Dimension Selection and Compression Reconstruction Algorithm of Measurement Matrix Based on Edge Density

Jiayin Yu and Erfu Wang(✉)

Electronic Engineering College, Heilongjiang University, Harbin, China
efwang_612@163.com

Abstract. This paper is based on the sparse representation of signals in orthogonal space. Data collection and compressed are combined by compressed sensing theory. The image signal can be reconstructed by fewer observations which we obtained it under the measurement matrix. Compressive sensing theory breaks through the limitation of data sampling. In the theory of compressed sensing, the selection of measurement matrix plays a key role in whether the compressed signal can be reconstructed or not. In this paper, different measurement matrices are selected to achieve the compressive sensing and their similarity coefficient matrices are analyzed to compare the different performance. This paper focus on the coefficient selection of random measurement matrix. To find the relationship between the image structure similarity coefficient and the other characteristic indexes. An algorithm for dimension design of measurement matrix is proposed. A high performance algorithm for image compression perception and image restoration is implemented.

Keywords: Compressive sensing · Sparse representation · Measurement matrix · Similarity coefficient

1 Introduction

Nyquist sampling theorem, also known as Shannon sampling theorem [1]. In this theorem, two processes of signal sampling and reconstruction are described: Firstly, the continuous time signal is converted to the discrete time signal, and then the discrete signal is restored. The key theory in the sampling theorem is that the sampling frequency must be higher than twice the maximum signal frequency [2]. Otherwise, the signal will be aliased. However, in practical application, this method requires a lot of computing resources.

Therefore, it is assumed that if a way can be found to realize the compression process while sampling and retain the effective information of the original signal [3], Moreover, it does not need to meet Nyquist's limit on sampling frequency to complete signal reconstruction, which can reduce the complexity of signal processing and the cost of calculation. Compressed sensing theory provides a new idea for signal processing [4]. Compressive sensing theory is to design a compression sampling algorithm

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aiming at the sparse nature of most signals in real life. Due to the sparse nature of signals, only a small number of observation values can be used for signal reconstruction during recovery.

Compressed sensing theory was proposed by Donoho et al. as a practical signal sampling coding theory [5, 6], it has been widely applied. Mallat and Zhang proposed the matching tracking algorithm (MP&OMP) in 1993, which was the first time to use the super-complete dictionary for sparse decomposition of the original signal [7]. Tropp proved that if select gaussian matrix or Bernoulli matrix [8], we can use the greedy algorithm to reconstruct the signal [9, 10]. Candes proved that the measurement matrix which is selected in compression observation should meet the property of finite equidistant, so several common measurement matrices have been widely used in compressive perception theory. Such as gaussian random matrix, Bernoulli measurement matrix, partial hada code matrix and so on. The above research results are the important foundation of CS theory. Based on OMP reconstruction algorithm [11], this paper analyzes the influence of different measurement matrices on image compression effect. Combined with the image feature information, we analyze the influence of parameters on the reconstruction effect under the same measurement matrix [12]. In this paper, a method for calculating the parameters of measurement matrix is proposed, which can better take account of the sharpness of image and the complexity of calculation.

2 Compressive Sensing Theory

If there are few non-zero elements in a signal, or the majority of signals in this signal are zero, the signal can be considered as sparse. In practice, the signals we come into contact with are generally not absolutely sparse, but they can be approximately sparse in a certain transformation domain. In other words, as long as the sparse space that meets the conditions is found, the data can be effectively compressed and sampled.

We set the length of the signal X is N, in the transformation domain, if there are K coefficients is not zero or much greater than the other coefficients, and $K \ll N$. Then, the signal is said to be K-sparse in the corresponding transformation domain. When we obtain M observations $(K < M \ll N)$. We can compress the signal. And this M observation can reconstruct the original signal X. Because the loss is some smaller coefficient, so we can get an approximation of X. Set the orthonormal basis of the transformation domain is $\psi_i = \{\psi_1, \ldots \psi_n\}$, X can be represented linearly by $\{\theta_1, \ldots \theta_n\}$:

$$X = \sum_{i=1}^{N} \theta_i \psi_i = \psi s \tag{1}$$

Where ψ is a $N \times N$ matrix, θ is $N \times 1$ matrix. s is sparse coefficient. Domain selection is the basis of signal sparse representation. At present, classical algorithms include discrete cosine (DCT) algorithm, Fourier transform (FFT) algorithm, wavelet transform (DWT) algorithm, etc. DWT algorithm is used in this paper.

The measurement $M \times N$ matrix is used to transform the original signal of N dimension into the Y observation vector of M dimension. Then the signal information can be restored as much as possible through reconstruction algorithm. It's essentially the projection of the original signal X onto the measurement matrix that we set up to get the projection value Y. The purpose of the observation matrix design is to better realize the reconstruction of the original signal or obtain the sparse coefficient vector. In order to achieve this, the observation matrix must satisfy the RIP characteristics. The one-to-one correspondence between the original space and the sparse space is guaranteed [13]. To meet the RIP characteristics, the measurement matrix ϕ is required to be unrelated to sparse basis ψ. In this way, two different K-sparse signals will not be mapped to the same set. The expression of measured value is:

$$Y = \phi X \tag{2}$$

Where ϕ is $M \times N$ order matrix. According to formula (1), it can be obtained:

$$Y = \phi X = \phi \psi_S \tag{3}$$

In the above formula, the number of equations is far less than the number of unknowns, and the system should have infinite solutions. However, since the original signal has been sparse transformed, there are only non-zero values and we know their positions. So we can get the solutions when $M > K$. Then the original signal is recovered by nonlinear reconstruction algorithm. In this paper, several measurement matrices are compared and their parameters are further verified.

When the measurement matrix meets the RIP characteristics, we can decode the projected value according to the method of solving the norm. Thus, the signal reconstruction process is transformed into the process of solving the optimization of norm, and the process of solving the minimization of norm is a linear process. At present, reconstruction algorithms can be roughly divided into two categories. One is greedy algorithm, including matching tracking algorithm and orthogonal matching tracking algorithm. The second is convex optimization algorithm, which includes gradient projection method, base tracking method, minimum Angle regression method and so on. In this paper, we use the orthogonal matching tracking algorithm.

3 Measurement Matrix and Reconstruction

In the realization of compressed sensing, measurement matrix, as a very important part, directly affects the accuracy of image restoration [14]. When selecting the measurement matrix, in addition to meeting RIP principle [15], as the key to encrypt the information to be processed, the measurement matrix also needs to have good randomness.

Different measurement matrices with different randomness have different measurement effects on different information sources. Therefore, when the image is projected by the measurement matrix, some parameters will have a certain impact on the recovery effect in the reconstruction process.

In this paper, random matrix, gaussian matrix and Parthadamard matrix are used as measurement matrix. The elements in the random channel matrix are generated randomly and have great uncertainty, so the confidentiality is also strong. In this experiment, the dimension of semi-determined random channel is adopted. Gaussian matrix is unrelated to the sparse basis of most signals. Besides these two matrices, it is also widely used in the process of compressed sensing. In this paper, part of the hadamar matrix is selected for comparative analysis with the above two matrices.

Lena grayscale map in the image standard database is selected as the information source, and the implementation process is shown in Fig. 1.

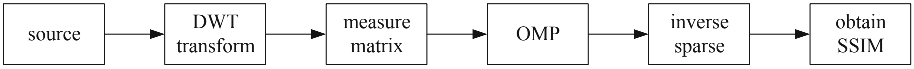
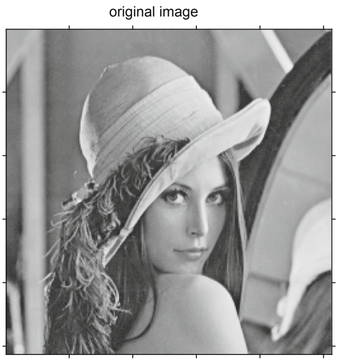


Fig. 1. The principle diagram

The observation signal can be expressed as:

$$Y = X * R \tag{4}$$

Where X is the sparse signal after DWT transformation, R is the measurement matrix. Y is reconstructed by using OMP algorithm. The reconstructed signal is transformed by inverse wavelet transform to get the restored image. In Fig. 2, (a) is the original Lena diagram, (b), (c) and (d) are respectively recovered images obtained when random matrix, gaussian random matrix and part of hadamar matrix are used as measurement matrix. Judging from the visual observation effect, images are compressed and reconstructed under all three measurement matrices, and some hadamar matrices used in (d) have better effect. In order to quantitatively evaluate the reconstruction effect of compressive perception, SSIM, the structural similarity coefficient, was used as the index to compare the similarity coefficient between the reconstructed signal and the original signal under different measurement matrices and observation values, and to obtain the performance curve. Since the value of M is too small, the restored picture is seriously distorted. Therefore, in this paper, the SSIM of the reconstructed image and the original image was calculated from $M = 128$. The simulation results are shown in Fig. 2.



(a)

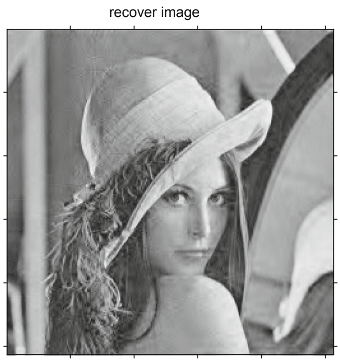
Original image

(b)

random matrix

(c)

Gauss matrix



(d)

PartHadamard matrix

Fig. 2. Image recovery under three matrices

As can be seen from the graph, the similarity coefficient between the reconstructed image and the original image under the measurement of PartHadamard matrix is larger than the other two overall, so the performance of some hadamard matrices in processing one-dimensional image information is better than that of other two random measurement matrices. In addition, due to the absence of noise, the simulation results of random gaussian channel and random channel are not different, and the performance of the two channels is similar under the conditions set in this paper. Combined with the reconstructed images above, we can more intuitively see that when some hadamard matrices are used as measurement matrices, the restored Lena graphs have clearer contour and clearer picture quality.

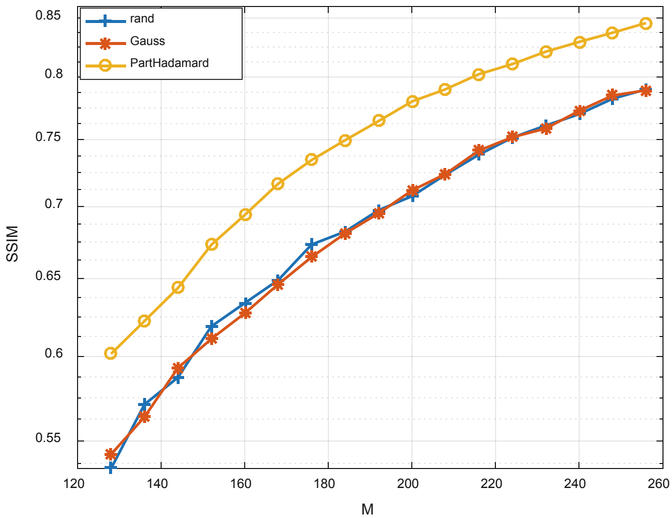


Fig. 3. SSIM of the three matrices is compared with the line graph

4 Dimension Selection Algorithm of Measurement Matrix Based on Edge Density

In compressive sensing theory, the selection of observation matrix has an important impact on reconstruction performance [13]. In the previous simulation, the observation matrix is a semi-random matrix of parameters, where is an adjustable parameter. The larger the size, the better the recovery, but the longer the program takes to run. Lena diagram and Lake diagram in the standard grayscale image library were still selected for experiment. PartHadamard matrix was selected as the measurement matrix, parameters were adjusted, and structural similarity coefficient data of two images were simulated and analyzed, as shown in Table 1.

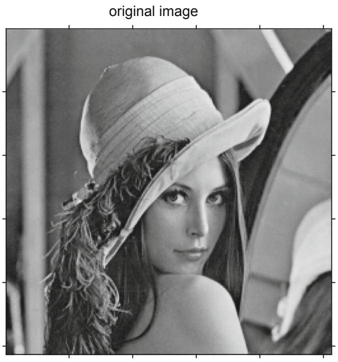
Table 1. Structural similarity coefficient

SSIM	Lena	Lake
M = 128	0.6007	0.4542
M = 136	0.6207	0.4886
M = 144	0.6522	0.507
M = 152	0.6766	0.5346
M = 160	0.6974	0.5572
M = 164	0.7063	0.5739
M = 168	0.7169	0.5884
M = 176	0.7352	0.6057
M = 184	0.751	0.6302

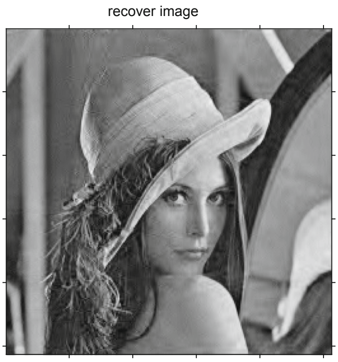
(*continued*)

Table 1. (*continued*)

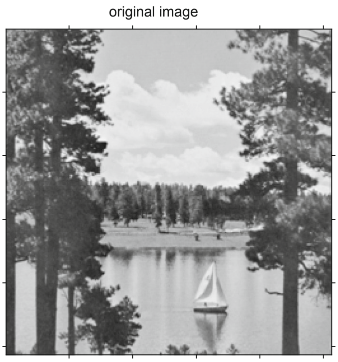
SSIM	Lena	Lake
M = 192	0.766	0.6466
M = 200	0.7779	0.6645
M = 208	0.7898	0.6821
M = 216	0.7997	0.6961
M = 220	0.8043	0.7025
M = 224	0.8095	0.7131
M = 232	0.8214	0.7238



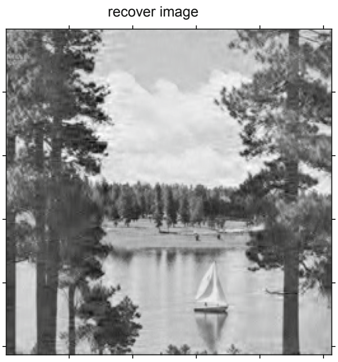
(a)
Original image



(b)
Reconstructed image



(a)
Original image



(b)
Reconstructed image

Fig. 4. The comparison between the two images under the m-value

It can be seen that, with the increase of M, the structural similarity coefficient between images also increases, which is reflected in the clearer image after reconstruction. Usually, the value of M is selected empirically. Among the edge information, edge density is an important method to evaluate the edge information. Therefore, this paper designs a dimension selection method of measurement matrix based on edge density. Define the function f(x):

$$f(x) = \lambda X + \mathrm{q} \tag{5}$$

For different images, there is always M value range suitable for them to achieve a certain recognizable effect in reconstruction. Different images have different M value range.

Lena diagram used in this experiment has an edge density of 72.6808, and 164 is taken according to the formula M. The edge density of the Lake graph is 99.6158. According to the formula, when M is equal to 220. Lena and Lake were compressed and reconstructed with the designed M value. As can be seen from the comparison in Fig. 4, the picture quality of the reconstructed signal is clear, which can fully reflect the information of each part of the image for clear identification. The validity of the dimension selection algorithm of measurement matrix based on edge density is verified in this paper.

5 Conclusion

In recent years, compressive sensing theory has been studied and improved, great progress has been made in practical application. By introducing the analysis of measurement matrix, this paper finds that when processing one-dimensional image information, PartHadamard matrix have better performance than others. Under the same matrix dimension, the original image can be recovered better by the measurement matrix. This paper studies the problem that the dimension value of random matrix can only be determined empirically. We designs the dimension selection principle of measurement matrix based on edge density and determines the M value through the integer of defined linear function. Simulation experiments show that each part of the image can be clearly identified under the dimension of the measurement matrix designed under this criterion, so we can extend the application of compressed sensing in image processing.

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