



An Investigation of Transmission Properties of Double-Exponential Pulses in Core-Clad Optical Fibers for Communication Application

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Abstract. In this paper, a comparative analysis of the propagation of double exponential and Gaussian ultra-short pulses in fused-silica core-clad optical fibers has been presented. The present study has taken the non-linear propagation parameters from Schrodinger's equation and for silica fiber into consideration. The analysis has been carried out for single-mode and multi-mode fibers, to study the effects of variation in pulse parameters and it has been observed that the double-exponential pulses have a bandwidth-efficiency $\sim 23\%$ over Gaussian pulses and may be useful as femtosecond-laser pulse shapes. It is found that double exponential pulses offer more resistance to dispersive effects than Gaussian pulses at longer distances and retain more power levels for higher input powers, while Gaussian pulses continue to decay. Finally, rapid decay in double-exponential pulses may make them suitable for time-and-wavelength-division-multiplexed passive optical networks (TWDM-PON) applications in optical communication.

Keywords: Double-exponential pulse · Gaussian pulse · Optical fiber

1 Introduction

Pulse shapes play an important role in optical fiber based communication and signal processing applications [1–3] in which non-linear properties of optical pulses [4] and those of the optical fibers [5] contribute significantly. It is a known fact that several useful application can be obtained from non-linear optical fibers carrying or generating pulses e.g., pulse compression, optical fiber communication, optical signal processing and quantum application [6–9]. Due to the increase in use of silica optical fibers due to their special properties, we have considered the fibers in this paper to be fused-silica fibers [5]. Gaussian pulses have thus far been the most common pico-second ultra-short pulse shape used for characterizing and utilizing non-linear properties [10] in an optical fiber, along with its variants e.g. super Gaussian pulses [11]. Various useful and detrimental non-linear properties e.g., chirping, group velocity dispersion or chromatic dispersion leading to pulse broadening etc., in an optical fiber, may be studied using Gaussian pulses. Other than Gaussian, transmission properties of several other pulse shapes e.g., hyperbolic secant, parabolic, etc., in various media, have been studied to some extent [12].

Another class of pulses called ultra wide band or double-exponential pulses [13] have not yet been studied much in the case of optical fibers, except for in a few medical experiments.

By virtue of the mathematical description of the double exponential function, it may be envisaged that ultra-short and temporally narrow pulses are possible for fiber-optic applications using the double exponential function. Analyses of double-exponential pulses are possible starting from the Schrodinger equation [14, 15]. Thus, in the present work, we have endeavoured a thorough initial study of the characteristics of double-exponential pulse propagation in regular core-clad silica single mode [16] and multimode optical fibers [17]. Throughout the work, the results for double exponential pulse transmission have been compared to the characteristics of Gaussian pulse propagation in optical fibers. The work has been analytically carried out with simulations performed using Matlab package.

Section 2 presents the simulation work and related results. Subsection 2.1 presents a confirmation test for the adopted simulation methods by repetition of published results for Gaussian pulse properties. Subsection 2.2 presents the pulse equations and shapes used in the current paper. Subsections 2.3 and 2.4 presents the simulation of double exponential pulse propagation in silica core-clad single and multi-mode optical fibers respectively. Section 3 discusses obtained results and necessary mathematical results and characteristics. Finally, Sect. 4 concludes the work.

2 Characterization of Gaussian and Double Exponential Pulses

This section presents the step-by-step development of double exponential pulse transmission characteristics via silica communication fibers.

2.1 Verification of Simulation Technique Using Gaussian Pulse

The non-linear effects of silica optical fibers limit their capacity to send signals without degradation [18–20]. These effects include Four Wave Mixing, Self-Phase Modulation and Cross Phase Modulation, which are useful in areas such as wide-band and low-noise optical amplification, supercontinuum (SC) light sources, optical signal processing, strain/temperature sensors, frequency/time/length measurement, and near-infrared spectroscopy [21]. In the present work, a fused silica based optical fiber has been used as a medium for propagation for the Gaussian and double exponential pulses. A step index fiber is considered in this section with refractive indices of the core and cladding, respectively, as 1.447 and 1.444 at 1550 nm wavelength [22]. Waveguide and Material dispersion are both observed in optical fibers. The dispersive effects of interest in this paper are chromatic and modal dispersions [23]. Pulse propagation through such fibers (see Fig. 1) can be simulated using non-linear Schrodinger equation (see Eq. 1) [11].

$$i \frac{\partial A(z, t)}{\partial z} = -\frac{i\alpha}{2} A(z, t) + \frac{\beta_2}{2} \frac{\partial^2 A(z, t)}{\partial t^2} - \gamma |A(z, t)|^2 A(z, t) \quad (1)$$

Equation (1) describes the propagation of optical pulses inside single-mode fibres [11], where, ‘A’ is the amplitude of the pulse envelope and ‘t’ is measured in a moving frame of reference (with respect to the group velocity). The terms on the right side of the equation control effects of fibre loss (α), dispersion (β) and non-linearity (γ) such that $\alpha > 0$, $\beta \neq 0$ and is real, $\gamma > 0$. Dispersive or non-linear effects dominate along the fibre depending on the initial values of initial width (T_0) and peak power (P_0) of the initial pulse. The dispersion length L_D ($=T_0^2/\beta_2$) and the non-linear length L_{NL} ($=1/\gamma P_0$) provide the lengths after which these effects are observed. Herein, β_2 represents pulse dispersion, causing broadening of pulse in time domain by causing a phase shift in frequency domain of the pulse. Again, γ accounts for the material dispersion of the fiber causing a phase shift in time domain leading to frequency shift in frequency domain. For the present work, Split Step Fourier method has been used, which is a pseudo spectral method that employs the use of the Fourier Transform [11].

For the Split-Step Fourier method, the Eq. (1) is rewritten as in Eq. (2):

$$\frac{\partial A}{\partial z} = (L + N)A \quad (2)$$

where, L is a linear operator that accounts for the dispersive effects and N is an operator that accounts for non-linear effects.

We have considered a Gaussian pulse having width $5T_0$ and observed its propagation through an optical fiber with its dispersion parameters $\beta_2 = 1$ and $\gamma = 1$ [11]. We have simulated the said pulse for a distance of one characteristic dispersion length (L_d) (distance where the pulse width increases by a factor of $\sqrt{2}$) and observed the trend in drop in intensity of Gaussian with respect to distance covered, as shown in Fig. 1, in agreement with the result in literature [11], which confirms the correctness of our adoption of the Split-step Fourier transform algorithm.

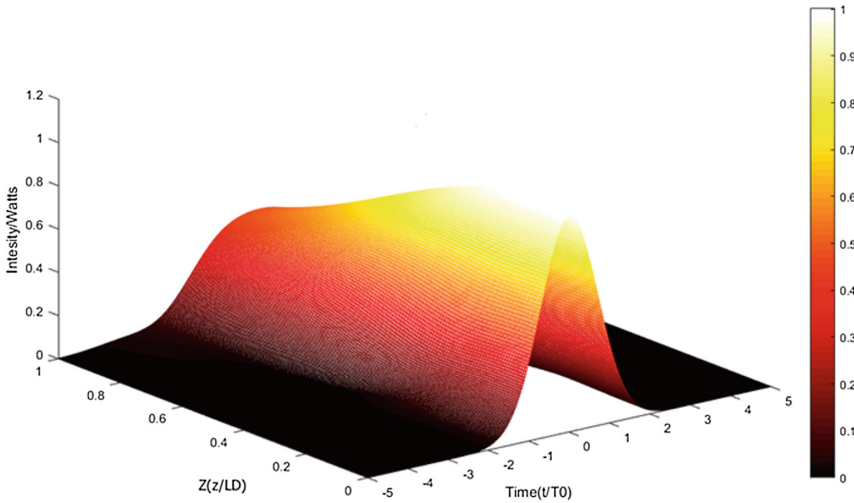


Fig. 1. Simulation of propagation of Gaussian pulse ($\beta_2 = 1$ and $\gamma = 1$).

2.2 Gaussian and Double-Exponential Pulses Used

The pulses of interest- in this study are Gaussian and double exponential pulses. A Gaussian pulse is one in which the pulse envelope is a Gaussian function whose Fourier transform is also a Gaussian function. This is unlike that of a plane wave whose Fourier transform is a Dirac-delta function [24]. This causes the plane wave to have lesser bandwidth than that of a Gaussian pulse, which is why the Gaussian pulse is of use in applications involving wideband communication. The Gaussian pulse is described by Eq. (3):

$$E(z = 0, t) = A \exp\left(-\frac{t^2}{2t_0^2}\right) \quad (3)$$

where, t_0 is standard pulse deviation in picoseconds, A is amplitude, t is time in picoseconds, and z is distance in kilometres.

The motive of this study is to find a clear juxtaposition between the Gaussian and double exponential pulses and explore the capabilities of the double exponential pulses for similar applications. The double exponential pulse can be described using Eq. (4) [13]:

$$E(t) = E_0(e^{-\alpha t} - e^{-\beta t}) \quad (4)$$

Where, E_0 is the amplitude of the pulse, the rise time and fall time of the pulse are given by α and β , respectively. The rise time is the amount of time taken for a pulse to reach 90% of its maximum value from 10% of its maximum value [13]. The pulse width is the interval of time during which the amplitude of the pulse is higher than 50% of its maximum amplitude. The α and β terms can be calculated using the equation set (5) [25]:

$$\alpha = \frac{\ln(2)}{t_{FWHM}}; \beta = \frac{1}{t_r} \quad (5)$$

t_{FWHM} is the full width half maximum amplitude and t_r is the rise time of the given pulse.

Figures 2 and 3 show the initial conditions for both pulses before propagation. The pulse parameters are chosen in a way that they can be propagated to the maximum extent inside a silica based fiber of maximum operating wavelength = 1550 nm [26]. Figure 2 is the time-domain presentation of the Gaussian and double exponential pulses used in the present study, followed by the wavelength representation of the same in Fig. 3, where both types of pulses are shown to be centred at 1550 nm ($\sim 1.55 \mu\text{m}$).

2.3 Dispersion Characteristics of Double Exponential Pulses in Single Mode Fibers

Dispersion characteristics of double exponential and Gaussian pulses have been observed for a distance of one characteristic dispersion length (L_d) and intensities have been studied with the help of plots to understand the comparative decay of Gaussian and double exponential pulses in a single mode fiber, with core radius $\sim 5 \mu\text{m}$.

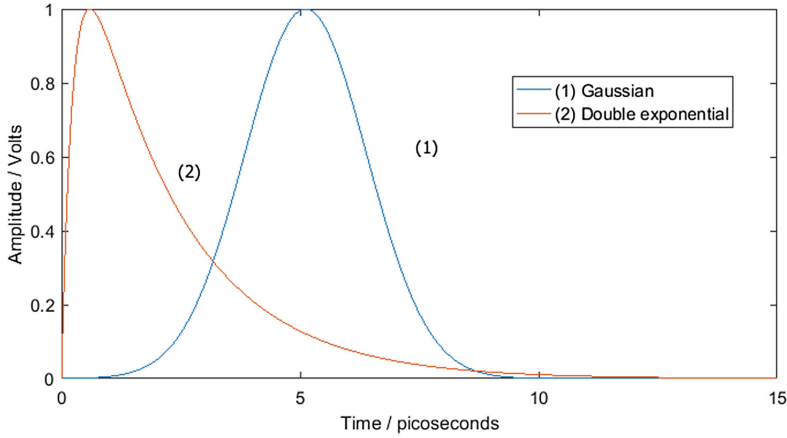


Fig. 2. Comparative time-domain plot of Gaussian and double exponential pulses.

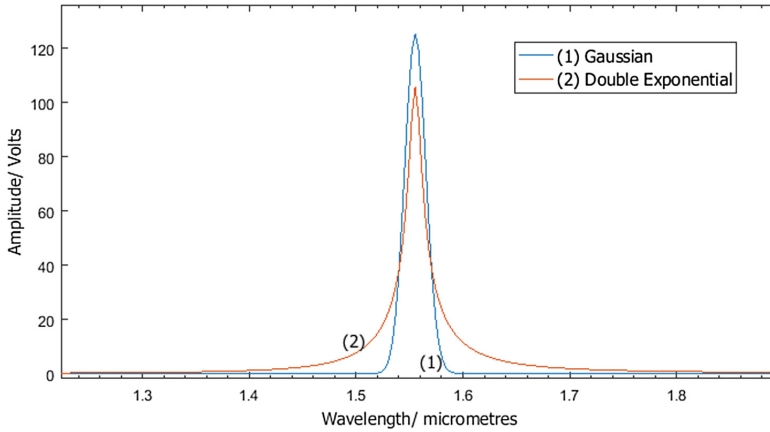


Fig. 3. Center wavelengths of Gaussian and double exponential pulses.

We have observed that rate of decay for a double exponential pulse is higher in the initial stages of propagation (as a function of propagation, ‘z’) and lower in later stages as compared to Gaussian. These dispersive effects are chromatic in nature. Figure 4 shows the dispersive nature of Gaussian pulse and distance-wise intensity profile of the same along the single mode fiber in consideration. Again, similar study for a double exponential pulse is presented in Fig. 4. We have observed the intensities at $z = 0, 0.33L_d, 0.66L_d$ and L_d . The pulse and fiber parameters are identical to those from [11] ($\beta_2 = 1$ and $\gamma = 1$).

We find a major part of the double exponential pulse gets exhausted from initial stage by comparing Figs. 4(a), (b) and Figs. 5(a), (b), respectively. However, the change in intensity stabilises for double exponential pulse at $\sim 0.8L_d$ (Fig. 5), compared to

Gaussian pulse (Fig. 4), where the latter continues to decay. Double-exponential pulses was generally found to stabilise after decaying over a range of ~ 1 to 5 times of dispersion length, L_d .

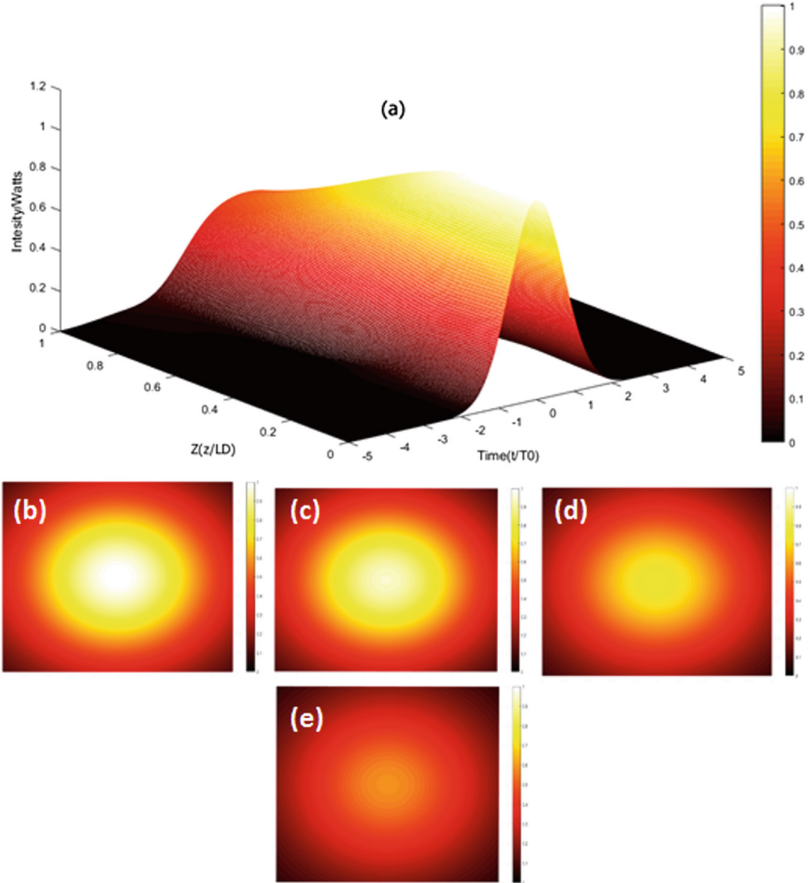


Fig. 4. (a) Intensity analysis of Gaussian pulse in Single mode fiber. (b) Intensity plot at $Z = 0$. (c) Intensity plot at $Z = 0.33L_d$. (d) Intensity plot at $Z = 0.666L_d$. (e) Intensity plot at $Z = L_d$.

2.4 Dispersion Characteristics of Double Exponential Pulses in Multi-mode Fibers

The analysis of dispersive effects in single mode fibers has been carried out in Sect. 2.3. This section deals with the analysis of pulse propagation in multi-mode fibers. Multi-mode fibers have a higher radius than single mode fibers which allow for more number of modes to propagate simultaneously. The β_2 parameter is a function of the modes in this case and the multi-mode fiber dimensions have been adopted from literature [29], with core-radius $\sim 25 \mu\text{m}$ and fiber radius $\sim 67.5 \mu\text{m}$.

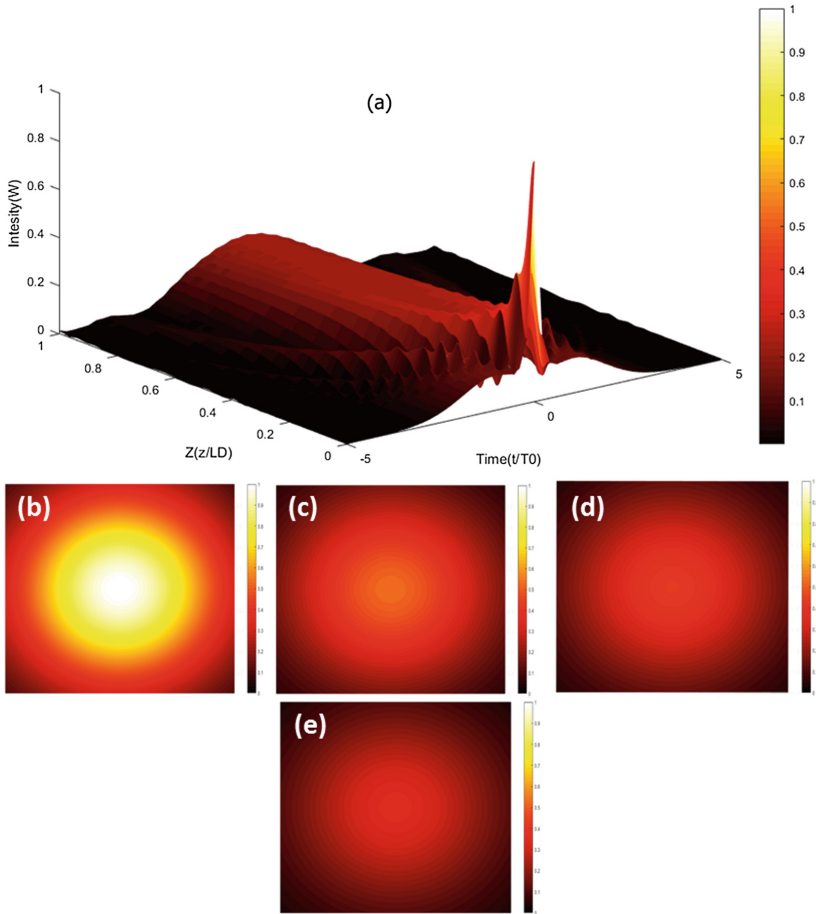


Fig. 5. (a) Intensity analysis of double exponential Pulse in Single mode fiber. (b) Intensity plot at $Z = 0$. (c) Intensity plot at $Z = 0.33L_d$. (d) Intensity plot at $Z = 0.666L_d$. (e) Intensity plot at $Z = L_d$.

In addition to chromatic dispersion, modal dispersion in Gaussian and double exponential pulses is observed. We have considered LP_{02} mode for the purpose of this simulation. A comparative study of the pulse propagation was done as shown in Figs. 6 and 7, representing LP_{02} modes for a Gaussian pulse and a double exponential pulse, respectively. Double exponential pulse is observed to have larger spread compared to Gaussian indicating a larger variation in pulse broadening among the two pulses. From this we infer that the double exponential pulses are better suited for applications involving modal dispersion based sensors [27].

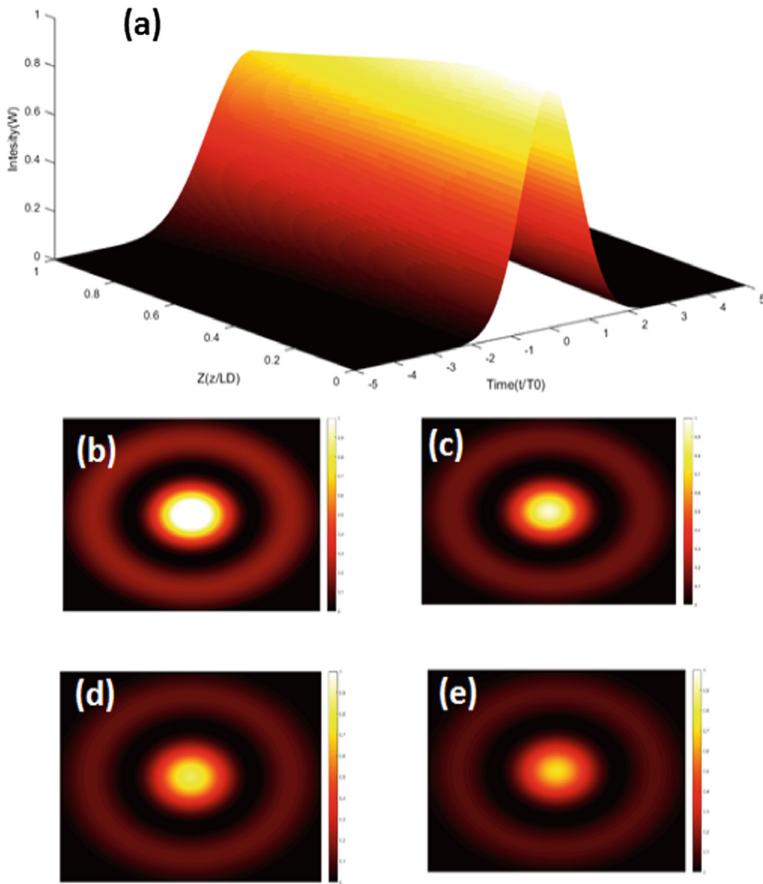


Fig. 6. (a) Intensity analysis of Gaussian pulse in Multimode mode fiber. (b) LP₀₂ Intensity plot at Z = 0. (c) LP₀₂ Intensity plot at Z = 0.33L_d. (d) LP₀₂ Intensity plot at Z = 0.666L_d. (e) LP₀₂ Intensity plot at Z = L_d.

3 Results and Discussion

Based on the simulation results presented in Sect. 2, an analysis of bandwidth efficiency has been carried out to draw a comparative conclusion on the effectiveness and area of application for double exponential pulses as compared to Gaussian pulses, transmitting through a non-linear silica fiber.

The Gaussian and double exponential pulse equations have been presented in Eqs. 3 and 4, respectively. Taking Fourier transform of the Gaussian equation (see Eq. 3), the expression in Eq. 6 was obtained:

$$E_g(z, \omega) = F(E_g(z, t)) = \frac{t_0}{\sqrt{\pi}} A \exp\left(-\frac{t^2}{2t_0^2}\right) \quad (6)$$

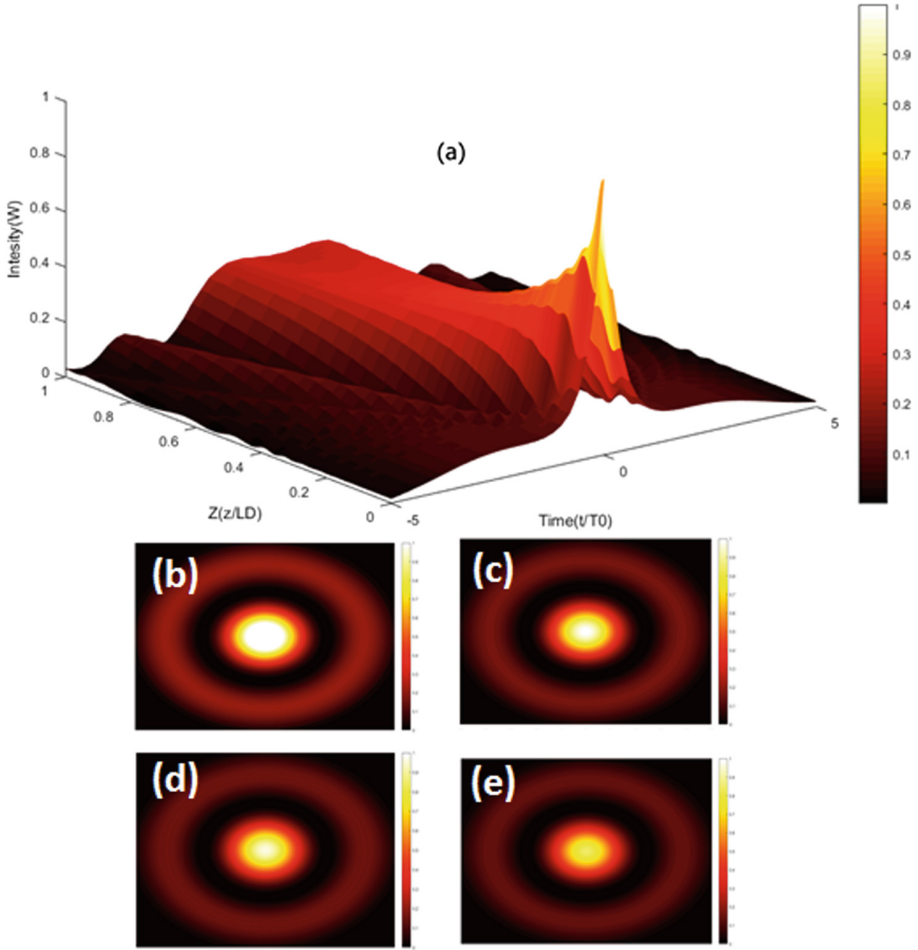


Fig. 7. (a) Intensity analysis of double exponential pulse in Multimode mode fiber. (b) LP₀₂ Intensity plot at Z = 0. (c) LP₀₂ Intensity plot at Z = 0.33L_d. (d) LP₀₂ Intensity plot at Z = 0.666L_d. (e) LP₀₂ Intensity plot at Z = L_d.

At half power bandwidth, $\mathcal{F}(Eg(z, t)) = \text{Max}(\mathcal{F}(Eg(z, t)))/\sqrt{2}$ [30].

Solving for Half-power bandwidth (HPBW) for Gaussian pulse, ω_{HPBW} was obtained as in Eq. (7):

$$(\omega_{\text{HPBW}})_{\text{max}}^{\text{Gaussian}} = \frac{0.832}{t_0} \quad (7)$$

Similarly, taking Fourier transform of Eq. (4), for the double exponential pulse, gives the expression as in Eq. (8):

$$E_e(z, \omega) = F(E_e(z, t)) = \frac{2\alpha}{\alpha^2 + \omega^2} - \frac{2\beta}{\beta^2 + \omega^2} \quad (8)$$

At half power bandwidth, for double exponential pulse, a similar approach as in Gaussian case was followed giving,

$$F(E_e(z, t)) = \text{Max}(F(E_e(z, t)))/\sqrt{2}$$

Solving for $(\omega_{\text{HPBW}})_{\text{max}}^{\text{exponential}}$, at $t_r = 0$; $\beta = \infty$ (found by second derivative test), the result was as in Eq. (9),

$$(\omega_{\text{HPBW}})_{\text{max}}^{\text{exponential}} = 0.6434/t_0 \quad (9)$$

For a given pulse-width, the ratio of HPBW for Gaussian to HPBW for double exponential pulse is as in Eq. (10):

$$(\omega_{\text{HPBW}})_{\text{max}}^{\text{Gaussian}} = 1.29(\omega_{\text{HPBW}})_{\text{max}}^{\text{exponential}} \quad (10)$$

From, the relation in Eq. (10), it can be said that double exponential bandwidth is 23% less than Gaussian, for a given pulse width in temporal regime. To observe the effect of pulse width on the pulse propagation we considered the propagation of the Gaussian and double exponential pulses with different pulse width ranging from femto pulse regime to ultra-short pulse regime, i.e., 10–15 to 10–12 s. Table 1 shows the effect of pulse widths on the rate of change in intensity of Gaussian and double exponential pulses over a distance and the effect of decreased pulse width on the pulses through parameters such as rate of change of slope in Gaussian and double exponential pulses, and, distance at which intensity of double exponential pulse is greater than Gaussian intensity.

Dispersion length is the length at which the pulse width increases by a factor of $\sqrt{2}$. Dispersion length is formulated by neglecting the non-linear effects in non-linear Schrodinger equation (NLSE) [28].

$$\frac{\partial A}{\partial z} - j\beta_2 \frac{\partial^2 A}{\partial t^2} = 0 \quad (11)$$

For the Gaussian pulse we solve Eq. (11) by using frequency domain for simpler analysis and then shifting back to time domain through Fourier analysis [28].

$$U(z, T) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} U(0, \omega) \exp\left(j\omega T - \frac{j\beta_2 z \omega^2}{2}\right) d\omega \quad (12)$$

Where $U(z, T)$ is the pulse at time T and position z , giving pulse width at z as,

$$T_1(z) = T_0 \sqrt{1 + \frac{z^2}{L_D^2}} \quad (13)$$

Where z is the distance travelled by the pulse; L_D is the dispersion length of pulse. Using equation and (11) and (12) we derive:

$$L_D^{Gaussian} = \frac{T_0^2}{|\beta_2|} = \frac{0.362 t_{FWHM}^2}{|\beta_2|} \quad (14)$$

For Gaussian pulse with pulse width, T_0, t_{FWHM} is the full wave half maximum time and dispersion coefficient β_2 .

For a double exponential pulse the parameters are as in Eq. (15)

$$a = \frac{\ln(2)}{t_{FWHM}}; b = \frac{1}{t_r} \quad (15)$$

Using Eqs. (11) and (12) we find L_D^{d-exp} ,

$$L_D^{d-exp} = \frac{1.18 t_p t_{FWHM}}{\beta_2} \quad (16)$$

$$t_p = \frac{\ln\left(\frac{b}{a}\right)}{(b-a)} \quad (17)$$

Table 1. Effect of pulse widths on the rate of change in intensity-slope

T_{FWHM} (pico-secs)	Z/L_d	$\Delta_{Gaussian}/(V^2/m)$	$\Delta_{Double Exponential}/(V^2/m)$
0.01	1.7618	0.0023	0.0172
0.1	1.8118	0.0023	0.0175
1	3.7437	0.0023	0.0199

Where, t_p is the peak time-instance of the pulse [13]. From this equation we infer that pulse propagates as a function of peak time and pulse width. We infer from the Eq. (16) that double exponential pulse decay depends upon peak time which is a function of its rise time or 'b' (see Eq. 15). When the rise time of the pulse increases, 'b' decreases till a certain value such that the dispersion length of the pulse will be less than that of Gaussian. After a certain order of 'b' we observe an increase in the dispersion length of double-exponential pulse and then it remains nearly constant that is why the double-exponential femto-second pulse has a longer dispersion length compared to pico-second pulse, as shown in Fig. 8.

From Table 1, we have observed that rate of change of slope of double exponential pulse improves with decrease in pulse width, which is graphically shown in Fig. 8. Double exponential pulse attains higher intensity compared to Gaussian faster with decreasing pulse width. This makes double exponential pulses useful as very short pulses, especially in femtosecond regime. TWDM [26] requires pulses with wavelength range in μ meters which boils down to femto second range. Due to the steeper slope and its suggested significance in femto second regime, double exponential pulses can be used to manufacture fiber-laser for TWDM purpose. From the results of the simulation, the decay rate of double exponential pulse is lower as in Figs. 8(a) and (b). We observe that the rate of change of intensity of the Gaussian pulse is higher than the double exponential pulse after a certain distance and rate of change of intensity of double exponential pulse is more than that of Gaussian pulse initially, but when the pulse width is decreased we observe higher amplitudes and lower attenuation of the double exponential pulse as compared to the Gaussian pulse.

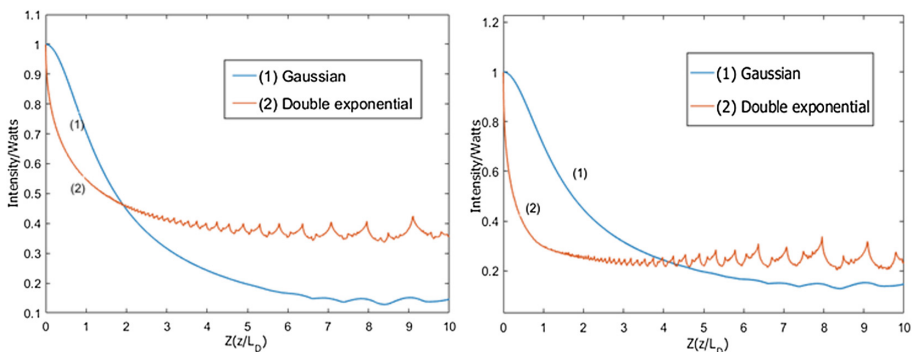


Fig. 8. (1) Gaussian and (2) double-exponential pulses at pulse width (a) 1 picosecond; (b) 0.1 picosecond.

Pico second pulses can be used in short range applications such as sensing [28]. TWDM-PON is the main solution to the NG-PON2 standard [26]. This system involves the use of multiple XG-PON systems to improve the bit rates in upstream and downstream and was used to meet the NG-PON2 standards. As observed in Fig. 8(b) femtosecond pulses have high dispersion lengths which is favourable for long range communication. The response to fiber parameters is non-linear and it resists noise better with higher amplitudes. As observed in Fig. 2 in Sect. 2 under Subsect. 2.2, the overlap with consecutive pulses is minimal due to their shape and small pulse width. This enables the Inter symbol interference (ISI) to be low and hence reduces margin for error. As observed from Fig. 4 in Sect. 2 under Subsect. 2.3 the half power bandwidth efficiency of these pulses is 23% hence it favours higher bit rates.

4 Conclusion

The propagation of double exponential and Gaussian pulses in optical fibers were compared for single as well as multimode cases. The effects of dispersion and non-linear effects on the pulses were studied and observations were made. It was found that double exponential pulses are better suited for applications such as femto lasers as it is more resistant to non-linear effects than the Gaussian pulse in that range. But it has a higher half power bandwidth efficiency than that of a Gaussian pulse. The rising and falling edges of the curve are steeper in the case of double exponential pulse and this makes it favourable for applications involving TWDM. The analysis in the case of multimode fibers shows that the Gaussian pulse is more resistant to the Modal Dispersive effects than the double exponential pulses.

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