



# Dynamics and Synchronization Analysis of Chaotic Characteristic Interconnected Electrical Power System

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**Abstract.** In this paper, the chaos characteristics of interconnected electrical power system under different cycles load disturbance are analyzed by the phase diagram, bifurcation diagram and Lyapunov exponent spectrum. Then synchronization characteristics of the interconnected electrical power system are analyzed by coupling synchronization algorithm. The analysis results shown that the system under different cycle load disturbance, the system have more complexity state of motion, such as, appear periodic state, chaos oscillation, when system with different coupling coefficient of shock, the system arrives at the synchronization time is different. The analysis results have certain guiding significance to maintain the safe to operation of power system.

**Keywords:** Interconnected electric power system · Chaos characteristics · Synchronization characteristics

## 1 Introduction

Chaos is a deterministic nonlinear dynamics system, which are sensitive to initial values and system parameters. It is a complex state of motion that occurs in deterministic systems, and a natural phenomenon that exists extensively in nature and human society [1, 2].

With the development of chaotic systems, the idea of chaotic control comes into being. At present, chaos control has taken a breakthrough result. An important research direction of chaotic control is chaotic synchronization. Chaotic synchronization refers to two chaotic systems, which, over time, tend to converge in some way [3]. Since 1990 Pecora and Carroll [4] proposed a drive-response synchronization method, which the synchronization of chaotic systems becomes a hot research field. Then a variety of chaotic synchronization methods are proposed, for example, time-delay feedback method, adaptive control method [5, 6], and observer method [7].

Due to the rapid development of science and technology, the scale of power system has undergone great changes, especially, the transformation of power system to power grid interconnection. Power system is also a kind of nonlinear system with dynamic behavior, when the system parameters change, the system will show complex nonlinear dynamic characteristics. The interconnected power system is a typical nonlinear power

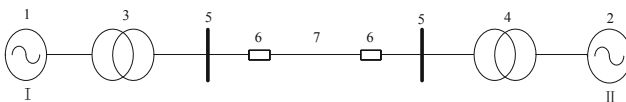
system, and the system has complex dynamic characteristics, such as, low frequency oscillation state, synchronous harmonic vibration state behavior, chaotic state behavior [8]. If there is a non-periodic load disturbance in the power system, when the load disturbance reaches a certain condition, the system will produce chaotic oscillation, so that the normal operation of the power grid is affected. If the chaotic oscillation is serious, it will cause the interconnected power system to crack directly, thus blocking the normal transmission of electricity [9]. Therefore, it is of special significance to study the chaotic phenomenon of power system to maintain the safe operation of power system.

The abnormal operation of the power grid is mainly caused by the chaotic oscillation in the power system, and the chaotic oscillation cannot make the power grid run properly by adding damping stabilizer to the system. Since the 20th century, many scholars have begun in-depth research on the causes of chaotic phenomena in power systems. Subsequently, unprecedented achievements have been made in the chaotic characteristics of power systems and the control of chaos in power systems [10–17]. However, so far, there is not a lot of expansion of the synchronous research of power system. Therefore, this paper will use the coupling synchronization algorithm to analyze the chaotic synchronization characteristics of interconnected power system, and use phase diagram, bifurcation diagram, Lyapunov spectrum and complexity to analyze the chaotic dynamics of the system.

In this paper, the chaotic characteristics and synchronization characteristics of interconnected power system are analyzed. The structure of the rest of article as follows, in Sect. 2, the mathematical state equation of the interconnected power system is established. The chaotic characteristics of interconnected power system are analyzed in Sect. 3. In Sect. 4, the coupling synchronization algorithm is detail described, and the synchronization characteristics of the interconnected power system are analyzed. Some conclusions are summarized in Sect. 5.

## 2 Model of Interconnected Electrical Power System

The structural model of the interconnected electrical power system is shown in Fig. 1. From the Fig. 1, we can see that the system model including system I, system II, connecting line and 7 elements. Where, the components 1, 3, 5 and 6 are equivalent generators, main transformers, loads and circuit breakers, respectively in system I. The components 2, 4, 5, 6 are the equivalent generators, main transformers, loads and circuit breakers in system II.



**Fig. 1.** Model of interconnected electrical power system

The dynamic mathematical model of a simple interconnection system containing two sets of generators is as follows:

$$\begin{cases} \frac{d\delta}{dt} = w \\ \frac{dw}{dt} = -\frac{1}{H}P_s \sin \delta - \frac{D}{H}w + \frac{1}{H}P_m + \frac{1}{H}P_e \cos \alpha t \end{cases} \quad (1)$$

where,  $\delta$  represent the relative angle between the equivalent generator potential of two power systems, the unit is rad.  $w$  means that the relative angular velocity, the unit is rad/s.  $h$  represent the moment of inertia, the unit is  $\text{kg}\cdot\text{m}^2$ .  $D$  means that the damping coefficient, the unit is  $\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$ .  $P_m$  represent the equivalent generator of the system I mechanical power, the unit is  $\text{W}$ .  $P_s$  means that the maximum electromagnetic power value transmitted by system I to System II, the unit  $\text{W}$ .  $P_e$  is the load disturbance power amplitude, the unit is  $\text{W}$ .  $\alpha$  represent the load disturbance frequency, the unit is  $\text{Hz}$ .

Set  $x_1 = \delta$ ,  $x_2 = w$ ,  $x_3 = \cos(\alpha t)$ ,  $x_4 = \sin(\alpha t)$ , Eq. (1) can be written in the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = bx_2 + dx_3 + c - a \sin x_1 \\ \dot{x}_3 = -\alpha x_4 \\ \dot{x}_4 = \alpha x_3 \end{cases} \quad (2)$$

where,  $a = P_s/H$ ,  $b = -D/H$ ,  $c = P_m/H$ ,  $d = P_e/H$ .

The  $a, b, c, d, \alpha$  represent the system parameter,  $x_1, x_2, x_3, x_4$  means that the state variable of the system. Makes the system parameter  $a = 1$ ,  $b = -0.4$ ,  $c = 0.2$ ,  $d = 0.755$ ,  $\alpha = 0.8$ , the initial value  $x_1 = 0.5$ ,  $x_2 = 0.5$ ,  $x_3 = 1$ ,  $x_4 = 0$ , and gets the chaotic attraction phase diagrams of system (2) in  $x_1-x_2$  plane,  $x_1-x_3$  plane, and  $x_1-x_4$  plane are shown in Fig. 2. The Lyapunov exponents  $LE_1 = 0.0255$ ,  $LE_2 = -0.4255$ ,  $LE_3 = 0$ ,  $LE_4 = 0$ , and Lyapunov exponent dimension  $D_L = 3.0560$  are obtained by calculated. It is obvious that the system has a positive Lyapunov exponent and the sum of all Lyapunov exponents less than zero, so the system is chaotic.

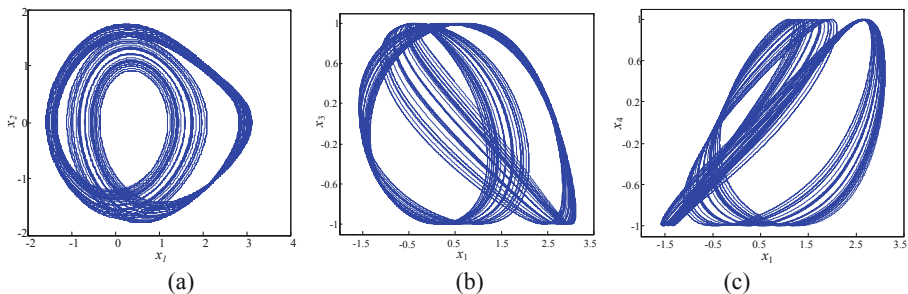
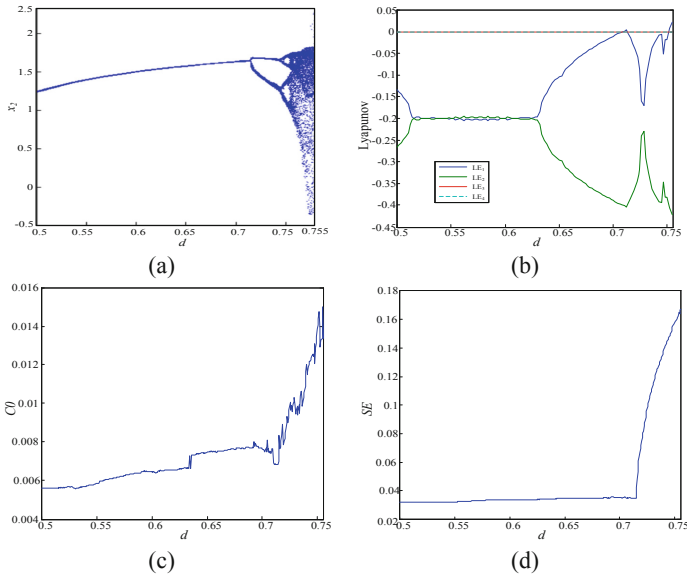


Fig. 2. Plane attractor diagram, (a)  $x_1-x_2$  plane, (b)  $x_1-x_3$  plane, (c)  $x_1-x_4$  plane

### 3 Chaos Characteristics Analysis of Interconnected Electrical Power System

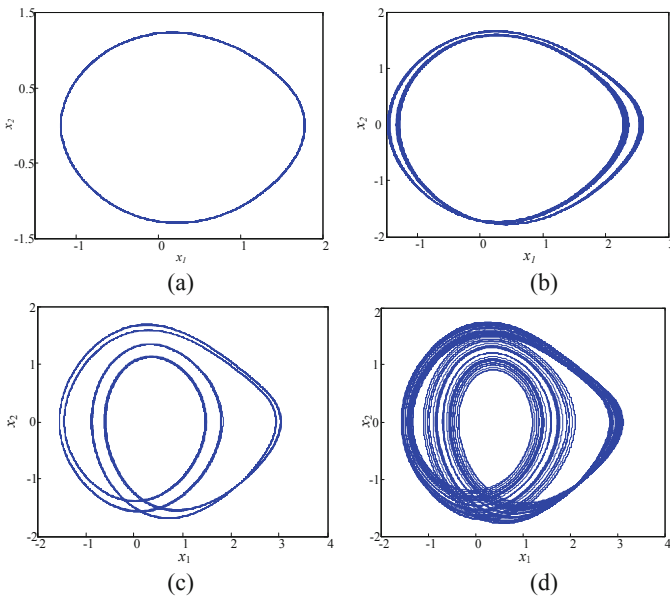
Keeping the above initial value and parameters values, when the parameter  $d \in [0.5, 0.755]$ . We can get the corresponding bifurcation diagram, Lyapunov exponent spectrum, SE complexity and C0 complexity are shown in Fig. 3.



**Fig. 3.** (a) Bifurcation diagram, (b) Lyapunov exponent spectrum, (c) C0 complexity, (d) SE complexity

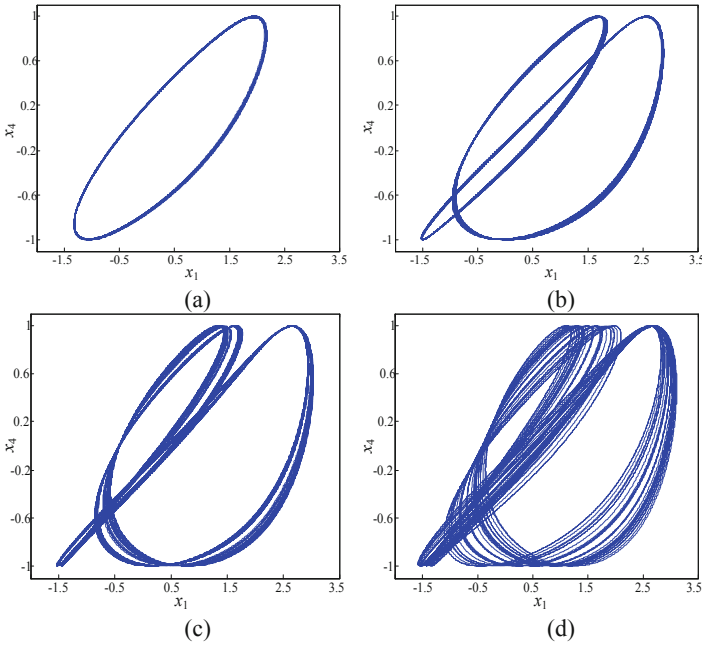
As can be seen from Fig. 3(a) and (b), when the system parameter  $d$  increases gradually from 0.5 to 0.755, the Lyapunov exponents spectrum and bifurcation diagram of the system can be visualized to see the change of the motion state of the interconnected power system. The interconnected electrical power system is transformed from a series of times periodic bifurcation process to chaotic state transformation. Figure 3(c) and (d), when the parameters  $d$  of the system increase gradually from 0.5 to 0.755, the complexity of the system is increasing, and the complexity of the periodic state of the system is much small than the chaotic states. Because the complexity of the system is the greatest in chaotic state, the complexity can visually see the transformation of the interconnected electrical power system from the periodic state to the chaotic state.

For the general nonlinear system, the change of the system parameters determines the state of the system, and the state transformation of the system can not only be displayed intuitively through bifurcation diagram, Lyapunov exponents spectrum and complexity, but also through the visual display of the phase diagram. The influence of the parameter change of the interconnected electrical power system on the system, and the parameters  $a = 1$ ,  $b = -0.4$ ,  $c = 0.2$ ,  $\alpha = 0.8$ , changes the amplitude of the parameter load disturbance, let the  $d = 0.5$ ,  $d = 0.715$ ,  $d = 0.745$  and  $d = 0.755$ . We get the limit ring of period 1, 2 and 4, and the chaotic attraction phase diagrams are shown in Fig. 4.



**Fig. 4.** Phase diagram, (a)  $d = 0.5$ , (b)  $d = 0.715$ , (c)  $d = 0.745$ , (d)  $d = 0.755$

The influence of parameter load disturbance frequency change on system in interconnected electrical power system. The parameters  $a = 1$ ,  $b = -0.4$ ,  $c = 0.2$ ,  $d = 0.755$ , change the parameter  $\alpha$ . When  $\alpha = 0.9$ ,  $\alpha = 0.82$ ,  $\alpha = 0.806$  and  $\alpha = 0.8$ , and the corresponding limit ring cycles 1, 2, 4 and chaotic attraction phase diagram are shown in Fig. 5.



**Fig. 5.** Phase diagram, (a)  $\alpha = 0.9$ , (b)  $\alpha = 0.82$ , (c)  $\alpha = 0.806$ , (d)  $\alpha = 0.8$

## 4 Synchronous Analysis of Interconnected Power System

### 4.1 Coupled Synchronization Algorithm

Coupled synchronization is all coupling variables or some variables in the drive system into the response system, so that the response system is synchronized with the drive system after coupling. A chaotic system is

$$\dot{X} = AX + f(X), \tag{3}$$

where, state vector  $X \in R^n$ , the constant matrix  $A \in R^{n \times n}$ ,  $f(X)$  is a continuous non-linear function of the linear part of the system. For the system with the different initial value and the same system parameters, they are synchronized by coupled synchronization algorithm. The drive-response synchronization equation is

$$\dot{X} = AX + f(X), \tag{4}$$

$$\dot{X}' = AX' + f(X') + K(X - X') \tag{5}$$

where,  $K$  is the coupling matrix.

The error vector is

$$E = X - X' \tag{6}$$

$$f(X) - f(X') = M_{X,X'}(X - X') \tag{7}$$

The error system is composed of drive system and response system by

$$\dot{E} = (A + M_{X,X'} - 2K)E \tag{8}$$

When the system error is zero, the system is in equilibrium state, that is, select the appropriate coupling coefficient matrix, make the system balance point is gradually stable, two chaotic systems to reach synchronization. In general, there are two kinds of coupling of similar variables between chaotic systems, single coupling and bidirectional coupling. Some chaotic systems can be synchronized by single-variable coupling, while some chaotic systems require two or even multiple variable coupling to achieve synchronization. Among them, the more coupled the number of variables, the stronger the synchronization ability. If it is a global variable coupling, as long as the number of coupling is large enough, the chaotic system can be coupled.

### 4.2 Coupling Synchronization Analysis of Interconnected Electrical Power Systems

The drive system is Eq. (2), the response system is obtained by the coupling synchronization algorithm as Eq. (9), and the error system is Eq. (10).

$$\begin{cases} \dot{x}_5 = x_6 + k_1(x_1 - x_5) \\ \dot{x}_6 = bx_6 + dx_7 + c - a \sin x_5 + k_2(x_2 - x_6) \\ \dot{x}_7 = -\alpha x_8 + k_3(x_3 - x_7) \\ \dot{x}_8 = \alpha x_7 + k_4(x_4 - x_8) \end{cases} \tag{9}$$

$$\begin{cases} e_1 = x_5 - x_1 \\ e_2 = x_6 - x_2 \\ e_3 = x_7 - x_3 \\ e_4 = x_8 - x_4 \end{cases} \tag{10}$$

where,  $k_1, k_2, k_3, k_4$  represent the couple coefficients,  $x_5, x_6, x_7, x_8$  means that state variable of response system.

When the drive system the same as the response system parameter  $a = 1, b = -0.4, c = 0.2, d = 0.755, \alpha = 0.8$ , the initial value of the drive system is (0.5, 0.5, 1, 0), the initial value of the response system is (1, 1, 1, 0.5), and the coupling coefficient  $k_1 = k_2 = k_3 = k_4 = 3$ . Obtain the system synchronization error curve as shown in Fig. 6(a). As can be seen from Fig. 6(a), after 2.5 s error variable  $e_1, e_2, e_3, e_4$  all tend to 0, which indicates that in the 2.5 s time, the system parameters are the same, the initial values are different of two connected electrical power systems have reached the synchronization.

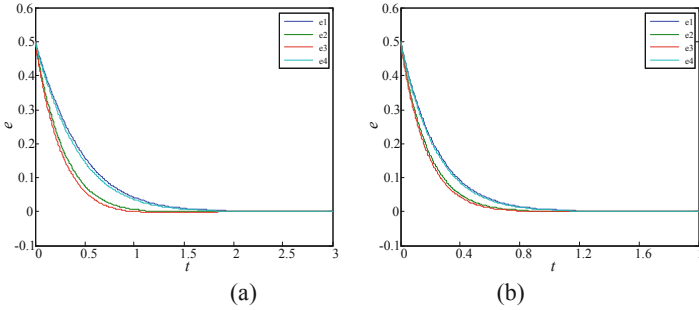


Fig. 6. Synchronization error curve, (a)  $k = 3$ , (b)  $k = 5$

If the parameters and initial values of the response system and the drive system are unchanged, and the values of the coupling coefficients are changed, we get the synchronization error curves are shown in Fig. 6(b) when the coupling coefficients are  $k_1 = k_2 = k_3 = k_4 = 5$ . Through the comparison of Fig. 6(a) and (b), it can be clearly seen that with the increase of coupling coefficient, the time of synchronization of the system is decreasing. In the practical application process, it is sometimes necessary a long time for the two systems to reach synchronization, sometimes it takes a short time to make the two systems synchronized, and the coupling synchronization algorithm realizes the synchronization time by controlling the coupling coefficient for the two systems applied in practice, which is one of the advantages of the coupling synchronization algorithm.

## 5 Conclusions

In this paper, the chaotic characteristics of interconnected power systems are analyzed by using phase diagrams, bifurcation diagrams, Lyapunov exponent spectrum and complexity. The synchronous characteristics of interconnected power systems are analyzed by coupling synchronization algorithm. The analysis results show that there is no chaotic oscillation when the periodic load disturbance of the system is small, and when the perturbation amplitude increases gradually, the system has the phenomenon of double periodic bifurcation to chaos. By changing the amplitude of load disturbance and the frequency of load disturbance, different chaotic attractor phase diagrams are obtained, and the transformation of interconnected power system between periodic state and chaotic state under different disturbance loads can be clearly seen. The time of the interconnected power system reaches synchronization is different, when the coupling synchronization coefficient of interconnection power system with different values. If the coupling coefficient of the interconnected power system is controlled to a certain range, the system can control the synchronization time in the actual application, which will provide some ideas and methods for the safe operation of the power system. Next, we will continue to study the chaotic characteristics and synchronous control of interconnected power systems.



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