Dynamical Analysis of Nose-Hoover Continuous Chaotic System Based on Gingerbreadman Discrete Chaotic Sequence

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Abstract. Apply the discrete chaotic sequence of Gingerbreadman System to the only one control parameter of Nose-Hoover continuous chaotic system, can get completely different simulation results. Namely, extracting a part of sequence of Gingerbreadman discrete system randomly, and take this sequence to control Nose-Hoover continuous chaotic system, then make analysis of this new system. Dynamic analysis of the new system, which is based on Nose-Hoover continuous chaotic system under the control of the discrete chaotic sequence of Gingerbreadman system. Compared with the original system carefully, find that phase diagram arising from new system produce obvious changes. We also calculate Lyapunov exponents, compared with the Lyapunov exponents computed from original system, find it also changed. It proved that our new system has chaotic characteristics, provide new method for the chaotic system which are used in the fields of cryptography, secure communication and information security etc.

Keywords: Gingerbreadman discrete chaotic sequence · Nose-Hoover chaotic system · Phase diagram · Lyapunov expontents

1 Introduction

In recent years, chaos theory has been widely applied in the fields of cryptography, secure communication and information security. It is reported that chaos coding technology and decoding technology have entered the U.S. defense department [1]. Chaotic systems can be divided into continuous and discrete systems, and continuous systems used for encryption often need to be discretized. Information security was studied in the early 1990s [2].

Habutsu [3] firstly used the discrete chaotic dynamic system to construct the encryption algorithm in 1991. Bianco [4, 5] used logistic map to generate a floating-point sequence in 1991 and 1994, then converted it into binary sequence which is Exclusive OR [XOR] with plain text. In 1991, Deffeyes [6] described a method to generate a two-dimensional N-by-M region from a one-dimensional password, which is similar to the generalized two-dimensional back-mapping in the long distance and

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mainly relies on geometry. and also the chaotic encryption methods based on synchronization proposed by Caroll and Pecora [7–13], Cuomo and Oppenheim [14, 15], Murali [16], Koearev [17], KHZ [18], Papadimitriou [19] et al. Bernstein and Lieberman [20] established a pseudo-random sequence generator with chaos circuit in 1991, and Gutowicz [21, 22] described an encryption scheme based on one-dimensional cellular automata. Firstly in 1994 and 1995, Pichler and Scharinger [23] introduced the encryption method with two-dimensional discrete chaotic systems. In 1997, Götz et al. proposed a new one-dimensional iterative method [24], and Kotulski proposed the inverse iterative algorithm [25]. Study of continuous chaotic system is a hot topic in recent years scientists to explore, from 1963 the American scientists Lorenz found chaos, 1975 the Chinese scholars Tien-Yien Li and the American mathematician Yorke published the famous article "period three implies chaos" [26] in "America Mathematics" magazine, to now, there is no need to repeat because it is too familiar to us.

Due to the defects of the encryption algorithm itself or the insufficient security of the inherent characteristics of the discrete chaotic system, the cryptographic system is actually a process of reversible transformation from plaintext space to ciphertext space determined by the key [27]. In recent years, people have stayed on the study of chaotic attractors generated by a single continuous system with parameter changes, and further analyzed on the basis of a system that has proved to be chaotic. Such as the research of Wang Fanzhen et al. [28]. based on a four-wing attractor of Qi et al. [29], or fine-tuning the original equation, increasing or decreasing the dimension to find new chaotic phenomena, In the field of chaotic circuit engineering, many techniques have used non-smooth nonlinear terms to generate multi-volume chaotic attractors [30–34]. Based on the above considerations, the key question discussed in this paper is: Can a chaotic sequence generated by a discrete system be used to control a certain parameter of a continuous system, so that it can change with discrete sequences, and whether new chaotic phenomena can be generated under such conditions? This paper gives a certain dynamic analysis through calculation and simulation, and gives certain conclusions at the end of the article.

The main method of time series chaos determination [35] analyzes the dynamic characteristics of chaotic characteristics from different angles. This paper analyzes the Nose-Hoover system controlled by the sequence generated by the discrete system from the aspects of Lyapunov exponent analysis and direct observation of phase diagram trajectory, and compares it with the simulation results of the original Nose-Hoover system to analyze the phase diagram. The formation of the trajectory is used to determine the effect of the sequence on the Nose-Hoover system.

So far, only a few literatures have studied and illustrated the phase diagram of the Gingerbreadman discrete chaotic system. This paper adds a study on the variation of the Lyapunov exponent with parameters. Discrete systems are analyzed on the basis of logistic map, and one-dimensional chaotic maps are the most studied. The chaotic behavior of logistic mapping was first proposed by American mathematical biologist R. May. In 1976, he published a review article in the American magazine Nature [36]. This article will not describe the basic principles.

Nowadays, many theorems and inferences have been proposed for the prediction of chaotic attractors [37, 38]. The method of this paper provides a new idea for judging the existence of chaos and analyzing the formation of chaotic attractors, and this assumption is proved by calculation and simulation.

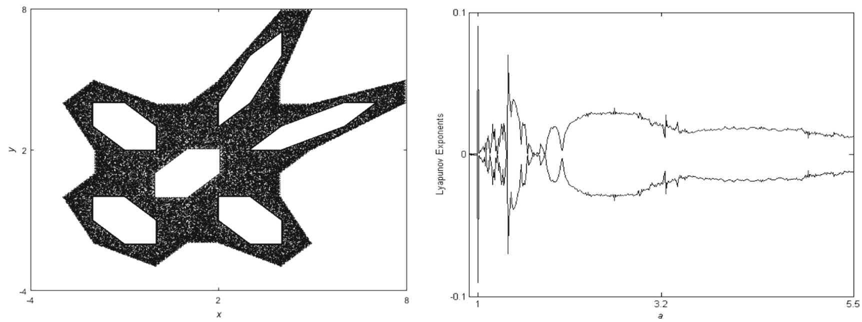
2 Dynamic Analysis of Discrete and Continuous Chaotic Systems

2.1 Dynamic Analysis of Gingerbreadman Discrete System

The mathematical model of system Gingerbreadman is:

$$\begin{cases} X_{n+1} = 1 + |X_n| - aY_n \\ Y_{n+1} = X_n \end{cases} \quad (1)$$

When the initial value of the system is $X_0 = 0.5$, $Y_0 = 3.7$, and $a = 1$, Gingerbreadman's attractor phase diagram on the x-y plane is shown in Fig. 1(a), The Lyapunov exponent of the system is $LE_1 = 0.09054$ and $LE_2 = -0.09054$, the system is in a chaotic state. It can be seen from Fig. 1(a) that the value of phase diagram parameter x is within the interval (−4,8]. In this paper, the discrete sequence generated by Xn is shown in Table 1.1 for simulation of Nose-Hoover system under the control of discrete sequences in 2.1 and 2.2. the relationship curve of Lyapunov exponent of Gingerbreadman discrete system with parameter a, as shown in Fig. 1(b). When the value of parameter a is between [0.95, 5.5], the maximum Lyapunov exponent of the system is always greater than 0, indicating that the system is in a chaotic state within this interval.



(a) *x*-*y* plane phase diagram (b) Lyapunov exponents with *a*

Fig. 1. Gingerbreadman system simulation

2.2 Dynamic Analysis of Nose-Hoover Continuous System

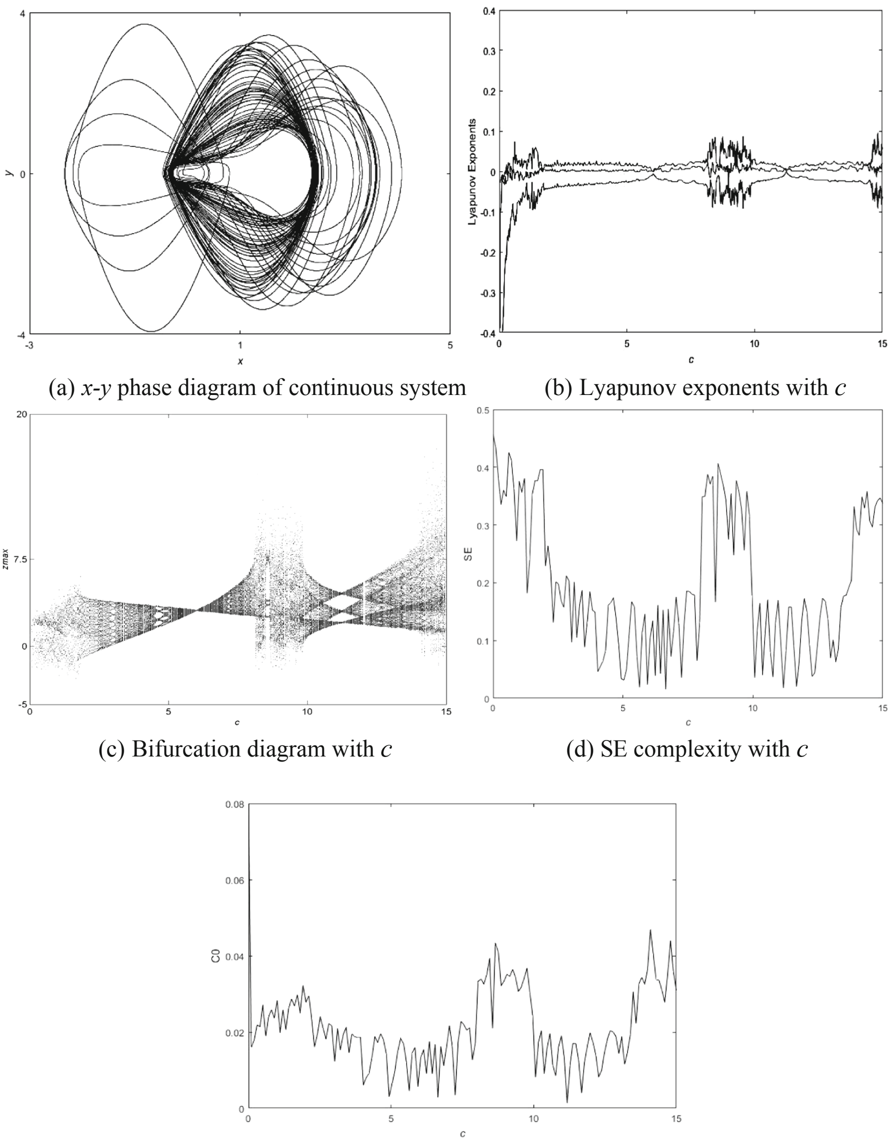
The modeling process and feature principle of the Nose-Hoover system can be found in the specific literature [39].

The mathematical model of system Nose-Hoover is:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + yz \\ \dot{z} = c - y^2 \end{cases} \tag{2}$$

When the initial value of the system is $x_0 = 0$, $y_0 = 5$, $z_0 = 0$, $c = 0.99$, and the simulation time step is 0.05 s, the attractor phase diagram on the x-y plane is shown in Fig. 2(a). The Lyapunov exponent of the system is $LE_1 = 0.0218$, $LE_2 = 0$, and $LE_3 = -0.0466$, so the Lyaounov dimension can be calculated as 2.4678. As shown in Fig. 2(b), you can see how the Lyapunov exponent of nose-hoover system changes with parameter c. When parameter c is within the interval [0, 15], the Lyapunov exponent always has a part greater than 0, which indicates that nose-hoover system within this interval is in a chaotic state. It can be seen from this that system Gingerbreadman's discrete sequence is in the range where parameter c can make the system have chaotic characteristics. In order to more accurately analyze the chaotic feature of Nose-Hoover continuous system, a bifurcation diagram of the continuous system can be made. The bifurcation diagram of Nose-Hoover system varying with parameter c is shown in Fig. 2(c). By comparing the curve of Lyapunov exponent of the system changing with parameter c with the bifurcation diagram of the system changing with parameter c, it can be seen that when $c = 8.5$ and $c = 12.13$, the system has a periodic window, and the system in the range of these two values is in a periodic state when c is in the interval (0,6.05], the whole Nose-Hoover system is in the period doubling bifurcation state. When $c > 6.05$, it enters the first bifurcation of the system, generates three periodic states, and again appears the more obvious periodic window at $c = 9$, and the second period-doubling bifurcation of the system at $c = 11.28$. In Fig. 2(b) and (c) shows that the system's Lyapunov exponent and bifurcation diagram is consistent, further contrast C0 complexity and SE complexity of x vary with parameters c, as shown in Fig. 2(d) and (e). Parameter c, for example, in the interval [7.852, 10.07], when $c > 7.852$, with the increase of c value, the SE and C0 complexity of system as the rising trend. When $c > 10.07$, with the increase of c value, the SE and C0 complexity of system as the declining trend. When c is in the interval [10.07, 13.19], the system enters into the anti-period-doubling-bifurcation state, which is consistent with the results verified by the corresponding Lyapunov exponents and the bifurcation diagram of the system. The stability of chaos dynamics of Nose-Hoover system can be explained, which further proves that this continuous system can be used as the control carrier of Gingerbreadman discrete sequence.

When analyzing Nose-Hoover system, Wolf algorithm [40] was used to calculate Lyapunov exponents, draw the spectrum graph of Lyapunov exponents changing with system parameters, make bifurcation diagram, and analyze the complexity of the system changing with parameter c. Through the above basic dynamics analysis, chaos characteristics of Nose-Hoover system can be explained. The preliminary proof is given for analyzing the control of discrete sequence based on this method.



(a) *x*-*y* phase diagram of continuous system

(b) Lyapunov exponents with *c*

(c) Bifurcation diagram with *c*

(d) SE complexity with *c*

(e) C0 complexity with *c*

Fig. 2. Simulation diagram of Nose-Hoover

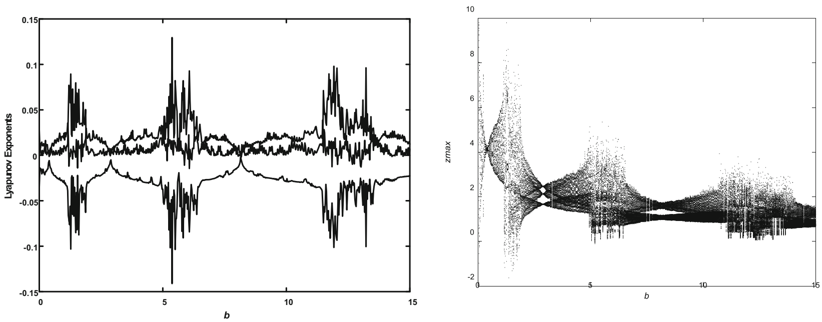
3 The Nose-Hoover Continuous System Under the Control of Discrete Sequence of Gingerbreadman System

3.1 Dynamical Analysis

In this paper, the basic mathematical model of nose-hoover system under Gingerbreadman discrete sequence control parameters is shown in Eq. (3).

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + bxy \\ \dot{z} = c - y^2 \end{cases} \quad (3)$$

As can be seen from Eq. (3), a parameter b is introduced in the second line and the second item of the original equation. If b is equal to 1, it is not fundamentally different from the mathematical model of nose-hoover system. However, this part of this paper also considers the influence of system parameter b. This article will illustrate the advantages and disadvantages of the new system through the following introduction. Gingerbreadman discrete system was numerically simulated to obtain a series of Xn discrete sequences, some of which are shown in Table 1. When the time interval t = 0.5 s is taken, the Lyapunov exponents of the system can be obtained by calculating that LE_1 = 0.1295, LE_2 = 0, and LE_3 = −0.1411. The Lyapunov dimension of nose-hoover system under the control of discrete sequence is further calculated to be 2.9179, and the parameter c under the control of discrete sequence is finally shown to be 7.7. In the experimental simulation, it was found that when the time-varying parameter c in the system was moved to the numerical solution algorithm of nose-hoover equation, the parameter c did not change finally, but it affected the simulation results, indicating that the stability of the chaotic system was affected by many factors. Figure 3(a) shows the curve of Lyapunov exponents changing with parameter b, by comparing Fig. 2(b) of the original Nose-Hoover system, it is found that the peak value of Lyapunov exponents shown in Fig. 3(a) increases with parameter b, and Lyapunov exponent value is always greater than 0, indicating that the Nose-Hoover system found in this paper under the control of Gingerbreadman discrete system has better chaotic characteristics. In addition, it can be seen that, in the same region [0, 15], the exponential waveform of Lyapunov exponents in the range of [1.025, 8.15] as shown in Fig. 3(a) is roughly the same as the waveform in the range of [0, 11.28] as shown in Fig. 2(b). The exponential curve of the former is half as compressed as that of the latter, and the entire change period is twice as long as that of the original system. By referring to the bifurcation diagram of the system changing with parameter b, as shown in Fig. 3(b), the time step is still 0.5 s. However, by comparing Figs. 3(b) and Fig. 2 (c), it is found that the chaotic characteristics of the system are different even under the control of changing parameters. In order to study how this graph is formed, and under two conditions (control parameters before numerical solution and control parameters after numerical solution), the number of plot points is about 300 ~ 500, which can be simulated in Fig. 3(b), and the trajectory shown, it can be seen that even if the same parameter c, the value is located before or after the solution of the system numerical value, it will affect the trajectory of the system, and finally the overall phase diagram of the drawing will be different. We can see that the shape of the whole system has changed, which is an attractor with two diamond-shaped torus, and from the basic observation of chaotic attractors, it can be judged that the system still has chaotic characteristics [41, 42].



(a) Lyapunov exponents with *b* (b) Bifurcation diagram with *b*

Fig. 3. Analysis of Nose-Hoover system under discrete sequence control

Table 1. The *X*n sequence generated by the discrete system corresponds to the value of the continuous system parameter C_i.

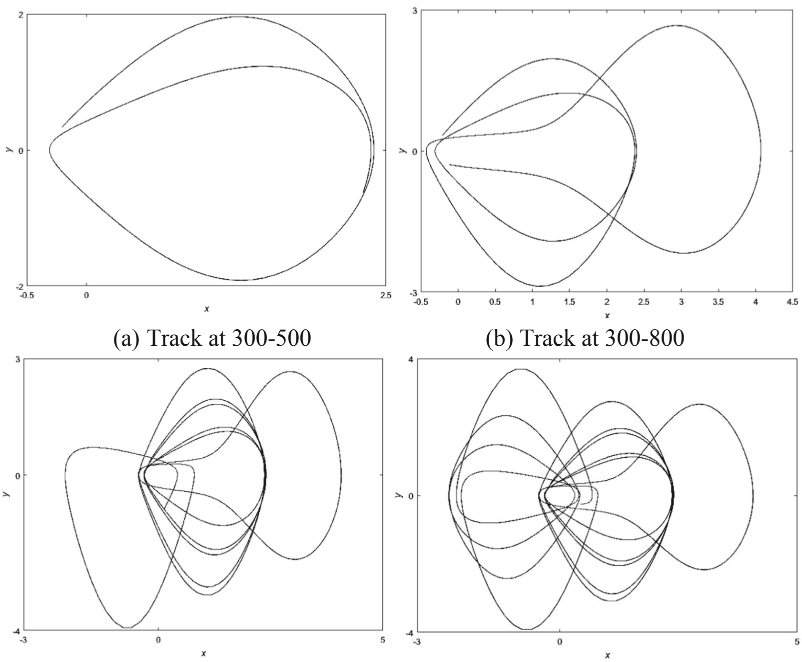
*X*n(n = 1, 2, 3……)	Continuous system corresponding parameter *C*i (i = 1, 2, 3……)
0.5	C1 0.5
−2.2	C2 −2,2
2.7	C3 2.7
5.9	C4 5.9
4.2	C5 4.2
−0.7	C6 −0.7
−2.5	C7 −2.5
4.2	C8 4.2
7.7	C9 7.7
……	……

3.2 Comparative Analysis of the Nose-Hoover System Under the Condition of Parameter *c* in Two Cases

It can be seen from the above analysis that the phase diagram of the Nose-Hoover system has different chaotic characteristics under different conditions. The original hypothesis can be proved: The Nose-Hoover continuous system under the discrete sequence control of the discrete system Gingerbreadman has chaos characteristic.

Comparing phase diagram Fig. 2(a) with Fig. 3(a), it is found that the Nose-Hoover system under discrete sequence control is more convergent. In order to further analyze the trajectory changes of the Nose-Hoover system, the trajectories formed by the Nose-Hoover system at different stages are successively simulated. The trajectory phase diagrams drawn by different points in the simulation can be clearly understood and formed as shown in the Fig. 2(a). Of cause the process of the attractor shown in Fig. 2 (a) can be judged based on the periodic orbital theory of chaotic attractors [42, 43]. When analyzing the formation of the phase diagram of the Nose-Hoover system, as shown in Fig. 2(a), the trajectory first forms two cycles from the middle part (the number of drawing points is around 300–500), and then enters a cycle of the right half (The number of drawing points is around 300–800), then the middle part is carried out for five cycles, and then enters the left half (the number of drawing points is around 300–1300). After completing four cycles (the number of drawing points is around 300–1900), it starts to enter the three periods of the middle part (the number of drawing points is around 300–2400), and then enters the period of the right half (the number of drawing points is around 300–2600), then enter the middle part of the three cycles (the number of drawing points is around 300–3150), then enter a cycle of the right half, then enter the three cycles of the middle part, and then enter the middle part, which can be repeated and a complete attractor phase diagram of the Nose-Hoover system as shown in Fig. 2(a) is obtained. When the plotting points number of this system is about $300 \sim 500$, $300 \sim 800$, $300 \sim 1500$, $300 \sim 1900$, the trajectory of Nose-Hoover is shown in Fig. 4(a), (b), (c), (d) respectively.

Comparing the trajectories of the Nose-Hoover system under discrete sequence control Fig. 3(b) and the trajectory of the Nose-Hoover system Fig. 4(a), (b), (c), (d), found by discrete system, it is found that the trajectory of Nose-Hoover system under the control of sequence generated by discrete system is simpler, the graph is more convergent, and the shape is more stable.



(a) Track at 300-500

(b) Track at 300-800

(c) Track at 300-1500

(d) Track at 300-1900

Fig. 4. Trajectory map of the formation process of Nose-Hoover system in each cycle

4 Conclusion

When studying the chaotic characteristics of a certain system, it can be studied by comparing the Lyapunov exponent, bifurcation diagram, complexity and other methods of the system. It can also be directly observed and compared with the original ones that have proved chaotic, and the formation process of the chaotic system can be understood. Many mathematicians and scientists have specific research on the formation control analysis of specific chaotic phenomena. For example, the literature [39] is the discovery of the Nose-Hoover system and the preliminary analysis process. Based on the original chaotic system, this paper makes further research and discovery by using new methods and new developments in chaotic systems in recent years. The proof process for specific mathematical models will not be described here.

The steady state value of the Lyapunov exponent of the Nose-Hoover system mentioned in this paper is larger under the condition of discrete sequence control, and the specific shape of the phase diagram also changes in macroscopic observation. An idea can be put forward: in the process of a series of changes in parameters, the Nose-Hoover system converges toward the periodic trajectory of the middle part of the Nose-Hoover system in Fig. 4.

The above experimental simulations show that the chaotic characteristics of the Nose-Hoover system change under the control parameters of the discrete sequence generated by the discrete system Gingerbreadman, and a chaotic attractor phase diagram different from the previous one is found in this sequence. The figure is in the shape of two side by side diamonds. It is found that the Nose-Hoover system under the control of Gingerbreadman discrete sequence has more stable chaotic characteristics. and has some chaotic characteristics of Gingerbreadman discrete system and Nose-Hoover continuous system as a whole. For example, it has the symmetry of the Gingerbreadman discrete system to some extent. The Lyapunov exponent of the Nose-Hoover continuous system changes with the parameters, the bifurcation diagram of the system with the parameter changes, and the complexity of the system with the parameter changes have certain similarity. However, in the detailed analysis, it is found that the continuous system controlled by discrete sequence has larger Lyapunov exponent value and better complexity under some discrete sequences, and the chaotic characteristics are more stable.

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