.01$ mW. It can be seen from Fig. 3 that the total power consumption of secondary users increases with the increase of secondary user target SINR. Secondary users need to meet service probabilities, while secondary users also need to gradually increase transmission power to achieve higher SINR. It can be seen from the figure that the non-robust algorithm consumes the least power and the Worst-case algorithm consumes the maximum power. The Worst-case algorithm needs to transmit higher power to suppress the SINR drop caused by the worst parameter uncertainty in the system. The non-robust algorithm assumes that the system parameters are known accurately and does not need to adjust the transmission power to suppress the influence of uncertain parameters such as the Worst-case algorithm, so the power consumption is the minimum, but the robustness is the worst. The energy consumed by the probabilistic constraint algorithm lies between the above two algorithms. The power consumption of the proposed double constraint algorithm is between probabilistic constraint algorithm and non-robust algorithm.

The dual constraint algorithm takes full account of the uncertainty of channel gain between primary and secondary users, and reduces the use of the method between secondary users.

Figure 4 shows the actual SINR performance curve received by sub-users of different algorithms. The perturbed range of channel parameters is determined in $\Delta s \in [-0.05, 0.05]$. In a cognitive radio network where 2 primary users coexist with 3 secondary users, the interrupt probability of secondary users is The target SINR is $\gamma_i^d = 2\,\mathrm{dB}$.

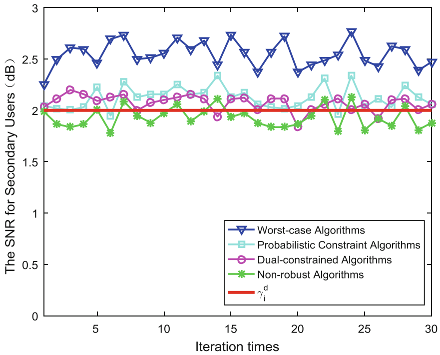

Fig. 4. Contrast curve of robustness for different algorithms

As can be seen from Fig. 4, there is no direct mathematical relationship between the non-robust algorithm and the variation of channel gain, so the variation of channel parameters will not affect the transmission power, so that although the power consumption is the minimum, it is the best protection for the primary user. However, the robustness is the worst, which can not satisfy the service probability of the secondary users well, and the robustness is the worst compared with other algorithms. In imperfect channels, as shown in Fig. 4, the non-robust algorithm leads to higher outage probability for secondary users. In order to realize the normal communication for all secondary users, Worst-case adjusts the transmission power based on the worst uncertain parameters, and the probability of the secondary user satisfaction is equal to 100. The probabilistic constraint algorithm uses the service probability of different users as the constraint condition, as shown in Fig. 4, this algorithm leads to 6.7% outage probability of the secondary user, which is greatly improved compared with the non-robust algorithm. In this paper, the probability of service between secondary users is used as the constraint condition, and the worst-case uncertain set of channel gain is fully considered between primary and secondary users. In order to suppress the uncertainty of gain, the transmit power of secondary users will be reduced. Indirectly, the probability of secondary user service is reduced.

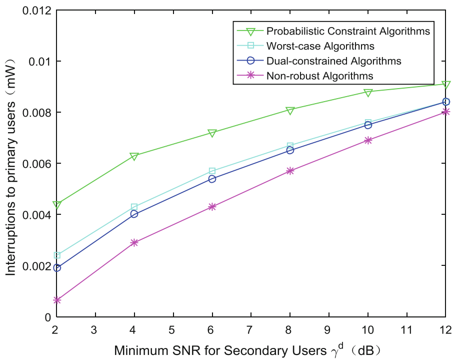

Fig. 5. Interference curve caused by different algorithms for primary users

It can be seen from Fig. 5 that the non-robust algorithm with minimal interference to the primary user and the probabilistic constraint algorithm with the greatest interference on the primary user. The non-robust channel is based on the excellent channel state. With the increase of channel information accuracy, the probabilistic constraint algorithm can more accurately control the interference to the primary user, and it is also the closest control method to the primary user interference. Compared with the Worst-case algorithm, the dual constraint algorithm has less interference to the primary user. In order to suppress the influence of channel uncertainty on the secondary user SINR, the secondary user in the Worst-case algorithm is bound to increase the power more, while the dual constraint algorithm adopts the method of satisfying the service probability among the secondary users, which provides more protection to the primary user.

The non-robust algorithm does not need to overcome the uncertain influence to increase the power consumption, the power consumption is minimum, the primary user protection is the best, but the robustness is the worst. The Worst-case algorithm considers all the worst uncertain sets. The transmission power is used to ensure that the user can communicate normally under the uncertainty of the worst-case parameters.

For cognitive systems, the Worst-case algorithm is applied to the power control between secondary users, and the algorithm will consider increasing the transmission power to meet the higher SINR requirements of the secondary users, resulting in the maximum power consumption.

Probabilistic constraint algorithm mainly makes primary user and secondary user satisfy certain interrupt probability constraint. It is a compromise method based on Worst-case algorithm and non-robust algorithm. However, the above method assumes that the cognitive system knows the distribution of uncertain parameters. However, for the diversity and time-varying of the communication environment between primary and secondary users, it is difficult to obtain a statistical model with uncertain parameters, which has the greatest interference to primary users.

It can be seen from the above results that it is important to ensure that the primary users in cognitive radio systems are not interfered with, taking into account the power consumption of the algorithm and the robust performance of the secondary users. The double optimization algorithm proposed in this paper is a compromise method between several algorithms, which not only fully protects the primary user, but also takes into account some robustness, and at the same time is not as conservative as the bounded uncertainty design method.

6 Conclusion

In this chapter, the existing uncertain parameters in cognitive radio systems are fully listed, and the reasons for these uncertainties are analyzed in depth, and the significance of considering these uncertain parameters in practical applications is clarified. The channel parameters between primary and secondary users are cross-system parameters. In non-cooperative mode, the channel parameters are difficult to obtain, while the channel parameters between secondary users are intra-system parameters, and channel parameters are easier to obtain. On the one hand, the channel gain uncertainty between primary and secondary users is described by additive uncertainty method, and the existence of worst-case estimation error is considered. Worst-case algorithm is used in power optimization. On the other hand, the probability distribution function is assumed to be known among secondary users, and probabilistic constraint algorithm is used in power optimization. In this chapter, two optimization methods, bounded uncertainty and probabilistic constraints, are organically combined, and a double optimization algorithm is proposed, which solves the problem of the interruption probability of primary users caused by the probabilistic constraint algorithm. At the same time, the problem of Worst-case algorithm is improved too conservatively. The simulation results show that the double constraint algorithm proposed in this paper has more practical value.

References

1. Setoodeh, P., Haykin, S.: Robust transmit power control for cognitive radio. Proc. IEEE **97** (5), 915–939 (2009)
2. Han, S., Liang, Y.-C., Soong, B.-H.: Robust joint resource allocation for OFDMA-CDMA spectrum refarming system. IEEE Trans. Commun. **64**(3), 1291–1302 (2016)
3. Dantzig, G.B.: Linear programming under uncertainty. Manage. Sci. **1**(3–4), 197–206 (1955)
4. Beyer, H.G., Sendhoff, B.: Robust optimization-a comprehensive survey. Comput. Methods Appl. Mech. Eng. **196**(33), 3190–3218 (2007)
5. Parsaeefard, S., Sharafat, A.R.: Robust worst-case interference control in underlay cognitive radio networks. IEEE Trans. Veh. Technol. **61**(8), 3731–3745 (2012)
6. Parsaeefard, S., Sharafat, A.R.: Robust distributed power control in cognitive radio networks. IEEE Trans. Mob. Comput. **12**(4), 609–620 (2013)
7. Akyildiz, I.F., Lee, W., Vuran, M.C., et al.: Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey. Comput. Netw. **50**(13), 2127–2159 (2006)
8. Xu, Y., Zhao, X., Liang, Y.-C.: Robust power control and beamforming in cognitive radio networks: a survey. IEEE Commun. Surv. Tutorials **17**(4), 1834–1857 (2015)

9. Xu, Y.J., Zhao, X.H.: Robust probabilistic distributed power control algorithm for underlay cognitive radio networks under channel uncertainties. Wireless Pers. Commun. **78**(2), 1297–1312 (2014)
10. Xu, Y.J., Zhao, X.H.: Robust power control for underlay cognitive radio networks under probabilistic quality of service and interference constraints. IET Commun. **8**(18), 3333–3340 (2014)
11. Soltani, N.Y., Kim, S., Glannakis, G.B.: Chance-constrained optimization of OFDMA cognitive radio uplinks. IEEE Trans. Wireless Commun. **12**(3), 1098–1107 (2013)